Auctions with Arbitrary Deals

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Abstract. To come to a deal, a bargaining process can sometimes take a long time. An auction may be a faster, but existing auction models cannot cope with situations where money is not an issue, or where it is difficult to express the utility of all participants in a monetary domain. We propose a modified Vickrey auction based only on preferences over the possible bids. This approach also allows for situations where a bid is not just a price or some fixed set of attributes, but can be any possible offer. We prove that in this flexible, generalized setting, the Vickrey mechanism is still incentive compatible and results in a Pareto-efficient solution.

1 Introduction

In some trading situations, the discussion about a deal does not concern the price, but only other qualitative attributes. The price may be fixed, or money might not play an important role. A common approach is to map all such attributes to a cost value, leading to direct relation between these attributes and price. However, it can be difficult to express qualitative attributes in price. For example, a preference for a certain color, or for a certain befriended relation can be difficult to formulate as a price. In such situations, one may try to use a negotiation technique to find the right attribute values, but negotiation methods tend to take a long time [1]. Auctions can be more efficient (especially one-shot auctions) [2][3], but they rely on utility functions. The main contribution of this paper is a model and some auction protocols for which utilities do not need to be expressed in money anymore; only a preference order is required.

To illustrate the basic ideas behind the model and the mechanisms, we use an example where a travel agency specializes in the organization of corporate day-trips. Corporate day-trips are team-building events, where employees of a company engage in some group activities of leisure. The kind of activity actually chosen depends on, among others, the type of people, their number, and on the budget. In our example, the budget is fixed, and the travel agency has to provide the best choice of activity for that given amount. Suppose there is a company of approximately 40 persons, but not all of them can participate on the same day. The two most popular days are a Tuesday (with 30 people) and a Wednesday (with 35 people). Unfortunately, the preference over the activities of choice is different in the two groups. The people who are available on Tuesday prefer indoor skiing, while those available on Wednesday, being of older age, would rather visit a museum. The company wishes to take as many employees as possible to the day-trip, so there is a slight preference towards the museum visit on Wednesday.

One could try to model this case as a negotiation scenario, or alternatively as a multi-attribute auction, where the attributes cover all possible dimensions of a deal [4][5]. In the latter setting, the attributes in our case would be the type of activity, the number of participants, the date, and whether an additional lunch is included or not. The company should provide an evaluation function that expresses the value of a certain bid in money. Day-trip service providers would then submit bids in the predefined format of higher and higher value. Given the fixed budget, the combination that has the highest value would be selected in the end. If the company decides to follow the Vickrey auction protocol [6], then the competing day-trip service providers should submit one closed bid, of which the one with a higher value is selected as winner. The service provider submitting the winner bid can then provide the day-trip service, and the value of the provided service should be equal to the highest non-winning bid. This allows the winner to choose a combination of attribute values that maximizes his utility. In such traditional approaches, like at Parkes and Kalagnanam [7], the utility of the winner bidder and the buyer is supposed to be opposing in all attributes. The buyer should want higher quality and lower price, while the service provider prefers to sell low quality for high price. In reality, however, the preferences may have a more complex relation: sometimes opposing, sometimes being the same. Moreover, since the attributes have to cover all possible offers, it might very well be, that certain attributes are meaningless for the winner (for example he can never offer additional lunch).

The model we introduce in this paper generalizes the traditional multi-attribute auctioning model in two ways. It does not assume that the bidders and the auctioneer have fully opposing preferences, and it does not require that all bids have the same dimensions. These changes make modeling of some real-world cases possible (and others easier), while certain properties of the original model and mechanisms stay valid.

In Section 2 we will introduce our model as a generalization of the traditional auctioning model. This is exemplified by showing how the preferences of the company and the tour operators can be represented. Then the applications of different auction protocols to this model are discussed (Section 3), along with the decisions made by the service providers and the company in the example. Section 4 analyzes the Vickrey mechanism, and proves that the most elementary properties of a Vickrey mechanism still hold in our general model. Advantages, disadvantages and consequences of the proposed model are discussed in Section 4, related work can be found in Section 5, and concluding remarks in Section 6.

