Simulated Analysis and Optimization of a Three-Antenna Airborne InSAR System for Topographic Mapping

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Abstract—A three-antenna synthetic aperture radar interferometer (InSAR) with a statistically optimal data processor for three-dimensional (3-D) terrain mapping has been proposed recently to reduce the phase ambiguity and data-noise drawbacks of the conventional two-antenna SAR interferometry technique. In this paper, a numerical simulator is developed to assess the achievable performance and various design tradeoffs of the three-antenna InSAR. The most critical conditions for the new reduced-ambiguity system operating on realistic scenes are taken into account. The phase-unwrapping procedure is included in the simulator to compare the new and the conventional technique in terms of both phase and height-estimation accuracy. The performance achievable by a three-antenna airborne InSAR system on a given site are analyzed, and the parameter optimization of the new system is investigated. The results of several case studies show that the new technique can outperform the conventional one significantly for a typical airborne configuration, especially for high-slope terrain steepness. It provides reduced-phase aliasing and better estimation accuracy. So, the phase unwrapping is simplified and high-quality maps of terrain height can be obtained. As a limit, absolute phase retrieval can be achieved with good accuracy and the unwrapping procedure can be avoided.

Index Terms—Maximum likelihood estimation, multibaseline radio interferometry, phase unwrapping, simulation, synthetic aperture radar (SAR), terrain elevation mapping.

I. INTRODUCTION

SYNTHETIC aperture radar interferometry (InSAR) is a well-established radar technique to obtain high-resolution maps of the terrain height by using the signals received by two antennas separated by a distance (baseline) along the cross-track direction [33], [9]. The elevation of the terrain can be derived from the phase difference (interferometric phase) between corresponding pixels in the two SAR images by means of simple geometric relationships [27]. However, this phase is measured modulo 2π, whereas the terrain height is related to the absolute interferometric phase, so the phase must be unwrapped before the conversion to height [12]. This is one of the most critical steps of the conventional InSAR technique.

In fact, the interferometric phase can be unwrapped to within a constant, so at least one ground reference (or tie) point (GRP) is necessary [33], [27]. Then, the phase aliasing in steep areas [13] causes unwrapping errors and significant degradation of the height estimates. Moreover, phase unwrapping is complicated by foreshortening and layover [12]. Finally, around the condition of complete foreshortening, the two received signals are fully decorrelated (the baseline equals the critical baseline [27]), so no useful information can be retrieved (blind angles) [10]. In addition, the single, available baseline does not allow optimal operation [27] for highly varying terrain slope. In fact, the conventional two-antenna interferometric technique suffers from the intrinsic limitation of the conflict among the baseline requirements in terms of sensitivity, phase noise, and robustness to phase aliasing [13].

Most of the drawbacks of phase ambiguity and data noise can be reduced or avoided by using recently proposed multibaseline or multifrequency InSAR techniques [34], [11], [4], [14], [16], [24], [8]. In this framework, [18] proposed a three-antenna SAR interferometer with a statistically optimal maximum likelihood (ML) processor, which combines the complex data at three phase centers to retrieve the interferometric phase directly (with reduced ambiguity and possibly the absolute phase) and to enhance accuracy [20], [21]. This makes it possible to fully exploit the baseline diversity of the antenna array. The proposed technique is a pixel-based, elevation-angle estimation method (as opposed to the phase-only interferometric techniques in [34], [4], [15], [24], [8]) consisting of an array-processing algorithm that takes account of the presence of multiplicative noise (speckle) in the pixel triplets of the three SAR images (as opposed to the conventional beamforming techniques in [11] and [14]). In [18] and [21], the performance analysis of the three-antenna system with the optimal phase estimator has been carried out in terms of variance of the estimated interferometric phase as a function of the overall baseline to critical baseline ratio, the SNR, the number of looks and the three-antenna array configuration, and a comparison with the conventional single-baseline technique has been carried out. However, this analysis is limited to a single multilook cell, as in [27], and does not take into account the unwrapping errors.

This paper deals with a more realistic performance evaluation of the three-antenna system operating on extended areas, also taking account of the beneficial effects of the reduced-
ambiguity estimation on the phase-unwrapping procedure. Moreover, the optimization of the parameters of the new system is investigated. The performance study is focused on the evaluation of the potential of the three-antenna InSAR technique for single-pass airborne platforms which, to the best of our knowledge, has not yet been analyzed. The final performance of the conventional and the three-antenna systems is compared in terms of root mean-square (RMS) height error. This takes into account the joint effect of the two types of errors, the usual small height errors, and the absolute phase related or large errors, on the index of quality of the reconstructed height map. This is significant in most of the applications. A study of higher order on the probability density function of the error is out of the scope of the paper.

The rest of the paper is organized as follows. In Section II, the three-antenna airborne system geometry, the model of the received signals, and the pixel-based ML processor used to estimate the interferometric phase with reduced ambiguity are recalled. Section III presents the simulator used to analyze the height estimation performance of the new system, including an unwrapping procedure specifically tailored to handle the reduced ambiguity interferogram. Finally, in Section IV, the numerical results obtained by employing the proposed simulator on a given site are illustrated. The phase accuracy of the new system is evaluated and compared to that of the conventional single baseline InSAR system for $C$- and $X$-band and different terrain steepness. The reduction of the completely decorrelated areas operated by the three-antenna system also is assessed. Then, the final-height accuracy and the optimization of the unwrapping procedure is analyzed for different array configurations. So, the tradeoff between reduced aliasing and possible degradation of phase accuracy for large ambiguity reduction, low SNR, and high spatial decorrelation is derived. Moreover, the robustness of the reduced-ambiguity system against incorrectly tuned unwrapping is assessed, and the accuracy gain on the conventional system is evaluated in realistic operative conditions. Some conclusions are drawn in Section V.

