We present the first all-optical nonlinear joint transform correlator based on a square-law receiver in the Fourier plane. Our device uses a photorefractive limiting quadratic processor. The compressional nonlinearity associated with the transfer function of the limiting quadratic processor enables the correlator to detect signals embedded in Gaussian and non-Gaussian noise. In the limiting region this device correlates the phase-only information of the input. This is the first time to our knowledge that photorefractives or real-time holography has been used in the correlation of the phase-only information. We demonstrate the operation of this device experimentally, and we evaluate its performance through computer simulation for various forms of noise.

**1. Introduction**

The joint transform correlator is well known to be one of the most convenient devices for correlating two images in the sense that there is no need to fabricate separate holographic filters, such as the matched filter or the phase-only filter. The classical joint transform correlator requires a quadratic processor in the Fourier plane. For well-separated images in the input, with square-law operation, the classical joint transform correlator performs in all aspects as a matched-filter-based correlator. Recently it has been shown that adding additional nonlinearities such as thresholding, saturation, or fractional power nonlinearity improves the performance of these correlators in terms of discrimination as well as the peak-to-noise ratio (PNR) of targets embedded in non-Gaussian noise. In order to implement the nonlinear joint transform correlator with optoelectronic approaches, it has been necessary to use a camera, a computer, and digital processing. These serial operations have compromised many of the advantages sought in parallel optical signal processing. In this paper we present the first all-optical nonlinear joint transform correlator. Our implementation relies on using an optical photorefractive limiting quadratic processor in the Fourier plane.

This correlator is tunable in terms of performance by a change in the signal-to-reference-beam ratio incident upon the photorefractive crystal. As is discussed in Section 2, in the region in which the beam ratio is small the transfer function of the device is similar to a square-law receiver; thus the correlator performs similarly to the classical joint transform correlator. In the region in which the beam ratio is large the transfer function of the device is essentially flat and the correlator correlates the phase-only information of the input. To our knowledge, this is the first time that photorefractives have been used in correlating the phase-only information of the inputs.

The operation of this correlator in the square-law portion of its transfer function permits it to perform as well as the matched filter. Matched-filter-based correlators guarantee satisfactory performance when the signal is embedded in noise with Gaussian statistics. On the other hand there are many situations in which the matched filter is not satisfactory. These cases can occur when the signal is embedded in noise with non-Gaussian statistics. Therefore nonlinear processing becomes very attractive in cases in which signals are often embedded in noise with non-Gaussian statistics, such as in communication theory applications or in optical signal processing.

For example, in a recent paper by Javidi and Wang, it was shown that a hard-clipped joint transform correlator achieved good performance when the signal was imbedded in a background desert scene. Similarly, in communication theory, Aashang and...
Poor demonstrated that, for additive impulsive noise (non-Gaussian noise), hard-clipped correlators perform better in certain regimes than linear correlators.

The photorefractive correlator that we introduce operates by soft clipping rather than hard clipping; therefore the joint spectra of images are compressed rather than thresholded. Compression is a common technique for improving the signal-to-noise ratio and has recently been applied to optical images. In this technique the compression was implemented by energy transfer in photorefractive two-beam coupling.

This correlator, in contrast to previous photorefractive correlators reported in the literature (cf. Refs. 17 and 18), is based on square-law reception in the Fourier plane. In addition, it has the ability to compress the signal in the nonlinear regime.

2. Theory

For implementing the nonlinear joint transform correlator it is necessary to design a photorefractive limiting device that possesses the two nonlinearities discussed in Section 1 (i.e., quadratic as well as saturation). The architecture is shown in Fig. 1. The operation of the device can be understood qualitatively as follows: The reference and signal images are placed side by side on a transparency with magnitude \( s(x, y) \). Beam \( A_1 \) passes through the transparency and then through Fourier-transform lens \( L_1 \). The resultant Fourier transform appears at the photorefractive crystal, where it interferes with beam \( A_4 \). After passing through the photorefractive crystal, the exiting beam collinear with beam \( A_1 \) is Fourier transformed by lens \( L_2 \) onto a self-pumped phase conjugator. The retroreflected Fourier spectrum diffracts from the hologram that was written by beams \( A_1 \) and \( A_4 \) to produce the output \( A_3 \). The self-pumped phase conjugator permits the device to be self-aligning.