2 Model

Let us consider the situation where an auctioneer sells a certain item. Each bidder has a set of possible *bids* he can offer to the auctioneer. Let β be the set of all possible bids and $\beta_i \subseteq \beta$ be the subset of bids that bidder i can offer. In the process of setting the deal, each bidder i sends bids to the auctioneer from β_i. In the general model presented in this section, a bid is not necessarily a price, but can be any description of a possible deal. The auctioneer prefers some of the offered bids over others, and he uses his preferences to select the winning bid.

A bid ($b \in \beta$) may consist of simply the price, in which case the payment of the winner depends on the set of submitted bids. In a more complex case, when bids consist of multiple attributes, it is common to assume a valuation function that can convert a combination of attribute values to a price. The payment is then again dependent on the value of the other submitted bids. This valuation function plays a central role in the different auction models: it converts the bids to money.

In contrast to this traditional auction model [2], we suggest to generalize the selecting the "payment" to selecting a bid. The consequence of this change is that there is no need to have a valuation function to express the values in money. It is possible to select one of the bids as a contract even without assigning a monetary value to it. We show that the same selection rules can be used as in the original algorithms.

In the traditional model, utility functions of the players use the valuation function to express the utility in money: $U(p) = V - p$, where V is the valuation of the good, and p is the price. In a multi-attribute case, it is $U(a_1, \ldots, a_n, p) = V(a_1, \ldots, a_n) - p$, where $a_1, \ldots, a_n$ are the attributes and p is the price. In our model, the utility function expresses a total *order* over the possible bids and by that over the bids for any participant. Such preference orders make the distinction between more and less preferred bids just as the traditional utility functions do. But in this case, the bid is not expressed in money to make this comparison, allowing us to hold an auction without any money involved. The utility function can be defined as a mapping from bids to integer numbers: $U : \beta \to \mathbb{Z}$. Such a function expresses a preference order as follows: for any two bids c_1, c_2, $c_1 \preceq c_2$ if and only if $U(c_1) \leq U(c_2)$.

A bid in the simplest form contains only the price of a well-defined good or service. Alternatively, a bid may define multiple attributes of a deal, for example the multiple attributes of the good on auction, plus the price. From the most abstract point of view a bid defines everything both parties have to deliver to fulfill the deal. In principle, price is simply one of the attributes of the deal, not even indispensable as existing auction models may let us think. The utility function as we defined it is indifferent of the actual form of the bid. As a consequence, the submitted bids may consist of different attributes.

Even though the structure of the possible bids may be different, the preference orders induced by the utility functions put the bids in a well-defined order (per agent). It is also possible that some bids are equally preferred, thus the utility function assigns the same integer value to different bids. We call such a set of equivalent bids an *equivalence class*. Especially in a multi-attribute setting where some attributes have a continuous domain, there are often infinitely many of such equivalent classes, having often infinitely many members. Consequently, given a utility value, the bid is not uniquely determined.

In the price-only case a utility value of zero means that the bidder (or the auctioneer) neither gains nor loses on the deal. The price level that makes the utility to be zero is called the *private value*. Analogously, we define our utility function to be zero for bids that are indifferent for the player (either a bidder or the auctioneer), positive for bids that are desired by the player, and negative for bids that he will never agree to. The class of bids that belong to utility zero we call the *private class*.

In the corporate-day-trip example, we assume two competing day-trip-service providers: $bidder_1$ and $bidder_2$. Our company asks the travel agency (the *auctioneer*) to find the best option given the preferences. The bidders have different contracts to offer. $Bidder_1$ has good connections to an indoor-ski facility. He can offer the indoor skiing activity any day for any number of persons, and due to the good business relations he can also offer lunch at the skiing facility. In terms of museum, he can only offer the standard museum visit activity without lunch. $Bidder_2$ is in a similar situation, except that he has a good connection

Table 1. Preference orders of the bidders and the auctioneer over the possible contracts in decreasing order. Equivalence classes are grouped by the same shade of gray.