II. THREE-ANTENNA AIRBORNE INSAR SYSTEM

The system under investigation is a single-pass conventional airborne interferometer with the addition of a third receiving antenna in asymmetric position between the two other ones as shown in Fig. 1 (one antenna both transmits and receives, the other two only receive), where $\theta$, $\xi$, $H$, and $r$ are the look angle, the baseline tilt angle, the platform altitude, and the slant-range distance, respectively. In such a way, three baselines are available: the overall baseline $B_{13}$, a shorter intermediate baseline $B_{23}$, and a very short baseline $B_{23}$. The position of the additional antenna 2 is defined by the asymmetry factor $p$, defined as the ratio between the shortest available baseline and the overall baseline $p = B_{23}/B_{13}$.

Since the ambiguity problems of interferometric SAR systems are governed by the baseline-to-wavelength ratio, the baseline diversity of the three-antenna system can be used either to increase the unambiguous phase range or to estimate the absolute phase [15], [18], depending on the SAR system parameters and observational geometry. In fact, the unfolding of an ambiguous estimate is possible when estimates of the same variable with different ambiguity are available ([23], [34]), as is well known in the surveillance and weather radar area. In [18] and [21], it is shown that for an asymmetry factor of the form $p = 1/n$ with $n$-integer number, the phase unambiguous range magnification (URM) compared to conventional interferometry is $n$. As an example, an asymmetry factor $p = 1/5$ produces a URM = 5 (phase is estimated modulo $10\pi$ instead of $2\pi$). Depending on the triangulation geometry and system parameters (baseline-to-wavelength ratio), the extended unambiguous phase range can be enough to derive an interferogram without fringe lines (condition of absolute phase retrieval [23], [2]). When the absolute phase is obtained, we may reconstruct the surface height directly. The burdensome phase-unwrapping procedure is skipped, phase-aliasing problems are avoided and, in principle, the GRP can no longer be necessary if navigation parameters are known with enough accuracy. Otherwise, the phase-unwrapping process is simplified by the unambiguous phase-range magnification, because the interferometric fringes become wider. With the reduced-phase aliasing, the accuracy of the height estimate can be improved significantly, especially for steep terrain [5]. In fact, the slope limit set by the sampling criterion can be derived from simple geometric relationships, similar to [13], as

$$
\tau_{\text{max}} = \arctan \left( \frac{\lambda r \sin \theta}{2DB_{13} \cos (\theta - \xi)} \text{URM} \right)
$$

where $\lambda$ is the transmitted wavelength, $D$ is the ground distance between two samples in the interferogram, and URM is the unambiguous range magnification with respect to the conventional $2\pi$ phase range. So, the higher the URM is, the higher the slope limit. Moreover, thanks to the high correlation of the signals received by the two closest antennas, the three-element InSAR system also can operate when complete decorrelation occurs in the conventional system [19]. Therefore, the range of blind angles is reduced, and the areas without useful information shrink [5]. Under the assumption of flat surface in the illuminated patch [27] from the analysis in [20], it is easily derived that the range of blind terrain slopes in the cross-track direction shrinks by a factor of approximately $1/p$ for zero-terrain azimuth slope and $B/r \ll \lambda/R_p$, where $B$ is the same overall baseline for the three- and two-antenna
systems and \( R_c \) is the slant-range resolution. Finally, the proper fusion of the information available at the three phase centres allows a general improvement of the phase-estimate accuracy with respect to the conventional system.

The maximum-likelihood estimator \([30]\) derived in \([18]\) to optimally process the data acquired by the three antennas is recalled in the following. We consider the three complex SAR images obtained by processing the echoes from the three two-way phase centres. We choose \( \varphi = \varphi_{13} \), the phase difference between the two outer phase centres produced by the observational geometry and the topography, as the interferometric phase sensed by the three-antenna system. As is usual in SAR interferometry, we apply a multilook processing to reduce the statistical variations of the phase \([27]\). The complex amplitudes of the pixels, corresponding to the same area on the ground observed in the three SAR images, are arranged in the vector \( \hat{\mathbf{I}}(\omega) = (I_1^{(\omega)}, I_2^{(\omega)}, I_3^{(\omega)})^T \) where \( I_k^{(\omega)} \) is the pixel in the \( k \)th image for the look \( \omega = 1, \ldots, N \) with \( N \) number of independent looks \([27]\). According to the statistical model proposed in \([18]\) and \([21]\), which is an extension of \([27]\), the data vectors are complex Gaussian random vectors with Hermitian covariance matrix \( \mathbf{\Gamma} \). Normalizing to unit power, the diagonal terms of \( \mathbf{\Gamma} \) equal one, while the offdiagonal elements are

\[
\gamma_{\omega i,j} = \exp\{j\varphi_{i,j}\} \rho_{\omega i,j}
\]

where \( \varphi_{i,j} \) are the fractions of the overall baseline \( B = B_{13}, x_{13} = 1, x_{12} = 1 - p, \) and \( x_{23} = p, \) and \( \rho_{\omega i,j} \) is the corresponding coherence, which can be expressed as \([27]\)

\[
\rho_{\omega i,j} = \alpha(B_{i,j})\text{SNR}/(\text{SNR} + 1)
\]

where \( \alpha(\cdot) \) is the spatial correlation factor, which is a function of the pertinent baseline [as expressly indicated in (3)], of system and observational parameters, of the image misregistration, and of the local topography and scatterer distribution. SNR is the signal to thermal noise ratio.