Because the output of the photorefractive phase conjugator in the low coupling limit is given by

\[
A_3 = \frac{\gamma |A_1|^4}{I_0} A_4^* A_2, \tag{1}
\]

where \( A_i \) stands for the amplitude of the beam \( i \), \( \gamma \) is the coupling coefficient, \( l \) is the crystal thickness, and

\[
I_0 = |A_1|^2 + |A_2|^2 + |A_3|^2 = I_1 + I_2 + I_3, \tag{2}
\]

it is possible to develop the transfer function of this device. In our device, \( A_1 \) should be replaced by \( |A_1|/\lambda f \) and \( A_2 \) by \( \sqrt{\lambda f} \) \( A_1 S^* \) \( \exp(i \phi_1) \) where \( S \) is the Fourier transform of the object transparency \( s \), \( \lambda \) is the wavelength of the source, \( f \) is the focal length of the lens, \( \beta \) is the overall round trip reflectivity, and \( \phi_1 \) and \( \phi_2 \) are the phases of beams \( A_1 \) and \( A_2 \), respectively.

Substituting the above parameters into Eq. (1), we can then write

\[
f(S) = \frac{\gamma |A_1|^4 I_4 (\lambda f)^2 |S(u, v)|^2 \exp[i(\phi_1 + \phi_2)]}{I_1 (\beta + 1) (\lambda f)^2 |S(u, v)|^2 + I_4} \tag{3}
\]

or

\[
f(S) = Z_{\text{clip}} \frac{m_e S^2(u, v)}{1 + m_e S^2(u, v)}, \tag{4}
\]

where

\[
Z_{\text{clip}} = \frac{\gamma |A_1|^4}{(\beta + 1) \exp[i(\phi_1 + \phi_2)]}, \tag{5}
\]

and \( m_e \) is the effective beam ratio, equal to

\[
m_e = \frac{I_1 (\beta + 1)}{(\lambda f)^2 I_4}. \tag{6}
\]

For \( \sqrt{m_e} |S| < 1 \) or for a small signal, this device behaves as a square-law receiver, which is the required nonlinearity for implementing the classical joint transform correlator. However, for \( \sqrt{m_e} |S| > 1 \) or for a large signal the output saturates and becomes independent of the phase and the amplitude of \( S \). Figure 2 shows the transfer function for various values of the product \( (m_e)^{1/2} \).

At the clipping limit \( \sqrt{m_e} |S| > 1 \) it is possible to approximate the nonlinear transfer function

\[
f(S) = Z_{\text{clip}} \exp(|S|/\nu^2 |S|^2)^0. \tag{7}
\]

![Fig. 1. Schematic diagram of nonlinear clipping device: SP, self-pumped; PRC, photorefractive crystal; BS1, beam splitter.](image1)

![Fig. 2. Transfer function of the photorefractive nonlinear correlator.](image2)
where \( (\cdot)^0 \) indicates the zeroth-order nonlinearity.\(^6,20\) Suppose that \( S(v_x, v_y) \) consists of the summation of the Fourier transform of two images:

\[
S(v_x, v_y) = F(v_x, v_y) + G(v_x, v_y).
\]

Then through the nonlinear transform method described in Refs. 6 and 20 it is possible to develop the nonlinear transfer function for the zeroth-order nonlinearity for our device:

\[
f(S) = \sum_{k=1, \text{odd}}^n \epsilon_k \cos(2kx_0v_x + k\phi_F(v_x, v_y) - k\phi_G(v_x, v_y))
\]

where \( \Gamma_m \) is the mathematical gamma function, \( \phi_F(v_x, v_y) \) and \( \phi_G(v_x, v_y) \) are phases of \( F \) and \( G \), respectively, \( x_0 \) is the separation between the two images in the image plane, and \( \epsilon_k \) is given by

\[
\epsilon_k = \begin{cases} 
1 & k = 0 \\
2 & k > 0
\end{cases}
\]

The \( k = 1 \) terms in Eq. (9) correspond to phase-only correlation. The other terms expressed in the equation are higher-order terms.