$Bidder_1$

	activity	# people	date
b_6^1	ski	30	any
b_5^1	ski	35	any
b_4^1	museum	30	any
b_3^1	ski + lunch	30	any
b_2^1	museum	35	any
b_1^1	ski + lunch	35	any

$Bidder_2$

	activity	# people	date
b_6^2	museum	30	any
b_5^2	museum	35	Tuesday
b_4^2	ski	30	any
b_3^2	museum + lunch	30	any
b_2^2	ski	35	Tuesday
b_1^2	museum + lunch	35	Tuesday

Auctioneer

	activity	# people	date
b_2^1	museum	35	any
b_1^1	ski + lunch	35	any
b_3^1	ski + lunch	30	any
b_5^1	ski	35	any
b_2^2	ski	35	Tuesday
b_6^1	ski	30	any
b_4^2	ski	30	any
b_1^2	museum + lunch	35	Tuesday
b_3^2	museum + lunch	30	any
b_5^2	museum	35	Tuesday
b_6^2	museum	30	any
b_4^1	museum	30	any

to museums, so he can offer the museum visit in connection to lunch, while the skiing stays without lunch. Unfortunately, the museum restaurant has booked a lot of guests for Wednesday already, therefore on that day only 30 persons are accepted. The possible contracts of $bidder_1$ and $bidder_2$ are listed in Table 1 in separate tables in decreasing order of preference. Just as the preference order of the auctioneer, which, however, contains all possible bids. The auctioneer, on behalf of the company, prefers skiing on Tuesday, museum on Wednesday, having lunch, and prefers contracts that incorporate more people (even if there are less participant expected on a certain day, because it gives them more flexibility in accepting more people last minute). In this example, the private class of $bidder_1$ is $\{b_1^1, b_2^1\}$, that of $bidder_2$ is $\{b_1^2, b_2^2\}$ and that of the auctioneer is $\{b_4^1, b_6^2\}$.

To properly see the difference of the proposed model and the traditional auction models, let us make two observations. Firstly, in our model, the preference orders of the auctioneer and the bidders are not opposing. Usual utility functions are required to be opposing, but this does not always hold in practise. Secondly, we do not assume that the bids submitted by the bidders are of the same type. Every bidder can offer the type of contracts he likes, and there is no need to define utility functions that expect values of attributes that are not interpreted by the bidder. With these extensions our model is capable of describing trading scenarios that are more realistic than those described by the traditional models.

Having seen the model, in Section 3 we show how one-shot auctions can be extended to handle arbitrary bids.

3 A One-Shot Mechanism

An auctioneer can choose between many different auction protocols to find the best deal in a set of possible bids. The English (best bid, ascending) or Dutch (best bid, descending) auctions are popular in practice. The one-shot Vickrey offers several theoretical advantages, and therefore it is popular among scientists. In the following we show how a Vickrey auction protocol fits in our auctioning model and illustrate its workings by our corporate-day-trip example.

3.1 Generalizing the Vickrey Auction

A Vickrey auction is a one-shot mechanism, where every bidder submits only one bid. The auctioneer selects the best bid according to his preference order and the bidder who submitted that bid is the winner. In case there are more bids that are best, a random choice is made. Since the auction is a second-best bid mechanism, the deal is not defined by selecting the winning bid. For the case when bids consist of a single price, Vickrey has suggested that the second-best price should define the deal [6]. Along the same lines of thoughts, we propose that in the case when bids may consist of any attributes, the protocol consists of the following steps:

1. Every $bidder_i$ selects the bid b_i^0 that is highest according to the auctioneer's preference order $\preceq_a$, but not lower than the bid(s) in his private value class C_i^0: $b_i^0 = \max_{\preceq_a}\{b \mid C_i^0 \preceq_i b\}$. This bid is sent to the auctioneer.
2. The auctioneer selects the bid that scores highest in his preference order as the winning bid. The bidder to whom this bid belongs is the winner of the deal. In case of more than one equally preferred bids, one is chosen at random as the winning bid, and another as the second-best bid.