For the \( N \) pixel vectors corresponding to a multilook cell, the asymptotically optimal \([18]\) ML estimator of the interferometric phase \( \varphi \) has been derived in \([18]\) and \([21]\) from the model in (2) as

\[
\varphi \in [-\text{URM}_\pi, \text{URM}_\pi], \text{max} \left\{ P_{\omega 1} - \rho_{23} \right\} \text{Re} \left\{ \exp\{-j\varphi(1-p)\} \right\}
\]

\[
+ (\rho_{23} - \rho_{12} \rho_{23}) \text{Re} \left\{ \sum_{\omega=1}^N P_1^{(\omega)} P_2^{(\omega)} \right\}
\]

\[
+ (\rho_{23} - \rho_{12} \rho_{23}) \text{Re} \left\{ \sum_{\omega=1}^N P_2^{(\omega)} P_3^{(\omega)} \right\}
\]

where the phase interval \([-\text{URM}_\pi, \text{URM}_\pi]\) is the extended unambiguous phase range that results from the baseline diversity, with \( \text{URM} = 1/p \). A closed-form solution of problem (4) is not available. Moreover, the function has many local maxima, so we use a fast grid search with look-up tables for the values of the trigonometric functions. Conversely, the expression of the ML phase estimator for the conventional single baseline InSAR with unambiguous range \([-\pi, \pi]\) is

\[
\varphi = \arg\left\{ \sum_{\omega=1}^N \hat{I}_1^{(\omega)} \hat{I}_3^{(\omega)} \right\}
\]

Since the coherences \( \rho_{i,j} \) determining the optimal data fusion weights in the estimator (4) depend on generally unknown parameters (such as local slopes, look angle, and SNR), they are directly estimated from the acquired data with little loss, as shown in \([20]\), by the following sample coherence estimator \([29]\)

\[
\hat{\rho}_{i,j} = \frac{\sum_{\omega=1}^N P_i^{(\omega)} P_j^{(\omega)}}{\sqrt{\sum_{\omega=1}^N |P_i^{(\omega)}|^2 \sum_{\omega=1}^N |P_j^{(\omega)}|^2}}
\]

This makes the three-antenna phase estimator adaptive. Such a three-antenna system actually can be realized onboard aircraft platforms, the additional hardware and room requirement being somewhat limited. In fact, many current operating or planned airborne systems already use more than two antennas either for multipolarization or multifrequency analysis or for joint along-track and cross-track interferometric capability (for instance, see \([7]\), \([31]\)). At the time of the second revision, the authors were acquainted with ongoing experimental analysis with a three-antenna configuration at FGAN \([28]\).

In \([18]\), \([20]\), a comparison between the conventional and the three-antenna system has been carried out through a local performance analysis in terms of rms phase error before unwrapping for a single multilook cell. A typical example of the results of this analysis are summarized in Fig. 2. The curves show the local rms phase error at the sensor level versus the local SNR for the conventional and the three antenna InSAR system with antenna asymmetry factor \( p = 1/5 \) (URM = \(5\)). The reference conventional system consists of antennas 1 and 3 separated by a baseline equal to the overall baseline \( B_{13} \) of the three-antenna system. Results are reported for two different values of the spatial correlation factor \( \alpha \) at the
baseline $B_{13}$ (assumed to be the same for the two systems). The linear correlation model is $N = 8$ looks. For a better interpretation of the analysis, we recall that the third antenna acquires an additional sample of speckle that is partially correlated with those seen from the other two antennas. This enhances the information content intrinsic in the gathered data. Moreover, the thermal noise in the third receiving channel is independent from the other two noise measurements, so that the three-antenna system has an effective SNR higher than a conventional system. For low correlation at the overall baseline $\alpha(B_{13}) = 0.2$, the expansion of phase unambiguous range is achieved in conjunction with a sensible local phase accuracy gain when the local SNR is high enough (greater than 12 dB). For high correlation (0.8), there is no more gain on phase accuracy, because the information gathered (thanks to the additional antenna) correlates well to the information available at the overall baseline. The gain expected from the higher effective SNR is negligible when SNR is already high. Conversely, when the local SNR is low (less than 9 dB), the unambiguous range magnification is obtained at the cost of a loss of phase accuracy. This is caused by catastrophic phase errors [34], [15], [18], viz. absolute phase related or large errors at the sensor level (a phenomenon discussed in Section IV). Catastrophic errors, together with adaptivity losses [20], mask the gain expected from the higher effective SNR, resulting in a net loss for the sensor level accuracy compared to the conventional system (a gain for low SNR can be obtained if a more fully populated array is employed [19]).

The strong dependence of the three-antenna system performance on the joint effect of the spatial correlation and the SNR (which in turn depend on the joint effect of the local scattering condition and terrain slopes) suggests that a complete comparison between the two systems can be carried out only through: 1) the evaluation of the mean performance achievable on a given area, and 2) taking account of the height errors introduced by the phase-unwrapping algorithm. These are not included in the local analysis at the sensor level and are reduced for the system with ambiguity reduction or absolute phase retrieval with respect to the conventional InSAR system. In other words, a meaningful comparison of the two- and three-antenna system performance also must take into account the absolute phase related or large errors that arise in the conventional system as well, although at a different level (i.e., after the unwrapping of the 2π-modulus estimated phase at the sensor level). Therefore, in the following, we analyze the performance achievable by the three-antenna InSAR system by making use of a statistical model that simulates most of the operative conditions of the system on a given scene. We also include a modern unwrapping procedure in the processing chain. This simulator is described in the next section. It also allows us to investigate the various design tradeoffs of the new system.

It is worth noting that an asymmetry factor of the form $p = m/n$ with $m, n$ coprime integers is also possible. It produces URM = $n$ with reduced phase accuracy compared to the case $p = 1/n$. This has been proposed and analyzed in [34], [18]. The simulated analysis and system optimization for fractional $p$ is not included in the paper.