In materials with small electro-optic coefficients, in order to reach the clipping limit for a large range of spatial frequencies, it is required that \( I_4 \ll I_1 \) (or \( \sqrt{m_s} |S| \gg 1 \)). Therefore according to Eq. (4), \( |f(S)| \ll 1 \) if \( I_1 \) is fixed, which makes detection experimentally difficult to realize. In these cases it may be necessary to use materials with high electro-optic coefficients. However, by a change in the material, the nonlinear transfer function given by Eq. (4) is no longer valid because of the large energy-transfer coupling effects. Because we require that \( I_1 \gg I_4 \), the undepleted pump approximation is valid. In this case we can rewrite the clipping limit in terms of intensity\(^19\):

\[
|Z_{clip}|^2 = |A_4|^2 \left| \frac{\sinh(0.5\gamma l)}{\cosh(0.5\gamma l + 0.5 \ln \beta)} \right|^2.
\]

In the diffusion limit, \( \gamma \) is a real number; in this case, using the theory of the phase of the phase conjugate beam\(^12\) indicates that the phase of \( A_0 \) or the phase of \( Z_{clip} \) is flat. Therefore the clipping limit in terms of the field in the regions where \( I_1 |S|^2 \gg I_4 \) satisfies the relationship

\[
Z_{clip} = |A_4| \frac{\sinh(0.5\gamma l)}{\cosh(0.5\gamma l + 0.5 \ln \beta)} \exp[i(\phi_1 + \phi_2)].
\]

3. Computer Simulation

In order to evaluate the performance of this correlator in terms of discrimination, we examined its performance for very simple inputs. Our inputs consisted of two dissimilar and two similar circles, as shown in Figs. 3(a) and 3(b), respectively. At this stage in our computer simulation we have examined the transfer function in Eq. (4) (i.e., the transfer function with small coupling coefficient) and have assumed \( Z_{clip} = 1 \).

Figure 4 shows the correlation results in which the operating point is varied for the three operating regions: square law, intermediate, and clipping. We set the operating point by controlling the effective beam ratio to be \( m_s \) equal to \( 10^{-3}, 1, \) and \( 10^4 \). The correlation absolute value (amplitude) results for each region are shown in Figs. 4(a), 4(c), and 4(e) when the input circles were identical and Figs. 4(b), 4(d), and 4(f) when the circles were dissimilar.

In Figs. 4(a) and 4(b) we can see the case when the correlator performed approximately as a classical joint transform correlator (in the square-law regime). In the center we see the autocorrelation peak and, surrounding this peak, the cross-correlation peaks. Notice the high cross correlation when the circles are dissimilar in Fig. 4(b). Figs. 4(c) and 4(d) show the correlation results of the intermediate range \( (m_s = 1) \). Note here that second-order peaks begin to appear, and the cross-correlation peaks become noticeably sharper for similar circles and start disappearing for dissimilar circles. Finally in Figs. 4(e) and 4(f) the similar case [Fig. 4(e)] has developed sharp cross-

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Fig. 3. Inputs that represent our signal and reference images: (a) two dissimilar circles, (b) two similar circles.
Fig. 4. Full plane correlation results for similar (left column) and dissimilar (right column) circles: (a), (b) in the square-law regime; (c), (d) in the intermediate regime; (e), (f) in the clipped regime.
correlation peaks with strong higher-order components, and the dissimilar circle case has virtually no cross-correlation component. These results indicate the high discrimination ability of the new correlator operating in the clipped regime versus the square-law regime.

In Figs. 4(e) and 4(f) we can see that the correlation peak for similar circles is surrounded by rings, while for two dissimilar circles the correlation peak disappears and is replaced by two concentric rings. The origin of the rings can be explained as follows: As we mentioned previously in Eq. (9), this correlator operates on the nearly phase-only information of the inputs. The impulse response of the phase-only filter of a circle is a ring (for example, the impulse response of a phase-only filter is the edge-enhanced version in which the intensity of the spectrum plane drops with the increase in spatial frequency). In our simulation for the correlation results of two rings we constructed the rings from the impulse response of a nearly phase-only filter. Our nearly phase-only filter was defined as

\[ F(\mu, \nu) = \left| F(\mu, \nu) \right|^\alpha \frac{F(\mu, \nu)}{\left| F(\mu, \nu) \right|}, \]

where \( \alpha \) is a number approaching zero. The exact phase-only filter occurs if \( \alpha = 0 \). Figure 5(a) shows the correlation results for two dissimilar circles and Fig. 5(b) shows the results for two similar circles; the circles were the same as for the previous case and \( \alpha = 0.01 \). In the phase-only case (\( \alpha = 0.0 \)) the rings shown in Fig. 5(a) remained, while in Fig. 5(b) they completely disappeared.

