Adaptive Relay Selection for Regenerative OFDMA Relay Networks with Fairness Constraints

Harin Jeong and Jae Hong Lee
School of Electrical Engineering and INMC, Seoul National University Shillim-dong, Gwanak-gu, Seoul 151-742, Korea
Tel: +82-2-880-7045
E-mail: {hrjeong, jhlee}@snu.ac.kr

Abstract—We propose an adaptive relay selection scheme for a regenerative orthogonal frequency division multiplexing access (OFDMA) relay network with fairness constraints. The proposed scheme selects the best \(M\) relays out of a set of \(K\) potential relays to maximize the system capacity. Among these selected relays, subcarriers are reallocated to satisfy fairness constraints as well as to minimize the decrease of the system capacity. The simulation results show that the proposed scheme achieves significant performance improvement over direct transmission and opportunistic relaying with OFDM (OR-OFDM). In addition, multiuser diversity is achieved as the number of potential relays increases with proposed scheme.

Index Terms — Fading channel, relay selection, OFDMA, fairness, relay network.

I. INTRODUCTION

User relaying achieves virtual spatial diversity in wireless communication networks by capitalizing the inherent broadcast property of wireless transmission [1]. Its performance improvement depends on how to select relays and how to distribute power among the source and the relays [2], [3]. Also opportunistic relaying was introduced as an alternative to space time coded cooperative diversity in [4].

Orthogonal frequency division multiplexing access (OFDMA) exploits frequency selectivity to obtain multiuser diversity by intelligent resource allocation [5].

By combining user relaying with OFDMA, the system achieves higher capacity and broader cell coverage, if appropriate relay selection (i.e., subcarrier allocation) and power allocation strategies are designed.

In a wireless network, unbalanced utilization of relays leads to a shortened network lifetime [6]. To guarantee the balance of the relays utilization, fairness constraints need to be considered.

Optimal power allocation over the subcarrier was examined to improve the performance of a wireless relay network [3], [7], and [8].

In this paper, we propose a new relay selection scheme for a regenerative OFDMA relay network to provide diversity gain. In the proposed scheme, fairness constraint on each relay is imposed. The system capacity and the outage probability are computed by the computer simulation.

The remainder of this paper is organized as follows. In Section II, the system model is described. An adaptive relay selection scheme is presented in Section III. Simulation results are shown in Section IV and conclusions are drawn in Section V.

II. SYSTEM MODEL

Consider a regenerative OFDMA relay network which consists of a single source, \(K\) potential relays, and a single destination as shown in Fig. 1. Let \(d_{sd}\) denote the distance between the source and the destination. Assume that potential \(K\) relays are uniformly distributed in the circle with a radius of \(d_{sd}\) and the source is at the center.

Assume that channel of the network on each subcarrier is frequency-flat by adopting OFDMA. Let the instantaneous channel gain coefficients of the \(n\)-th subcarrier between the source and the relay \(k\) and the relay \(k\) and the destination be denoted by \(h_{sk}^{(n)}\) and \(h_{dk}^{(n)}\), respectively.

To ensure a half-duplex operation of the relay (i.e., relay cannot transmit and receive simultaneously), the transmission frame of the network is divided into two time slots of equal duration.

In the first time slot, the source transmits information data \(x^{(n)}\) with power \(P_{sk}^{(n)}\) on the \(n\)-th subcarrier to the relay \(k\). The received data \(r_{k}^{(n)}\) at the relay \(k\) in the first time slot is given by

\[
r_{k}^{(n)} = h_{sk}^{(n)}x^{(n)} + n_{k}^{(n)},
\]
where $n^{(n)}_k$ is independent, complex Gaussian noise on the $n$-th subcarrier at the relay $k$, with variance $\sigma^2$.

In the second time slot, the relay $k$ transmits the regenerated version of the received data $\hat{x}^{(n)}$ with power $P^{(n)}_{dl}$ on the $n$-th subcarrier to the destination. The received data $r^{(n)}_d$ at the destination in the second time slot is given by

$$r^{(n)}_d = h^{(n)}_{dl} \hat{x}^{(n)} + n^{(n)}_d$$

where $n^{(n)}_d$ is independent, complex Gaussian noise on the $n$-th subcarrier at the destination, with variance $\sigma^2$.

When the relay $k$ is selected, the capacity of the relay network on the $n$-th subcarrier is given by

$$c^{(n)}_k = \frac{1}{2} \min \left\{ \log_2 \left( 1 + d^{(n)}_k P^{(n)}_{sk} \right), \log_2 \left( 1 + b^{(n)}_k P^{(n)}_{ld} \right) \right\}$$

where $d^{(n)}_k = \left\| h^{(n)}_{sk} \right\| / \sigma^2$ and $b^{(n)}_k = \left\| h^{(n)}_{ld} \right\| / \sigma^2$.