III. NUMERICAL SIMULATOR

Now, we present the numerical simulator for a detailed performance analysis of the three-antenna SAR interferometer with adaptive ML data processor. A simple image simulator for the three-antenna InSAR has been developed, bypassing the complicated and time-consuming generation of the unfocused (raw) SAR images [13]. A block diagram of the simulator is reported in Fig. 3. The components of the phase for the corresponding pixels in the three focused SAR images due to the observational geometry and terrain height are generated with low computational load, starting from the radar and platform parameters and the topography of the illuminated scene. This latter information is obtained by a digital elevation model (DEM) of the area. These phase contributions are combined with the elements of complex three-dimensional (3-D) vectors taking account of partially correlated speckle and thermal noise in the three images. These vectors are generated with the proper correlation structure according to the multidimensional model in (3), exploiting the Cholesky decomposition [30] of the correlation matrix $(p_k)_{k,\ell \in \{1,2,3\}}$ as shown in [18]. The result of this operation is the corresponding pixels observed in the three complex SAR images.

The parameters of the speckle and noise statistical models of (3) are evaluated, taking into account the azimuth and range local slopes and variable look angles that affect the spatial decorrelation, the scattering coefficient as a function of the local grazing angle, and the variable illuminated area that determines the SNR. In particular, we employ the model for the spatial correlation coefficients derived in [27], which accounts for both local slopes

$$
\alpha_{ij} = 1 - \left\{ B_{ij} x_{ij} \left[ \frac{R_\theta \cos(\tau_r) \tan(\tau_\theta)}{\lambda r} \right] + \frac{R_r}{\lambda r} \tan(\vartheta - \tau_r) \right\} - B_{ij}^2 x_{ij}^2 \left[ \frac{R_\theta R_r \cos(\tau_r) \tan(\tau_\theta)}{\lambda r^2 \sin(\vartheta - \tau_r) \tan(\vartheta - \tau_r)} \right]
$$

where $B_{ij} = B \cos(\xi - \vartheta)$ is the overall baseline, projected onto the direction orthogonal to the look direction, and $x_{ij}$ are the pertinent baseline fractions, $\tau_r, \tau_\theta$ are the range (cross-track) and azimuth slopes, and $R_r, R_\theta$ are the range and azimuth resolutions. Misregistration and volumetric scattering effects currently are not included in the simulator. However, they easily can be considered by employing a complete model for the coherence [27]. Note that considering both the slopes, the correlation coefficients exhibit a quadratic dependence on the baseline in addition to the well-known linear term. This model for the simulated analysis is more complete than the one used in [18] and [20]. For the modeling of the variable scattering coefficient (and hence varying SNR), which is a critical parameter for the reduced ambiguity phase retrieval, we resort to a general land-clutter model, presented in [26].

The expression of the scattering coefficient is an extension of the Barton model [1], and it is reported here in parametric form for the sake of simplicity

$$
\sigma^0 = f(\psi_g, \lambda, A)
$$
where \( \psi_g \) is the local grazing angle (function of \( \theta, \tau_r, \) and \( \tau_a \)), and \( A \) is a constant depending on the terrain typology. Moreover, we approximate the varying terrain area corresponding to the SAR resolution cell, taking into account only \( \tau_r \). Then, we obtain the SNR by means of the radar equation [6]. The simulator also accounts for the joint effects of topography and viewing geometry, such as foreshortening and layover. Finally, the interferogram with increased unambiguous phase range is generated by the ML phase estimator from the simulated complex pixel vector data.

After the analysis of the phase accuracy, the reduced-ambiguity interferogram must be unwrapped to compare the performance of the new and the conventional InSAR systems in terms of height error. The expected phase of a topographically flat earth (FEP) is subtracted from the interferogram. Then, we employ a completely automatic region-growing, phase-unwrapping technique adapted from [17], which can detect and avoid unwrapping anomalous phases in the interferogram. First, for each pixel to unwrap, a preliminary phase estimate based on the unwrapped phase values of the nearby pixels is submitted to a confidence test. The consistency between predicted phase values is adopted as a measure of confidence of the estimated phase. If the confidence value is lower than a threshold \( t_{fp} \), a second estimate based on the wrapped phase of the investigated pixel is derived, adding to it multiples of \( 2\pi \text{URM} \). This estimate is compared to the previous one. If the absolute value of the difference between these two estimates is lower than a threshold \( t_u \), the second one is assumed as final estimate. Otherwise, the pixel is marked, and its unwrapped phase is determined by interpolation. In particular, interpolation is used for the areas without useful phase information because of complete decorrelation, then the FEP is added up again and the phase is converted into height. When the three-antenna system can retrieve the absolute phase, the unwrapping procedure is skipped, leaving just the test and filtering stage, and the phase-to-height conversion can be performed directly. Errors induced by platform roll are not included in the simulation model [32], [3]. Finally, the reconstruction errors are computed. It is worth noting that the areas affected by layover and shadowing are not included in the evaluation of phase and height accuracy. An analogous simulation procedure and the same unwrapping technique (\( \text{URM} = 1 \)) is employed for the conventional two-antenna system used as reference.

