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# Estimation of the Scale of Fluctuation for Spatial Variables of RC Structures

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**Abstract.** Dimensional and structural properties of RC structures are non-homogenous due to the quality of workmanship, environmental and material variability. One of the required statistical information for spatial variability analysis of RC structures includes the scale of fluctuation, $\theta$. This paper discusses the estimation of $\theta$ for two spatial variables; concrete compressive strength and concrete cover. Methods used to estimate the $\theta$ are the Curve fitting method and the Kriging Method. Kriging is an optimal interpolation method which uses the concept of randomness that allows the uncertainty of the predicted values to be calculated. Data measurements for concrete compressive strength and concrete cover were obtained from Peterson (1964) and Public Work Department of Malaysia respectively. The most reliable value for $\theta$ of $f_{cu}$ was determined and the value obtained for $\theta$ of $c$ was found unreliable due to the insufficient of data points from the available data.

## 1 Introduction

In the field of the structural reliability of reinforced concrete (RC) structures particularly located in marine environments (eg., for service life predictions), the issue of neglecting the spatial variability of the deterioration parameters across the structure has been raised. It is becoming increasingly difficult to ignore the spatial variability, as most modern structures possess a high degree of structural complexity. Hence, the materials and properties of RC structures are not homogeneous due to the variability of the environment and workmanship [1, 2]. Most of the main parameters in the chloride-induced cracking models are subject to uncertainty. The assumption that materials and dimensional properties of RC structures are homogeneous can lead to non-conservative predictions of failure for RC structures in corrosive environments [3, 4]. Neglecting such sources of uncertainty has a significant impact on the safety and assumed whole life durability performance of the structures [1].

The required statistical information for analysis involving spatial variability includes the mean value ($\mu$), standard deviation ($\sigma$) and scale of fluctuation ($\theta$). Data for the first two statistical parameters, $\mu$ and $\sigma$ are available in the literature, however, data for $\theta$ is very

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scarce and the insufficient data for $\theta$ has been reported by other researchers [1, 2, 5]. The main concern in the spatial variability of the reliability analysis is the lack of data for $\theta$ although some studies have included spatial variability for durability aspects such as Mullard and Stewart [2], Vu and Stewart [6], Karimi and Ramachandran [7], and Engelund and Sorensen [8]. However, these studies have been unable to provide accurate mathematical estimation for $\theta$. For example, $\theta$ used by Mullard and Stewart [2] was based on engineering judgment which was governed by the correlation function on a particular random field.

There are other important spatial variables that need to be included in the spatial variability of the reliability analysis, including concrete compressive strength, $f_{cu}$ and concrete cover, $c$. However, data for $\theta$ of $f_{cu}$ and $c$ are difficult to find elsewhere. Clearly, due to the limited presence of $\theta$ in the literature, $\theta$ needs to be further investigated in order to provide more accurate and reliable results for the spatial variability of the reliability analysis in the chloride-induced corrosion problem. This research takes this opportunity to carry out some analysis based on the available data in order to suggest new values of $\theta$ for $f_{cu}$ and $c$.

## 2 Definition and the development of the scale of fluctuation, *θ*

The scale of fluctuation, $\theta$ is defined as the approximate length over which strong correlation persists in the random field [9-11]. The scale of fluctuation is a parameter used for finding realistically accurate information about the variance of local averages of random fields. The scale of fluctuation of a spatial variable can be estimated from data measurements. For example, a data measurement of concrete compressive strength on a 3 m height of the concrete column. The important information required for calculating $\theta$ includes the value of concrete compressive strength along the height of the column and the distances between each measurement. This section summarises the description on the development of $\theta$, including the contribution and recommended values for $\theta$ used by previous researchers.