4. Experiment

The experimental arrangement for our saturated nonlinear joint transform correlator is shown in Fig. 6. Beam \( A_4 \) from an argon laser passes through the neutral-density filter (not shown in the figure) and is then expanded by a simple telescopic setup to a diameter of 4 mm. The intensity of beam \( A_4 \) was

![Fig. 5. Correlation of rings: (a) two dissimilar rings, (b) two similar rings.](image)
measured to be 0.001 mW/mm². Beam A₁ has an intensity of 1.8 mW/mm² and a diameter of 2 mm. Beam A₁ is configured so that it passes through a mask with many pairs of circular holes. Each pair represents a signal and a reference object. The apparatus is configured such that only one pair is present at a time.

These objects are Fourier transformed by a lens whose focal length \( f₁ = 16 \) cm into a barium titanate crystal of 7.5 mm on a side. Beams A₁ and A₄ interfere on the crystal, thereby forming a phase hologram. The intersection angle \( \theta \) between the two beams was 20°. As previously described, the resultant joint spectrum of the two images at the output of the crystal are Fourier transformed into a barium titanate self-pumped phase conjugator. The self-pumped phase conjugator is necessary in order to generate the conjugate Fourier spectrum, which acts as a self-aligning readout beam for the phase grating. The output beam A₃ was Fourier transformed by a lens of focal length \( f₃ = 15 \) cm into the camera.

Figure 7 shows the experimental results of correlating two circular holes. In Fig. 7(a) the two holes are of equal radii, 0.3875 mm, and the coupling coefficient is positive. Figure 7(b) is identical except that the crystal is rotated 180° so that the coupling coefficient is negative. The results are identical to those in Fig. 4(a) except that the efficiency is lower. Figures 7(c) and 7(d) represent the correlation results of dissimilar circles for positive and negative coupling coefficients, respectively; the radii of these circles were 0.3875 and 0.175 mm. In both cases the separation between the holes was 1.5 mm.

In the case of similar circles, as shown in Figs. 7(a) and 7(b), we can see that there are three peaks in the correlation plane: the central peak, which corresponds to the autocorrelation of the circles, and the other two peaks, which correspond to the cross correlation. In contrast to correlating with a quadratic processor in the Fourier plane (as shown in our computer simulation), we can also see that each peak is surrounded by an arc. However, when the two circles are not identical, as in Figs. 7(c) and 7(d), the correlation peaks disappear, and each arc is replaced by two arcs. The arcs are part of the complete rings discussed in Section 3 (see Figs. 4 and 5).

This correlator still has two major problems: lack of resolution caused by beam crossing and a slow response time. These problems can be overcome in two ways. The first way is to set up the crystal in a reflection geometry such that the incident beams are collinear. In this case the beam-crossing problem is eliminated and the grating spacing is small so that the resolution is high. In addition, the speed increases because materials with large electro-optic coefficients such as barium titanate have \( p \)-type carriers (holes) so that the speed increases with smaller grating spac-
This is in contrast with crystals of the sillenite family such as bismuth silicon oxide. Recently there have been reports on the ferroelectric crystal $\text{KNbO}_3$, which has a response time of 5 ms in the reflection geometry with large efficiency. The second way is to use a quantum-well structure in the longitudinal transmission geometry, which may increase the speed of the four-wave mixer to the microsecond range. These methods hold the promise of operating our proposed device at video frame rates.

5. Evaluation Performance with Noisy Inputs

For evaluating the performance of this correlator we examined its performance at three operational points for two classes of inputs. One class of signals was a tank in a desert scene, i.e., noise with non-Gaussian statistics, and the other class was a tank in additive Gaussian noise.