IV. RESULTS AND DISCUSSION

A performance analysis of the three-antenna airborne InSAR system on realistic scenarios, and a comparison with the conventional system (with the same overall baseline), has been carried out by means of the proposed simulator. We have applied the simulation model to an area covering about 3.25 km × 3.25 km placed in Tuscany, Italy, whose height varies from 0–280 m. The DEM of this area is shown in Fig. 4(a). The system and mission parameters are: overall baseline \( B = 2.75 \) m, \( \lambda = 6 \) cm (\( C \)-band) and 3 cm (\( X \)-band), \( \xi = 60^\circ (30^\circ \text{off-vertical}) \), nominal SNR (midswath, hilly terrain) \( 6, 12, \) and \( 18 \) dB, \( R_v = 7.2 \) m, \( R_a = 6.25 \) m, \( N = 8 \) looks (2 rg., 4 az.), \( \theta \) (midswath) = 35°, \( H = 4 \) Km.

A. Phase Accuracy

First, we compare the sensor-level rms phase error on the site under investigation for the conventional and the new system. Fig. 4(b) and (c) show the interferograms (after the FEP removal) derived by the conventional and the three-antenna system with \( \text{URM} = 5 \) for a nominal SNR (midswath) \( \geq 12 \) dB and \( C \)-band and with the DEM scaled by a factor of 1.5. The advantages in terms of fringe width of the new system with respect to the conventional one are evident. As a limit, absolute phase retrieval is achievable. As an example, for the system and mission parameters mentioned previously, we have obtained a nonwrapped interferogram without FEP with \( p = 1/15 \) (URM = 15), see Fig. 4(d).
Fig. 4. DEM of investigated area and corresponding interferograms for $C$-band. Nom. SNR = 12 dB and scene-scaling factor = 1.5: (a) original DEM, (b) interferogram derived by the conventional system (color cycle 2$\pi$), (c) interferogram of the three-antenna system for URM = 5 (color cycle 10$\pi$), and (d) absolute phase derived by the three-antenna system (after FEP subtraction) for URM = 15.

Fig. 5(a) reports the rms phase error of the new system for different values of URM and various nominal SNR, for $C$- and $X$-band, overall baseline 2.75 m, and unscaled DEM. The corresponding performance of the conventional system with the same overall baseline is shown by the horizontal lines. The average phase accuracy of the new system for low and medium URM’s (2–7), $C$-band, and nominal SNR = 12 dB, is higher than the conventional system thanks to the data fusion effect. This gain is more evident for higher SNR’s, while the effect is reduced slightly in $X$-band. When the rms phase error for the new system is lower than the conventional InSAR, the height estimates achievable by the three-antenna ML technique surely are better.

On the other hand, for higher URM’s, a degradation of the phase accuracy of the new system occurs. This is due to the increasing difficulty in solving the phase ambiguity, which determines the arising of catastrophic phase errors [18]. These particular errors are rare but large, and they assume values around multiples of 2$\pi$. The possibility of catastrophic errors is intrinsic in any problem of ambiguity reduction (see [25]). To give an intuitive physical explanation of this phenomenon in the framework of multibaseline SAR interferometry, we refer to the discussion on interferogram fusion in [34]. Although the estimator (4) makes use of the available complex data directly, with proper weights and by exploiting amplitude information, the underlying mechanism exploited for solving the ambiguity is the same as in the combination of phases separately estimated at different baselines discussed in [34]. As the value of the system interferometric phase $\varphi = \varphi_{13}$ increases, the graph of $\varphi_{12} = \varphi_{13} - \varphi_{12}$ (sensed at the intermediate baseline) versus $\varphi_{13}$ is a straight line passing through the origin. In terms of the wrapped phases $(\varphi_{12})_{\text{mod} 2\pi}$ and $(\varphi_{13})_{\text{mod} 2\pi}$, the straight line is folded within a $2\pi \times 2\pi$ square. In the absence of thermal and decorrelation noise, the point $(\varphi_{13})_{\text{mod} 2\pi}$ will lie on one of the line segments in the square. The presence of noise will cause the point $(\varphi_{13})_{\text{mod} 2\pi}$ to be shifted away from the line segments, and we will have to assign it to the nearest line segment and solve the ambiguity accordingly. Points that are assigned to a wrong line segment will show large errors, approximately of multiples of one cycle ("catastrophe" meaning drastic change). As an example, we can see the spikes of noise in Fig. 4(d) and the histogram in Fig. 9 showing errors mainly concentrated around $\pm 2\pi$ (the probability of catastrophic errors vanishes for increasing
Fig. 5. Phase accuracy: (a) rms phase error versus URM (C- and X-band, Nom. SNR = 6, 12, 18 dB and scene-scaling factor = 1) and (b) rms phase error versus scene-scaling factor (C-band, nom. SNR = 12 dB and URM = 2, 5, 7, 10, 15).

Fig. 6. Histogram of the spatial correlation factor at the baseline extremities α/l (C-band, scene-scaling factor = 0.1, 1, 1.5).

The previously discussed catastrophic phase errors at the sensor level do not necessarily imply a degradation of the final-height accuracy, as will be shown in the following. In fact, thanks to the high URM, the phase-unwrapping errors caused by foreshortening and phase aliasing surely are reduced [see the fringe widening in Fig. 4(c) compared to the conventional interferogram shown in Fig. 4(b)]. This may overcompensate for the possible degradation of the sensor level phase estimate. Moreover, it will be shown that the additional impulsive noise can be detected and filtered out, exploiting the benign characteristics of catastrophic errors (as envisaged by [34] and shown in an example in [15]). This is possible both when phase unwrapping is still necessary after the reduced-ambiguity phase estimation, and when absolute phase is estimated directly by the three-antenna sensor.