The development for $\theta$ was initially supported by Vanmarcke [9] which has proposed various methods for calculating $\theta$ for stationary random fields. Another study that contributed to the development of $\theta$ by Karimi [12]. This study used historical data from various engineering disciplines to apply to the chloride-induced RC corrosion problem. However, it was found that the probability of the onset of corrosion was underestimated. This is because the spatial variables, $C_s$ and $D_{app}$ were treated as random variables only. The study by Ramachandran et al. [13] has suggested the Kriging method to predict missing data for estimating $\theta$. A recent study done by Connor and Kenshel [1] provides detailed descriptions on the estimation for of $\theta$ for $C_s$ and $D_{app}$ from the analysis of experimental data recorded on a bridge in South East Ireland. Two methods were adopted in the study to estimate $\theta$ of both spatial variables. The first method is the Curve fitting method used to determine the values for $\theta$ through an associated model parameter, known as correlation length, $d$. The second method is Kriging method to determine the missing data in the specific unmeasured location from the recorded data.

This paper used data from the study by Peterson [14] to determine $\theta$ for concrete compressive strength. This research also has an opportunity to access data from the Public work department (PWD) of Malaysia to calculate $\theta$ for concrete cover. The purpose of using the concrete cover data from PWD of Malaysia is to study the degree of physical fluctuation of concrete cover on real structures. The new finding for $\theta$ were discussed here will serve as a base for further works and possibly be beneficial for providing discussion and information (e.g., limitation and procedures) of $\theta$ that is useful for future research.

## 2.1 The Kriging method

Kriging is a statistical interpolation method which applies a geostatistics method. Geostatistics is defined as the study of phenomena that vary in space and/or time. The Kriging is well known in the fields of mining engineering, geology, hydrology and social science. This method is very appropriate for finding the missing data as Kriging accepts irregular spaced data, calculates an error of estimates which can give an actual measure of the reliability of estimated points and uses the autocorrelation between known data values for estimation of unmeasured values. Kriging itself is an optimal interpolation based on regression against observed *z*-values of surrounding data points, weighted according to spatial values [15].

This method uses the concept of randomness which allows the uncertainty of the predicted values to be calculated. Kriging generates the best linear unbiased estimate at a specific location by employing a semivariogram model. As the estimation of $\theta$ uses a curve fitting method, therefore the most suitable interpolation method must be used to estimate the missing data.

Some advantages of Kriging as stated by Geoff Bohling [16], are that 1) it helps to compensate for the effects of data clustering, 2) it can assign individual points within a cluster less weight than isolated data points, 3) it gives an estimate of error (kriging variance), along with estimates of the variable, *Z*, itself (although the error map is basically a scaled version of a map of distance to nearest data point, and therefore not that unique), and availability of estimation error provides basis for stochastic simulation of possible realizations of *Z(u)*.

The basic linear regression estimator *Z*(u)* is

$$Z^*(u) - m(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u)\left[z(u_\alpha) - m(u_\alpha)\right] \tag{1}$$

In the Equation (1), *Z(u)* is treated as a random field and the aim of this equation is to calculate Kriging weights, $\lambda_\alpha$. Kriging calculates a residual at u as the weighted sum of residuals at neighbouring data points. Kriging weights, $\lambda_\alpha$, are derived from the covariance function or semivariogram which should characterize the residual component. Kriging weights, $\lambda_\alpha$, minimize the variance of the estimator $\sigma^2_E$ in Equation (2) under the unbiasedness constraint E {Z*(u)-Z(u)}=0.

$$\sigma^2_E(u) = Var\{Z^*(u) - Z(u)\} \tag{2}$$

## 2.2 Modeling the semivariogram

To model the semivariogram, the empirical semivariogram needs to be replaced with an acceptable semivariogram model. This is due to the kriging algorithm needing access to semivariogram values for lag distances other than those used in the empirical semivariogram. The five most frequently used models are given in Table 1.

The semivariance function $\gamma_{Exp(\tau)}$ used in the equation can be obtained empirically from a set of data points as follows:

$$\lambda_{Exp(\tau)} = \frac{1}{2N(\tau)} \sum_{\alpha=1}^{N(\tau)} \left[z(u_\alpha + \tau) - z(u_\alpha)\right]^2 \tag{3}$$

where $\gamma_{Exp(\tau)}$ denotes the empirical semivariance function and $N(\tau)$ denotes the number of pairs of data that are separated by a lag distance $\tau$.