In both classes our inputs for computer simulation were in a $1024 \times 512$ pixelated array. The reference tank was an array of $54 \times 26$ pixels, and our signal images were embedded in a $128 \times 102$ array. The separation between the signal and the reference images in the input plane was set to be equal to 128 pixels. Figure 8(a) shows our input for a tank in scene noise, and Fig. 8(b) shows the input for a tank in additive Gaussian noise. We chose three operational regions, $m_e$ very small ($10^{-3}$), $m_e$ intermediate (10), and $m_e$ large ($10^4$).

For small values of $m_e$ the correlator should be equivalent in its performance to the classical joint transform correlator, for intermediate values of $m_e$ the correlator should be equivalent to a joint transform correlator with the lowest spatial frequency suppressed, and for large values of $m_e$ the correlator should perform as the clipped correlator.

For evaluating the performance we assumed that $Z_{\text{clip}}$ was constant and hence assigned it the value of 1. We chose the metric of peak-to-noise ratio (PNR) as an evaluating criterion.

In calculating the PNR, we first subtract the mean value of the correlation pattern from the correlation array. The PNR is then defined as the ratio of the resultant peak amplitude to the root mean square of all of the other amplitudes.

Figure 9 shows the correlation results for a tank in a desert scene and the correlation results for a tank in additive Gaussian noise. Figures 9(a), 9(b), and 9(c) are the results for a tank in clutter for beam ratios $10^{-3}$, 10, and $10^4$, respectively. For $m_e = 10^{-3}$ (i.e., in the region in which the correlator operates as a matched filter) the correlation peak is embedded completely inside of the noise. Similar results have been reported by other researchers for the matched filter. However, with an increase in the beam ratio ($m_e = 10$) in such a way that the reflectivity of the four-wave mixer is reduced around the dc (low spatial frequency) value of the Fourier spectra, a major

![Fig. 8. Input for examining the performance of the correlator in the presence of noise: (a) the reference and the signal tank in non-Gaussian scene noise, (b) the reference and the signal tank in additive Gaussian noise.](image-url)
Fig. 9. Correlation results with a beam ratio $10^{-3}$, 10, and $10^{+4}$: results for a tank in additive Gaussian noise.

Reduction in the noise occurs (although the correlation peak is still smaller than some of the stray noise peaks). This improvement in the correlation results is equivalent to the result of dc blocking demonstrated by several researchers for enhancing correlation for objects embedded in clutter.

Increasing the beam ratio further such that $m_e$ is large, we can observe further improvement of the
PNR. We performed a search over a wide range of beam ratio values, from \( m = 10^{-4} \) to \( m = 10^{+8} \), and our results indicate that, for the given set of inputs, the optimal selection for \( m \) was \( m = 10^{+2} \), which yielded a PNR value of 75. For comparison the phase-only filter results for the same inputs would be 35.

At very high beam ratios both the signal and the noise are compressed in the Fourier plane. Compression of the noise in the Fourier plane causes the noise spectrum to spread over a larger area in the correlation plane. This effectively produces a reduction in the mean value. However, compression of the spectrum of the signal in the Fourier plane makes the correlation peak sharper and narrower. The combined effect is an improvement in the PNR. For full details on the effect of signal compressing in enhancing the signal-to-noise ratio the reader is referred to Refs. 14, 15, 16.

In evaluating our device in its performance with Gaussian noise we also observed an improvement in the PNR, as shown in Figs. 9(d)–9(f) (i.e., by increasing the beam ratio). The improvement in the PNR may also be attributed to the nonlinear compression associated with the transfer function. The respective PNR’s for this case for the three regions were 3, 8, and 50.

6. Conclusions

We have proposed and demonstrated the first all-optical holographic nonlinear joint transform correlator to our knowledge based on a limiting square-law receiver in the Fourier plane. This correlator has different operational points that can be tuned by varying the beam ratio. The performance of the correlator can vary from that of a matched-filter-based correlator for small beam ratios to almost a hard-clipping correlator for large beam ratios.

We have investigated, the performance for different beam ratios through computer simulation, and we have found that the PNR for different forms of noise can be improved by permitting the beam ratio to increase. We attribute this performance improvement to signal compression associated with the nonlinear transfer function of this device.

The new correlation design permits us to avoid the use of a camera, computer interfacing, and spatial light modulators commonly used with these types of architectures. This feature make this holographic device complementary to the optoelectronic counterpart in nonlinear correlation technology.

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References and Notes