To analyze the phase estimation performance over steeper terrain, the same reference DEM with height multiplied by 1.5 has been used (height from 0 to 420 m), while the case of a relatively flat area is modeled dividing by the DEM height values by ten (height from 0 to 28 m), as in [13]. Fig. 5(b) shows the rms phase error of the new system as a function of the scene height scaling factor for different values of URM, nominal SNR = 12 dB, and C-band. The phase accuracy of the conventional system is reported for a comparison. For low and medium values of the URM, the rms phase error is slightly affected by the scene orography, as occurs for the conventional system. In contrast, the performance is quite influenced by the scene-scaling factor for high URM’s. The degradation of the phase accuracy of the new system, with increasing terrain steepness, is due to the increasing areas in which the signal decorrelation is high (because of the terrain surface approaching the orthogonality to the look direction [10]), and catastrophic errors arise. As an example, Fig. 6 shows the histogram of the spatial correlation for the overall baseline of the three-antenna system, which equals the baseline of the conventional one. The plots refer to C-band and different scene scaling. It can be seen that for increasing terrain steepness, the occurrence frequency of the low values of spatial correlation increases. The effect of the catastrophic errors illustrated above is more evident for high URM’s, where their amplitude can be larger. This should be considered with the increased dynamic 2URMπ of the phase estimate for high URM’s, compared to the conventional 2π range.

Note that the histogram in Fig. 9 actually does not refer to the sensor level phase errors, it shows the phase prediction residual error instead, that is the difference between the error of the sensor level phase estimate and the error of the phase prediction during the filtering process (see Section III). However, when this process performs well, so that the predictions are good, the histogram of the sensor level phase estimate exhibits the same qualitative behavior of the histogram of the phase prediction residual error. A more detailed analysis of the probability density function of the sensor level error of the three-antenna phase estimator can be found in [22]. It also is worth stressing that one should not expect errors around ±URM2π (viz., one ambiguity interval) in the sensor level estimate. These might arise after unwrapping the URM2π-modulus interferogram (although with very low probability, since the unwrapping of a reduced-ambiguity interferogram is reliable).
B. Blind Angles

Another advantage of the new system with respect to the conventional InSAR, is the reduction of the range of blind angles around the condition of complete foreshortening (look direction orthogonal to the terrain surface), for a same overall baseline. In Fig. 7(a), the maps of the completely decorrelated areas for the conventional and the three-antenna InSAR systems operating in C-band are superimposed upon one another. The white areas are completely decorrelated for both the systems, while the black areas are completely decorrelated only for the conventional system. In other words, black refers to cured decorrelation. The total area corresponding to the blind angle range of the conventional two-antenna airborne system is significant in highly steeping terrain condition (DEM scaled by a factor 1.5) despite the range of blind terrain slopes in the cross-track direction for the assumed parameters, and zero-azimuth slope is only about around the orthogonal direction to the look angle. The areas without useful information for the three-antenna system are reduced significantly. The range of blind angles has shrunk, and the operational capability of the new system is extended with respect to the conventional one. As a consequence, the interpolation errors are reduced. The number of completely decorrelated cells of the investigated DEM for different scene-scaling factors, with cell ground size of $25 \times 25$ m, are reported in Fig. 7(b) for the conventional InSAR and the three-antenna InSAR (URM = 5) system. The number of cells of the DEM is 16,900. With the three-antenna system, the areas without useful information shrink by a factor ranging up to 4.3.

<table>
<thead>
<tr>
<th>$s$</th>
<th>0.1</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ant.</td>
<td>0</td>
<td>56</td>
<td>412</td>
</tr>
<tr>
<td>3 ant.</td>
<td>0</td>
<td>13</td>
<td>130</td>
</tr>
</tbody>
</table>

Fig. 7. Blind angles for C-band: (a) map of decorrelated areas (black: only for conventional InSAR, white: for both the systems; URM = 5, scene-scaling factor = 1.5) and (b) number of decorrelated cells ($25 \times 25$ m) versus scene-scaling factor (URM = 5).

C. Height Accuracy and Design Tradeoffs

We have seen that the phase accuracy of the new system is generally better than the conventional one, but in the case of high URM’s, the effect of locally low SNR and high spatial decorrelation can produce an rms phase error on the scene that is higher for the new system. Thanks to the reduced ambiguity, the phase-unwrapping errors are reduced compared to the two-antenna system. In the following, we analyze how the reduction of phase aliasing and the fringe widening (especially in conditions near to complete foreshortening) compared to the conventional system produce better final-height estimates after the phase unwrapping, despite a possible lower phase accuracy over the scene. Optimization of the thresholds of the confidence test in the unwrapping procedure and of the array asymmetry factor $p$ also is analyzed.

Fig. 8(a) shows the height accuracy of the new system as a function of the URM for different values of the thresholds ($t_u = t_p$). C-band, and nominal SNR = 12 dB for the scene scaling = 1. The height accuracy of the conventional system with optimized thresholds ($t_u = t_p = 1.75$ rad) also is reported. For low thresholds ($t_u = t_p = 1$, 1.5 rad), the height accuracy of the new system is always better than the conventional system. Moreover, the height error depends slightly on the URM. For medium threshold values ($t_u = t_p = 2.5, 3.5, 5$ rad), we can achieve even higher height accuracy.
for medium URM's. Finally, for \( t_u = t_p = 7.5 \text{ rad} \), a well-defined optimum URM appears. These results can be justified as follows. For low values of URM (\( \times 2 \)), the new system capability of reducing the drawbacks of foreshortening and phase aliasing compared to the conventional InSAR is not yet high. So, the thresholds must be low (1.5 rad) to avoid errors in the unwrapping procedure, as for the two-antenna system. The gain in height accuracy is due mainly to the enhanced phase accuracy. It is worth noting that the gain of height accuracy over the scene is lower than the corresponding phase gain (the two gains do not have to be necessarily the same; this is due to the fact that the coefficient that relates the local phase and height errors, apart from unwrapping errors, varies over the scene, depending on look angle and slope [27]). On the other hand, for medium values of URM (\( \times 5 \), \( \times 7 \), and \( \times 10 \)), the new system has a high potential of reduction of the phase aliasing, which can be exploited only by relaxing the unwrapping thresholds (2.5, 3.5, 5 rad). However, for thresholds that are too high (5, 7.5 rad), the catastrophic errors in the phase estimate for high URM are not filtered out by the unwrapping procedure, so that large height errors can occur.