To obtain the maximum capacity of the relay network, the received SNR at the relay $k$ and at the destination should be equal over each subcarrier [7]. In this case, the maximum capacity of the relay network on the $n$-th subcarrier is given by

$$c^{(n)} = \frac{1}{2} \log_2 \left( 1 + a^{(n)}_k P^{(n)}_{sk} \right) = \frac{1}{2} \log_2 \left( 1 + b^{(n)}_k P^{(n)}_{ld} \right).$$

The total transmission power of the source and the relay on the $n$-th subcarrier is given by

$$P^{(n)} = \sum_{k=1}^{K} \rho^{(n)}_k \left( P^{(n)}_{sk} + P^{(n)}_{ld} \right)$$

where $\rho^{(n)}_k$ is the selection indicator function for the relay $k$ on the $n$-th subcarrier (i.e., $\rho^{(n)}_k = 1$ when the relay $k$ is selected on the $n$-th subcarrier, while $\rho^{(n)}_k = 0$, otherwise) [3].

Assume that only one relay is selected on each subcarrier (i.e., $\sum_{k=1}^{K} \rho^{(n)}_k = 1$, $\forall n$) to guarantee the exclusiveness of subcarrier.

In this case, (5) reduces to

$$P^{(n)} = P^{(n)}_{sk} + P^{(n)}_{ld}$$

where $R^{(n)}$ is the selected relay on the $n$-th subcarrier. From (4) and (6), the maximum system capacity on the $n$-th subcarrier is given by

$$C^{(n)} = \sum_{k=1}^{K} \rho^{(n)}_k c^{(n)} = c^{(n)}_{R^{(n)}}$$

$$= \frac{1}{2} \log_2 \left( 1 + \mu^{(n)} R^{(n)} P^{(n)} \right)$$

where $\mu^{(n)} = \frac{a^{(n)}_k b^{(n)}_k}{a^{(n)}_k + b^{(n)}_k}$.

Notice that the maximum system capacity on the $n$-th subcarrier is determined by $R^{(n)}$ and $P^{(n)}$. As the optimal total transmission power depends on the channel gain coefficients of the selected relay [4], [7], and [8], the system capacity depends on the relay selection on each subcarrier.

III. ADATIVE RELAY SELECTION SCHEME

To maximize the system capacity, each subcarrier should be allocated to the best relay of its own. Simultaneously, fairness of each relay must be considered to prolong the network life time.

A. Problem formulation

Suppose that the system has total subcarriers $N$ and total power $P_{total}$. In this system, $M$ relays are selected out of $K$ potential relays depending on their distribution to maximize the system capacity. Let $\mathcal{R} = \{r_1, r_2, \ldots, r_K\}$ denote the set of the selected relay.

To maximize the system capacity, only a few relays in good channel conditions could be selected. This unbalance in relay selection causes poor relay utilization, as well as shortened network lifetime.

To mitigate this problem, we limit the maximum number of subcarriers allocated to the relay $k$ to $l^{max}_k$ which determines the fairness of the system. For example, by letting $l^{max}_k = \lceil N/M \rceil$, where $\lceil \cdot \rceil$ is the ceiling operation, maximum fairness is achieved for $M$ selected relays [6].
The optimization problem of relay selection with fairness constraint is formulated as

\[
\max C = \sum_{n=1}^{N} \sum_{k=1}^{K} \rho_k^{(n)} c_k^{(n)} \\
\text{s.t. } \rho_k^{(n)} \in \{0,1\}, \forall k,n \\
\sum_{k=1}^{K} \rho_k^{(n)} = 1 \ \forall n \\
\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_k^{(n)} (P_{sk}^{(n)} + P_{lk}^{(n)}) \leq P_{\text{total}} \\
\sum_{n=1}^{N} \rho_k^{(n)} \leq l^\text{max} \ \forall k .
\]

Note that this is an integer programming problem which is NP-complete having high computational complexity [9].

**B. Proposed adaptive relay selection scheme**

To reduce computational complexity, a suboptimal adaptive relay selection scheme is proposed. The proposed scheme consists of two steps. In the first step, \(M\) relays are selected to maximize the system capacity assuming uniform power distribution over subcarriers, without considering fairness constraints. In the second step, subcarriers are reallocated among the \(M\) selected relays to satisfy fairness constraints as well as to minimize the decrease of the system capacity.

**Step 1: Relay selection without fairness constraints**

Suppose that total power of the system is uniformly distributed over subcarriers (i.e., \(P_{\text{total}} = P_{\text{total}} / N\)). Let \(\mu_k^{(n)}\) denote the relay selection metric of relay \(k\) on the \(n\)-th subcarrier. The maximum system capacity is achieved at the maximum relay selection metric as \(\log_2(\cdot)\) in (7) is a monotonically increasing function.

Therefore the selected relay on the \(n\)-th subcarrier is given by

\[
R_k^{(n)} = \arg \max_k \left( \mu_k^{(n)} \right) .
\]

Then, set the selection indicator function: \(\rho_k^{(n)} = 1\) and \(\rho_k^{(n)} = 0 \ \forall k \neq R_k^{(n)}\).

Notice that the relay selected on each subcarrier by using (11), achieves the maximum system capacity considered as the upper bound of the proposed scheme.