**Table 1.** Analytical semivariogram models.

| No | Semivariogram name | Semivariogram model |
|---|---|---|
| 1 | Nugget | $g(h)=\begin{cases}0 & \text{if } h=0\\ c & \text{otherwise}\end{cases}$ |
| 2 | Spherical | $g(h)=\begin{cases}c\cdot\left(1.5\left(\frac{h}{a}\right)-0.5\left(\frac{h}{a}\right)^3\right) & \text{if } h\le a\\ c & \text{otherwise}\end{cases}$ |
| 3 | Exponential | $g(h)=c\cdot\left(1-\exp\left(\frac{-3h}{a}\right)\right)$ |
| 4 | Gaussian | $g(h)=c\cdot\left(1-\exp\left(\frac{-3h^2}{a^2}\right)\right)$ |
| 5 | Power | $g(h)=c\cdot h^{\omega}\quad \text{with } 0<\omega<2$ |

When the semivariance function is plotted against the corresponding lags, the plot is referred to as the semivariogram. The semivariogram finds the set of weight for estimating the variable value at the location u from values at a set of neighbouring data points. The weight on each data point generally decreases with increasing distance to that point, in accordance with the decreasing data-to-estimation covariances specified in the right-hand vector, k. However, the set of weights is also designed to account for redundancy among the data points, represented in the data point-to-data point covariances in the matrix K. Multiplying k by k-1 (on the left) will downweight points falling in clusters relative to isolated points as the same distance. Stated by Vanmarcke [9], there are two important elements required in order to estimate $\theta$ through associated model parameter, $d$. The important elements are 1) sufficient data measurement and 2) an analytical model for $\rho(\tau)$. Once these elements have established, nonlinear regression methods can be used to estimate the parameters of the chosen model (in this case, known as correlation length, $d$). On the other hand, as stated in the study by Connor and Kenshel [1], by curve fitting the analytical model for $\rho(\tau)$ to the correlation coefficients determined from the experimental data $\rho_{Exp(\tau)}$, the value for model parameter $d$ that produces the best fit can be obtained, and $\theta$ can then be determined based upon its relationship to model parameter $d$. Six types of analytical model (Gaussian ACF) for $\rho(\tau)$ are presented in Table 2.

## 3 Estimating *θ* for concrete compressive strength

Data set used to measure $\theta$ for concrete compressive strength is obtained from the study done by Peterson [14]. The data from this study found to be significant for estimating $\theta$ for concrete compressive strength. An experimental work carried out by Peterson [14] investigates the variation in strength in the longitudinal direction of the column. Data obtained from the study by Peterson [14] shows the variation of $f_{cu}$ from the top of the column to the foot of the column, ($f_{cu}$ in a column varies systematically over its height) and provides the measuring distance and the distance between measured points which are sufficient to produce a reliable value of $\theta$ for $f_{cu}$. The specimens, columns with a size of 0.305 m$^2$ × 3.05 m tall (12 in.$^2$ × 10-ft tall), were removed from the forms 24 hours after casting then totally immersed in a shallow water bath for 25 days. The columns were cut into 0.305 m (12 in.) segments and the ends of the segments were ground smooth. Cores with a diameter of 0.152 m (6 in.) were drilled in a direction parallel to the original column axis and tested when the concrete was 28 days old [17].

**Table 2.** Analytical semivariogram models.

| **Type of ACF** | **ACF Model** | **θ** | **Reference** |
|---|---|---|---|
| Square exponential (Gaussian) | $\rho(\tau) = \exp\left[-\left(\lvert\tau\rvert / d\right)^2\right]$ | $\sqrt{\pi}\, d$ | Vanmarcke (1983) |
| Exponential | $\rho(\tau) = \exp\left[-\left(\tau / b\right)\right]$ | $2b$ | Vanmarcke (1983) |
| Cosine exponential | $\rho(\tau) = \exp\left(-[\tau]/e\right)\cdot\cos\left([\tau]/e\right)$ | $e$ | Kim (2005) |
| Triangular | $\rho(\tau) = \begin{cases} 1-\lvert\tau\rvert / a, & for\ \lvert\tau\rvert \le a \\ 0 & for\ \lvert\tau\rvert > a \end{cases}$ | $a$ | Vanmarcke (1983) |
| Sinusoidal | $\rho(\tau) = \dfrac{\sin\left[-2.2\left(\lvert\tau\rvert / f\right)\right]}{-2.2\left(\lvert\tau\rvert / f\right)}$ | $f$ | Gomes and Awruch (2002) |
| Second-order autoregressive | $\rho(\tau) = \left[1+\left(\lvert\tau\rvert / c\right)\right]\cdot\exp\left(-\lvert\tau\rvert / c\right)$ | $4c$ | Vanmarcke (1983) |