This behavior can be understood better by looking at the Fig. 9, showing the histogram of the phase prediction residual error in the unwrapping procedure for \( C \)-band, nominal SNR = 12 dB, URM = 15, and scene scaling = 1. The prediction residual error is the difference between the predicted phase from the surrounding unwrapped pixels and the measured phase (unwrapped in a tentative way) [17]. It can be seen that by setting the thresholds of the confidence test at 3.5 rad, we can avoid accepting phase estimates affected by catastrophic errors, while by choosing thresholds equal to 7.5 rad, the catastrophic errors cannot be filtered out. Fig. 8(a) also shows that the optimum pair thresholds-URM is given by \( t_u = t_p = 5 \text{ rad} \) and URM = 7 (although URM = 5 and 10 produce almost the same performance). This corresponds to an rms height error of 1.86 m. Compared to the rms error of 2.93 m for the conventional system, this results in a height accuracy gain of 1.57. These optimal system settings derive from the tradeoff between reduction of the phase-aliasing effect and decrement of the phase accuracy for increasing URM.

With the highest URM (\( \times 15 \)), the phase after the FEP removal is not wrapped at all [see Fig. 4(d)], so the phase to height conversion can be performed without unwrapping and possibly, without the knowledge of GRP's. The phase accuracy is now quite degraded [see Fig. 5(a)] by the catastrophic phase errors. However, these still can be detected and filtered out with thresholds lower than 5 rad, so that a good height accuracy results (still better than the conventional system).

The results of the same analysis for \( X \)-band are reported in Fig. 8(b). Analogous system behavior for varying thresholds can be noticed. As a contrast, the optimized performance for the two-antenna system is significantly worse than for \( C \)-band (12.1 m). This is due mainly to the difficulty in the unwrapping arising from increased phase aliasing and foreshortening effects. On the other hand, unwrapping problems do not affect the three-antenna system performance thanks to the reduced ambiguity for medium and high URM's. As a result, for the optimum value of URM (\( \pm 7 \)), the increased interferometer sensitivity in \( X \)-band produces a slightly better accuracy (1.75 m) than in \( C \)-band, compensating the increased decorrelation [see the phase degradation in \( X \)-band of Fig. 5(a)]. The resulting maximum gain (optimal URM) of the three-antenna system over the conventional one is 6.93. This is due to the poor operation of the two-antenna system in \( X \)-band, as opposed to the flexibility of the new system.

### D. Analysis for Varying Steepness

In the previous section, we analyzed how the reduction of phase aliasing and data noise, operated by the three-antenna system, results in a height accuracy gain on the conventional one. In the following, we investigate the height-estimation performance of the two systems for different terrain steepness. Fig. 10(a) shows the rms height error of the new system as a function of the scene height-scaling factor for different values of URM, nominal SNR = 12 dB, and \( C \)-band. The performance of the two-antenna system is reported as a comparison. The best achievable performance of both systems is reported by using confidence thresholds optimized for each height-scaling factor, whose values are in Table I. This performance analysis is obtained under the assumption that unwrapping procedure can be optimally tuned for a given scenario. This assumption will be relaxed afterwards.

One can see that the benefits of the three-antenna system in \( C \)-band are more sensible for lower and higher steepness than the previous analysis. The optimal URM ranges from five to ten, as for scene scaling = 1. Moreover, absolute phase retrieval is still obtained for URM = 15 with satisfactory accuracy (better than the conventional system). The maximum achievable gain is from 1.28 to 1.57.

As seen in Fig. 8(b), the advantages offered by the three-antenna system are enhanced in \( X \)-band [see Fig. 10(b)]. It is evident that the conventional system performance is degraded for increasing scaling factor, while the new system exhibits an rms height error that is affected to a lesser extent by the varying terrain steepness. In particular, for scaling factor = 1.5, the two-antenna system cannot operate correctly while the three-antenna InSAR still performs well. This is due to the unwrapping errors that arise in the conventional system, starting from scene scaling = 1 and becoming dramatic for
scaling = 1.5. In contrast, the unwrapping difficulties are resolved in the new system for URM higher than two. In X-band, a more defined optimal URM can be noticed than in C-band, especially for high steepness. This can be justified by the higher weight of the phase-aliasing effect in the tradeoff between reduction of phase unwrapping errors and increment of phase accuracy. The maximum achievable gain is for scene scaling = 0.1 (mainly due to the data fusion effect) for scaling = 1 and becoming very high (12.8) for scaling = 1.5 (when the two-antenna system cannot operate well). Moreover, as seen in Fig. 8(b) for scaling = 1, for scaling = 0.1 the increased interferometer sensitivity in X-band produces in the new system a better accuracy (1.16 m) than for C-band (1.62 m), compensating the increased decorrelation. Instead, the conventional system performance is reduced in X-band.

### E. Robustness Against Nontuned Unwrapping

So far, the height accuracy analysis has been carried out by employing the unwrapping procedure optimized for the topography under investigation. This has been useful for a fair comparison of the height performance of the new and the conventional systems. In such a way, the best achievable performance of the two systems has been evaluated for varying terrain steepness. The results have shown the performance improvements of the three-antenna InSAR compared to the conventional one, especially in X-band. However, in actual applications, it is not possible to select the confidence thresholds optimally. As a consequence, an analysis of the robustness of the confidence threshold settings is necessary, and the height performance must be evaluated for mismatched thresholds. To this purpose, we have employed fixed thresholds optimized for scene scaling = 1 (see Table I), and we have evaluated the resulting height accuracy of both the systems for the other scalings.