3. The contract attributes are defined by the auctioneer's equivalence class of the second-best bid b_2. The winner $bidder_w$ selects the bid b_3 that scores highest according to his own preference order, and does not score lower than the bid(s) in the equivalence class of this second-best bid in the auctioneer's order, i.e., $b_3 = \max_{\preceq_w}\{b \mid b_2 \preceq_a b\}$. If the winner has bids that are equal to b_3, then the bid (b_3') that scores highest in the auctioneer's preference order is selected ($b_3' = \max_{\preceq_a}\{b \mid b_3 =_w b\}$).

In the example, the private class of $bidder_1$ consists of the offer of skiing and lunch for 35 persons and the offer of museum for 35 persons ($C_1^0 = \{b_1^1, b_2^1\}$). The private class of $bidder_2$ also contains two offers, both on Tuesday for 35 persons: museum and lunch or skiing ($C_2^0 = \{b_1^2, b_2^2\}$). The two submitted bids are the museum for 35 persons (b_2^1) and the skiing for 35 persons on Tuesday (b_2^2). Since the auctioneer prefers the museum for 35 persons (b_2^1) over skiing with 35 persons on Tuesday (b_2^2), $bidder_1$ is the winner. To set the deal, $bidder_1$ has to select a bid that is not worse for the auctioneer than the equivalence class with contracts for skiing with 35 persons on Tuesday or any other day ($\{b_2^2, b_5^1\}$). Since $bidder_1$ does not have a bid that is better for the auctioneer as well as for himself, the contract made will be skiing with 35 persons (b_5^1).

If all the usual assumptions of a price-only auction holds (especially the pseudo-linear utility functions), this mechanism reduces to the well-known Vickrey auction protocol. In that case equivalence classes consist of only one element, and the winner does not have a choice. In case of the more general preference orderings, however, it is possible that the auctioneer's equivalence class of the second-best bid consists of several possible bids, which are equivalent for the auctioneer, but may make a difference for the winner bidder. By allowing the winner to choose one of these bids a better deal can be made.

Other auction protocols can be generalized in a similar manner. In the Dutch auction the auctioneer announces contracts from more preferred toward less preferred contracts until a bidder stops him. The bidders consider every bid based on their preference orders and private classes. Similarly in the English auction protocol bidders use their own and the auctioneer's preference order to always submit better bids. The auction stops when there are no new bids. It is easy to see (via playing an example auction in all three protocols), that the outcome of the English and the Vickrey protocol is the same, while that of the Dutch is different, just like in traditional model.

The main advantage of the original Vickrey protocol is that it is *incentive compatible*. That is, the dominant strategy of the bidders is to bid according to their private value. We will prove that this property also holds for our modified auction mechanism, thus their dominant strategy is to submit a bid according to their private class.

4 Properties

An auction mechanism is called *optimal*, if the utility of the auctioneer is maximized by the deal it provides. In general it means that the deal is defined by the private value that is most preferred by the auctioneer. Vickrey mechanisms are not optimal, because the deals they result in are defined by the second-most preferred private value. This is true regardless of the existence of any valuation function, also in our case.

Multi-agent researchers are usually concerned about building systems that provide *Pareto-optimal* or *Pareto-efficient* solutions. A deal is Pareto efficient if it cannot be changed in a way that it provides higher utility for one party while not decreasing the utility of the other one.