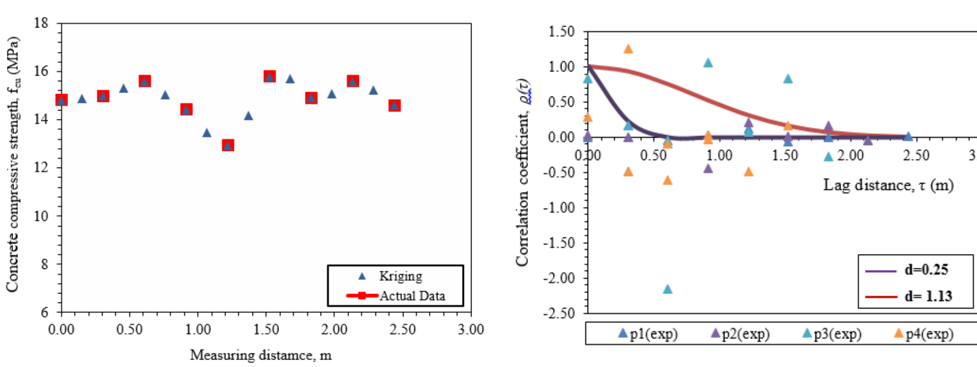
The concrete mixes were named A, B, C, D, E and F. However, only data from concrete mixes A and C were used in this study. As reported by Peterson [14], both concrete mixes (A and C) exhibited adequate cohesion, did not segregate and the concrete was poured in the vertical direction. Each column was divided into nine parts and each part was drilled to investigate the strength of the core cylinders. The variation of $f_{cu}$ from the top of the column to the foot of the column and $f_{cu}$ in a column often varies systematically over its height. The study found that the strength at the bottom is often greater than the strength at the top. The important information required for the study is the concrete compressive strength of the core cylinders with 0.667 m intervals drilled from column A and C as can be seen in Table 3.

## 3.1 Scale of fluctuation for concrete compressive strength

All the experimental correlation coefficients from column A1-A4 and column C1-C4 are combined in one plot to produce a better fit. The square exponential Gaussian was considered the best fit with experimental correlation coefficients for two columns (Column A and C). Fig. 1 (a) shows a complete set of data after Kriging and Fig. 1 (b) shows experimental correlation coefficient for concrete compressive strength fitted with Gaussian ACF for Column A. Two values of $d$ were chosen and the plot representing each $d$ was superimposed as shown in Fig. 1 (b). The square exponential (Gaussian) ACF with $d = 0.25$ m has a better fit than the square exponential (Gaussian) ACF with $d = 1.13$ m. Therefore, the square exponential (Gaussian) ACF model with $d = 0.25$ m and $\theta = 0.5$ m was chosen to be the best fit with the experimental correlation coefficient for concrete compressive strength for column A.