The results are shown in Fig. 11 for C- and X-band, nominal SNR = 12 dB, and URM = 7. In C-band, the height performance of the conventional system for high steepness is degraded compared to the case of optimized thresholds [11.9 m instead of 4.45 m, see Fig. 10(a)], while the accuracy of the three-antenna system is affected only slightly (3.68 m instead of 3.50 m). In X-band, the thresholds used for the conventional system are mismatched for scene scaling = 0.1, so the accuracy degrades from 2.91 to 7.03 m, while the three-antenna system performance slightly degrades for scene scaling = 1.5 (from 3.77 to 3.84 m). This analysis is not exhaustive, but it is certainly representative. The results show that the performance of the new system is slightly sensitive to mismatched thresholds used in the unwrapping procedure, while the conventional InSAR accuracy is affected heavily when the unwrapping is not properly tuned. This behavior derives from the quite broad optimum for the threshold settings of the three-antenna system [see Fig. 8(a) and (b)], as opposed to a well-defined optimum value of the threshold settings for the two-antenna InSAR. As a consequence of the higher robustness against mismatching of the three-antenna systems, the accuracy gains achievable on the conventional system in realistic conditions are higher than the ones computed for the two systems operating at their best. For URM = 7, the height-accuracy gain in C-band for optimized thresholds ranges from

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**Table I**

Optimal Thresholds (rad) Versus Scene-Scaling Factor. Nom. SNR = 12 dB. (a) C-Band and (b) X-Band

<table>
<thead>
<tr>
<th>Scaling Factor</th>
<th>Conv.</th>
<th>URM=2</th>
<th>URM=5</th>
<th>URM=10</th>
<th>URM=15</th>
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<td>1.75</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
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<td>s = 1</td>
<td>2.5</td>
<td>1.5</td>
<td>5.0</td>
<td>2.5</td>
<td>2.5</td>
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<tr>
<td>s = 1.5</td>
<td>2.5</td>
<td>5.0</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
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<td>1.25</td>
<td>3.5</td>
<td>2.5</td>
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<td>3.5</td>
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</tr>
<tr>
<td>s = 1.5</td>
<td>2.5</td>
<td>3.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

(a)

(b)
1.28 to 1.57, while for fixed thresholds, the gain is from 1.52 to 3.22. In X-band, the gain for no mismatch is from 2.51 to 12.8, while for mismatched thresholds the gain ranges from to 6.06 to 12.6.

The analysis of the benefits produced by the three-antenna system compared to the conventional interferometry has been carried out assuming same baseline lengths. A theoretical system comparison for different baselines has been carried out in [19] for URM = 2. The analysis has verified the gain of the three-antenna system with respect to conventional interferometry both with the same overall baseline and halved baseline. This confirmed the intrinsic flexibility of the new system compared to the single baseline InSAR. Similar results also have been obtained with the proposed simulator, including the effect of phase unwrapping errors, by using shorter (2 m) and longer (4 and 5.5 m) baselines for the two systems operating in C-band. The results show that the gain of the three-antenna system on the conventional system after tuning of the baseline lengths, is modified only slightly. A thorough analysis for an overall baseline of the new system different from the baseline of the two-antenna system assumed for performance comparison is out of the scope of this paper and is left as a matter of future research.

V. CONCLUSIONS

In this paper, a numerical simulator for performance analysis of the reduced-ambiguity, three-antenna InSAR system with adaptive ML pixel-based data processor has been presented. An analysis on given scenarios has been carried out in terms of phase and height accuracy, assuming a single-pass airborne platform, for different transmitted wavelengths and system configurations, including a region-growing unwrapping algorithm in the processing chain. The optimization of the three-antenna array configuration and of the unwrapping procedure also have been investigated, and a performance comparison with the conventional two-antenna interferometry has been carried out.

Results show that the new system can produce elevation maps significantly more accurate, through the multibaseline data fusion and the reduced unwrapping errors. In particular, the gain due to multibaseline data fusion turns out to be sensible despite the typically short airborne baseline. Moreover, it has been shown that the possibility of filtering out, in the unwrapping procedure, possible catastrophic errors of the phase estimates arising in the reduced-ambiguity system for low SNR and high decorrelation. As a consequence, absolute phase retrieval also can be achieved with satisfactory accuracy. The effectiveness of the three-antenna acquisition system and of the ML data processor for typical airborne platform and mission parameters has been assessed, especially for steep terrain and X-band. Finally, it has been shown that the gain of the height accuracy of the three-antenna system is particularly significant in actual operative conditions, when the tuning of the unwrapping procedure may not be practicable thanks to the higher robustness of the three-antenna system to mismatched unwrapping parameter settings.

Disadvantages of the three-antenna system are additional hardware (third antenna and receiving channel) and data processing (third image focusing). It is worth noting that the additional hardware is less than in a multifrequency system. This is an alternative approach to improve performance [34], [16]. The additional data processing should not be a problem given the continuous progress of DSP technology. Difficulties could arise in the installation of the additional antenna on the platform, including lack of room for low values of p. Future work will address the practicability of obtaining high phase ambiguity reduction without difficulties in the array implementation, by using different forms of the array asymmetry factor [18], [21], while achieving similar good height accuracy. Moreover, the increment of the accuracy gain on the two-antenna system that is achievable by employing more than three phase centers will be investigated.

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REFERENCES


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