Proposition 1. *The deals provided by the Vickrey mechanism in the proposed general auctioning model are Pareto efficient.*

Proof. Pareto efficiency follows from the last step in the auction algorithm. It prescribes that the winner chooses a deal that is the best of the bids that are better for him than the second best bid, but not worse for the auctioneer. This ensures that it is not possible to have a deal that is better for the winner, but not worse for the auctioneer.

Due to the selection of b'_3 over b_3 it is ensured that the auctioneer cannot have a better deal without violating the winner's preferences. If there was a b_4 that is not worse than b'_3 for the auctioneer ($b'_3 \preceq_a b_4$), but better for the winner ($b'_3 \prec_w b_4$), then in the final step of the protocol b_4 would have been chosen instead of b_3.

Note that in case of a price-only auction, the equivalence classes always contain only one element, therefore the last step of the protocol does nothing.

Beside Pareto efficiency, *incentive compatibility* is one of the most important properties of Vickrey mechanisms. Incentive compatibility means that if the bidder submits a bid according to his private class, then his expected utility is not less then in case of any other bid. This is equivalent to saying that the dominant strategy of the bidders is to bid according to their true valuation.

Proposition 2. *The Vickrey mechanism in the proposed general auctioning model is incentive compatible.*

Proof. The incentive compatibility of the traditional Vickrey mechanism originates from the fact that deviation from the true value either decreases the chance of winning the auction without increasing the expected price, or increases the chance of winning the auction while decreasing the expected price due to a risk of paying more than the true value.

Similar reasoning holds for the Vickrey mechanism in our model. According to this protocol, a $bidder_i$ should submit the bid b_i^0 that is highest according to the auctioneer's preference order, but not lower than the bid(s) in the his private value class C_i^0 (we repeat from the protocol description: $b_i^0 = \max_{\preceq_a}\{b \mid C_i^0 \preceq_b b\}$). Bidders can deviate from this protocol in two ways.

1. The submitted bid b_1 can be higher than (or in) its private value class, but *not the highest according to the preference order of the auctioneer* a, i.e.,

$C_i^0 \preceq_i b_1$ and $b_1 \preceq_a b_i^0$. In this case the bid b_1 has a smaller chance of winning, and if it wins, the bid values are not going to be better than they would be when the truthful bid b_i^0 was submitted, because those are based on the same second-best bid.

2. The bidder may also choose to submit a bid b_1 that he prefers *less than the bids in his private value class* C_i^0, i.e., $b_1 \preceq_i C_i^0$. If the bidder then still loses, this is clearly not a good idea. However, if the bidder wins it may select a bid b_3 based on the second-best bid $b_2 \preceq_a b_3$. We show that even then, the bidder is not better off than with bidding b_i^0. Consider the following proof by contradiction.

   Suppose that the bidder is strictly better off bidding this bid b_1 than bidding b_i^0. In that case, there should be a bid b_3 for which the following holds: $b_2 \preceq_a b_3$, because it must be at least as good as the second-best bid b_2, and also $C_i^0 \preceq_i b_3$, because otherwise the bidder would have a bid with less than zero utility. Clearly, the bid b_3 lies in the set $\{b \mid C_i^0 \preceq_i b\}$, and since b_i^0 is the maximum of this set according to the auctioneer's ordering, it holds that $b_3 \preceq_a b_i^0$. With $b_2 \preceq_a b_3$ it thus holds that $b_2 \preceq_a b_i^0$. Thus b_i^0 is also higher than the second-best bid, and if the bidder had followed the protocol, it would also have won, and have been allowed to choose a bid based on b_2. Contradiction. Consequently, a bidder is not better off by bidding lower than its private value class.

Since a bidder is never better off by not bidding according to the protocol, it is a weakly dominant strategy for the bidders to do so. Consequently, the modified Vickrey mechanism is incentive compatible.