**Table 3.** The compressive strength of core cylinders drilled from columns [14].

| Column No. | Compressive strength of core cylinders (MPa) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| A1 | 14.810 | 15.000 | 15.590 | 14.420 | 12.940 | 15.790 | 14.910 | 15.590 | 14.610 |
| A2 | 21.670 | 23.630 | 21.180 | 23.000 | 23.830 | 23.140 | 23.240 | 23.340 | 24.620 |
| A3 | 17.160 | 20.200 | 19.910 | 18.730 | 19.120 | 19.610 | 19.520 | 19.220 | 19.900 |
| A4 | 19.120 | 21.380 | 19.520 | 20.500 | 20.300 | 18.140 | 21.280 | 20.890 | 20.990 |
| | | | | | | | | | |
| C1 | 28.240 | 30.890 | 32.260 | 28.540 | 30.700 | 28.640 | 32.070 | 31.190 | 30.890 |
| C2 | 30.499 | 34.324 | 36.874 | 34.912 | 33.147 | 33.245 | 36.579 | 35.206 | 34.226 |
| C3 | 28.146 | 30.891 | 37.070 | 37.560 | 36.187 | 33.735 | 37.462 | 36.972 | 36.972 |
| C4 | 31.382 | 37.070 | 39.914 | 40.894 | 38.639 | 40.306 | 40.502 | 40.796 | 41.385 |




(a) (b)

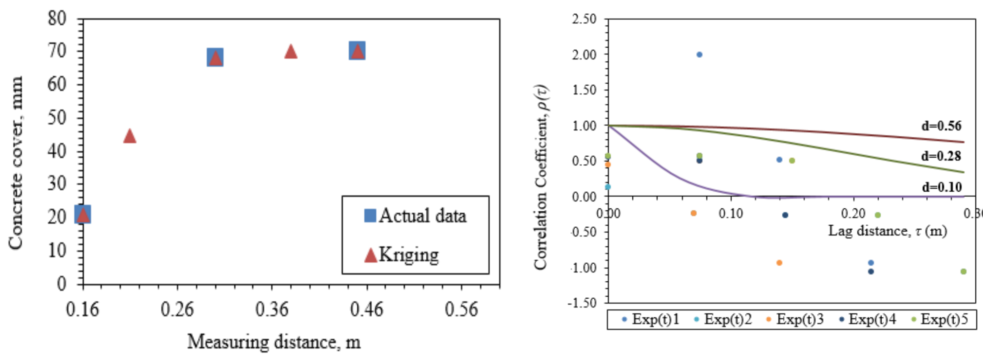
**Fig. 1.** (a) A complete set of data after Kriging and (b) Experimental correlation coefficient for concrete compressive strength fitted with Gaussian ACF for Column A.

## 4 Estimating $\theta$ for concrete cover

Data used for estimating $\theta$ for concrete cover, $c$, are obtained from the Public Work Department (PWD) of Malaysia. The data was recorded observations (data measurements) of $c$ on seven piers of a bridge of River Aur, North port of Klang, Selangor, Malaysia. The process of determining the reliable value of $\theta$ for concrete cover is the same as the process of determining $\theta$ for concrete compressive strength.

### 4.1 Scale of fluctuation for concrete cover

The experimental correlation coefficient, $\rho_{Exp(\tau)}$ was found not perfectly fit with the chosen ACF model (the square exponential ACF) as can be seen in Fig. 2 (b) and a complete set of data after Kriging shown in Fig. 2 (a). The model nearly fits with the Gaussian ACF model when $d$ = 0.1 m. However, with $d$ = 0.1 m and $\theta$ = 0.177 m is considered very small and this will produce high correlations between elements and concerns in the decomposition of the correlation coefficient matrix. The unreliable values of $\theta$ for c is due to the measurements of c were made in less than 1 m. The measuring distance should be at least 3 m in order to have ample information about the degree of fluctuation of the property.




(a) (b)

**Fig. 2. (a)** A complete set of data after Kriging and (b**)** Experimental correlation coefficient for concrete cover fitted with Gaussian ACF.

## 5 Conclusion

The purpose of this paper is to determine reliable values for the scale of fluctuation, $\theta$ of two important spatial variables: concrete compressive strength, $f_{cu}$ and concrete cover, $c$. Data from Peterson [14] provides the measuring distance and the distance between measured points which are sufficient to produce a reliable value of $\theta$ for $f_{cu}$. The results of this study support the idea that the increase in the number of observation will increase the accuracy of the value of estimated scale of fluctuation. For estimating missing data, a simple calculation method such as Kriging can be considered as a good starting point when dealing with a continuous record of data, however, the method would not be useful when only a small set of data points is available.

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