Let \(l_k\) denote the number of subcarriers that allocated to the relay \(k\), which is given by

\[
l_k = \sum_{n=1}^{N} \rho_k^{(n)} .
\]

The number of total selected relays out of \(K\) potential relays \(M\) is obtained as

\[
M = \sum_{k=1}^{K} \eta_k
\]

where \(\eta_k = \begin{cases} 0, \text{ if } l_k = 0, \\ 1, \text{ otherwise.} \end{cases}\)

**Step 2: Subcarrier reallocation**

Consider a relay \(k_0\) which does not satisfy fairness constraint (i.e., \(l_{k_0} > l^\text{max}\)). To mitigate unfairness on the relay \(k_0\), some of its subcarriers must be reallocated to other relays, which causes a decrease in the system capacity.

The cost function of reallocating the \(n\)-th subcarrier to the relay \(k\) instead of the initially selected relay \(k_0\) is given by

\[
e_k^{(n)} = \frac{\mu_{k_0}^{(n)} - \mu_k^{(n)}}{\mu_k^{(n)}} \ \forall n .
\]

Note that the cost function is depends on \(n\) and \(k\).

Define a set of subcarrier indices allocated to the relay \(k\) as

\[
S_k = \{n | \rho_k^{(n)} = 1, \ n = 1,2,\ldots,N\} .
\]

Define a set of relays that could be selected on reallocating subcarrier as

\[
\mathcal{W} = \{k | l_k < l^\text{max}, \ k = 1,2,\ldots,M\} .
\]

Then, the feasible set of the solutions for minimizing the cost function of relay \(k_0\) is obtained as

\[
\mathcal{F}_{k_0} = \{(n,k) | n \in S_{k_0}, k \in \mathcal{W}\} .
\]

The capacity decrease due to this subcarrier reallocation must be minimized to obtain suboptimal solution.

Among \(\mathcal{F}_{k_0}\), find \((n^*,k^*)\) that minimizes the cost function, that is,

\[
(n^*,k^*) = \arg \min_{(n,k) \in \mathcal{F}_{k_0}} e_k^{(n)} .
\]
Then the selected relay on the $n^*$-th subcarrier $R^{(n^*)}$ becomes $k^*$ instead of $k_0$.

Set the selection indicator function: $\rho_{R^{(n^*)}}^{(n^*)} = 1$ and $\rho_{k_0}^{(n^*)} = 0$.

Subcarriers are reallocated until all elements of $R$ become to satisfy (10). The detailed flow chart of the subcarrier reallocation scheme is shown in Fig. 2.

C. Power allocation (PA)

After a relay is selected for each subcarrier, power is allocated to subcarriers using the water-filling method [7]. Then, the optimal solution of the total transmission power allocated on the $n$-th subcarrier is given by

$$P^{(n)} = \left( \lambda - \frac{1}{\mu_{R^{(n)}}^{(\star)}} \right)^{\dagger}$$

where $(x)^{\dagger}$ stands for $\max(0,x)$ and $\lambda$ is a Lagrange multiplier which is chosen such that overall power constraint (9) is satisfied.

IV. SIMULATION RESULTS

Suppose that a frequency-selective channel in the network has four equally spaced independent paths with a uniform power delay profile.

The performance of the proposed scheme is evaluated via Monte-Carlo simulations for the total number of subcarriers $N$ is 128, the distance between the source and the destination $d_{sd}$ is 1000m, the path loss exponent is 4, and the maximum number of subcarriers allocated to each relay is $\lceil N/M \rceil$.

Fig. 3 shows the average system capacity per subcarrier versus signal to noise ratio (SNR) for $K = 100$. It is shown that the proposed scheme achieves approximately 0.2 $\text{bps Hz}$ and 0.6 $\text{bps Hz}$ higher average system capacity per subcarrier than the OR-OFDM and direct transmission at $\text{SNR} = 10 \text{ dB}$. Also it is shown that the proposed scheme provides only about 0.1 $\text{bps Hz}$ smaller average system capacity per subcarrier than its upper bound. It is also shown that as SNR increases, the performance gap between the proposed scheme and other schemes increases.

Fig. 4 shows the outage probability for $K = 100$ and $R = 1 \text{ bps Hz}$. It is shown that the proposed scheme achieves approximately 2 $\text{dB}$ SNR gain over the OR-OFDM PA at the outage probability of $10^{-3}$. It is also shown that the performance of proposed scheme is almost the same outage probability with or without the power allocation. It shows that the proposed scheme has negligible performance degradation with fixed power allocation, which is not for the OR-OFDM or direct transmission.

![Fig. 2. Flow chart of subcarrier reallocation scheme.](image)

![Fig. 3. Average system capacity vs. SNR (K = 100).](image)
V. Conclusions

In this paper, we propose a new adaptive relay selection scheme for a regenerative OFDMA relay network. Due to the prohibitive complexity of the ordinary optimization problem, a simple and efficient suboptimal scheme is presented. Simulation results show that the proposed scheme achieves significant improvement in average system capacity and outage probability, over the conventional schemes. Furthermore, the proposed scheme has negligible performance degradation with fixed power allocation.

However, the relay selection in this scheme is based on the instantaneous channel state information, non-negligible signaling overhead is inevitable. Therefore, relay selection in a distributed way will be major subject of our future work.

REFERENCES