5 Related Work

Auctions are well-studied allocation mechanisms that are used in a wide range of areas from e-markets [8][9] to resource allocation [10]. The Vickrey auction mechanism [6] is particularly interesting for use in multi-agent systems because it possesses several desired properties like incentive compatibility and efficiency. However, the Vickrey mechanism has also certain limitations. Humans tend not to use this auction very often, because the auctioneer needs to be fully trusted, people's private values in real-life scenarios are often not (completely) private, or because it gives a lower revenue for the auctioneer. Sandholm [11] also showed some limitations of the Vickrey auction for computational agents. For example, when agents have local uncertainty, or when several auctions are held for items that are interrelated.

The situation where such multiple related items are auctioned is studied as a so-called combinatorial auction [12]. In this case a combination of bids is selected as a winner. The resulting *winner selection problem* is NP-hard, and the focus of some interesting research (see e.g. [3]).

Another extension of the original (single-issue) auction model is to allow bids to contain multiple attributes beside the price. In such auctions, an item with configurable attributes is auctioned. The bidders submit bids that specify the attribute values and the price. Che analyzed an auction mechanism with two attributes: price and quality [13]. He compared different second-price methods and concluded that the one where the winner can select a deal in the end (in contrast to implement exactly the second-best bid) can implement an optimal trade. We generalize this result for non-opposing preferences that order arbitrary bids.

Later David, Azoulay-Schwartz and Kraus extended Che's work to more dimensions, and analyzed the mechanism from the auctioneer's point of view (revenue maximization, rather than efficiency) [5]. In their case utility functions also express the value of bids in money. Although the value of a certain bid might be different for every bidder, these utility functions represent the same preference order for every bidder.[3] In this paper, we generalize their setting on two levels. On the conceptual level, we generalize the notion of bids to not only contain price versus a good or price versus attributes of a good, but to contain arbitrary attributes of a deal. On the practical level this means that the bid selection is not based on money, so we do not require the existence of utility functions that convert all attributes to money. Another difference is that we allow the utility functions of the bidders to represent different preference orders.

Recent research on multi-attribute auctions includes the work of Parkes and Kalagnanam [7], who have introduced iterative mechanisms for multi-attribute Vickrey auctions with pseudo-linear and non-linear utility functions. Teich et al. have recently extended their earlier work on multi-attribute e-auctions by a bidder-support module that can suggest new value combinations to help the bidders to elicit the auctioneer's utility function [4]. For a general overview of existing work on multi-attribute auctions, please see the review paper of Teich et al. [14].

6 Discussion

In this paper we have introduced a generalized model of auctioning that can handle bids with different, possibly non-monetary, attributes submitted by bidders who may have different preferences. The preferences in the model are expressed as orderings instead of the usual utility functions. We have shown how a standard Vickrey auction needs to be modified to cope with arbitrary bids and not-strictly opposing preferences, and have proved that the mechanism is still incentive compatible and that it is Pareto efficient.

An advantage of preference orders over multi-attribute utility functions is that the bids do not have to consist of the same attributes. It is possible to

[3] Actually the requirement is that the utility functions of the bidders should be diametrically opposed to the utility function of the auctioneer. In terms of the order this is equal to requiring strictly opposing orders. Since all bidders have to have a strictly opposing order to the auctioneer's, their orders are then necessarily the same.

order different kinds of bids because preference orders in general do not depend on the structure of the bids.

Another advantage is that here, in contrast to using a utility function, we do not have to express the value of a bid in money. Sometimes it might be difficult to express certain attributes in money, while a preference order can still be defined. In case the attributes include monetary as well as non-monetary attributes it is possible to mix the preference order model with the traditional model. If a valuation function exists that can summarize the value of some of the attributes, then this summarized value can substitute the attributes it is derived from. Then the preference orders have to consider this single value instead of the multiple original attributes.

A practical issue with the new auctioning model is the representation of the preferences. The auction protocols assume that the preference order of the auctioneer is known by the bidders. How such orders can be expressed in a compact form, however, is left for future work.

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