Self-focusing of intense laser beam in magnetized plasma

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In this paper, evolution of spot size of an intense laser beam in cold, underdense, magnetized plasma has been studied. The plasma is embedded in a uniform magnetic field perpendicular to both, the direction of propagation and electric vector of the radiation field. Nonlinear current density is set up and the source dependent expansion method is used to determine the evolution of the spot size of a laser beam having a Gaussian profile. It is shown that transverse magnetization of plasma enhances the self-focusing property of the laser beam leading to reduction in critical power required to self-focus the beam. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2357715]

I. INTRODUCTION

Interaction of high power laser fields with ionized plasma is important for many applications including laser fusion,[1,2] laser wakefield acceleration,[3,4] and x-ray lasers.[5,6] At high laser intensities nonlinear interactions between plasma and laser become significant. The increasing use of high power laser beams in various applications has aroused worldwide interest in laser-plasma interaction in the nonlinear regime.

The interaction of intense laser pulses with magnetized plasma is an important and relatively new area of study. It has been seen that intense magnetic fields are generated in the laser plasma interaction.[7] These fields affect the propagation of laser pulses in plasma since the canonical momentum for magnetized plasma interacting with radiation is not conserved, as in the case of unmagnetized plasma. This finds application in the fast ignition scheme in inertial confinement fusion (ICF),[8] where quasistable self-generated magnetic fields may be present in the underdense corona region close to the critical surface of the ignition pulse. Nonlinear interactions,[9] wake excitation and its reaction on nonlinear evolution of laser pulses[10] and modulation instability[11] are some of the processes that have been recently studied for intense laser pulses interacting with magnetized plasma. When a laser pulse propagates through plasma embedded in a uniform magnetic field the plasma electron motion will be modified due to the magnetic field and will give rise to changes in the dispersion of the laser beam and nonlinear current density. The new contribution to the source driving the laser beam in magnetized plasma is expected to significantly affect the self-focusing[12–14] property of the laser beam.

In the present paper, we have analyzed for the first time, the effect of the magnetic field on self-focusing property of an intense laser pulse propagating in a cold, underdense, homogenous plasma. The magnetic field is transverse to the electric vector and the direction of propagation of the radiation field. The study is motivated by the fact that transverse magnetic fields are generated in the laser-plasma interaction and in many applications, modification of the propagation characteristics of the laser beam due to the presence of these fields becomes important. The organization of the paper is as follows: In Sec. II the nonlinear current density generated due to propagation of a moderately intense laser beam through transversely magnetized plasma and the linear dispersion relation have been obtained. In Sec. III, a nonlinear wave equation for a laser beam propagating in magnetized plasma is set up. In Sec. IV, solution of the envelope equation for the laser spot is obtained and critical power required to self-focus the laser beam in magnetized plasma is defined. Conclusions are presented in Sec. V.

II. FORMULATION

Consider a linearly polarized laser pulse propagating in a uniform plasma embedded in a constant external magnetic field $\mathbf{b}=b\hat{e}_y$. The electric vector of the radiation field propagating along the z direction is represented by

$$\mathbf{E}=\hat{e}_x E(r,z,t)\cos(k_0 z-\omega_0 t), \tag{1}$$

where E, k_0, and ω_0 are amplitude, wave number, and frequency of radiation field, respectively. The wave equation governing the propagation of the laser pulse through the plasma is given by

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t}. \tag{2}$$

The plasma current density is given by

$$\mathbf{J}=-ne\mathbf{v}, \tag{3}$$

where $\mathbf{v}$ is the velocity of the plasma electron and n is the plasma electron density. The equations governing the relativistic interaction between the electromagnetic field and plasma electron are the Lorentz force equation

$$\frac{d(\gamma\mathbf{v})}{dt}=-\frac{e\mathbf{E}}{m}-\frac{e}{mc}\mathbf{v}\times(\mathbf{B}+\mathbf{b}) \tag{4}$$

and the continuity equation

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0, \tag{5}$$

γ is the relativistic factor and $\mathbf{B}$ is the magnetic vector of the radiation field. Here plasma is assumed to be cold, so that initially the plasma electrons are at rest and the external magnetic field plays no role.

Using the perturbative technique all quantities can be simultaneously expanded in orders of the radiation field. From Eq. (4) the first order equations for velocity along x and z directions are, respectively, given by

$$\frac{\partial v_x^{(1)}}{\partial t} = -\frac{e}{m}E_x + v_z^{(1)}\omega_c \tag{6a}$$

and

$$\frac{\partial v_z^{(1)}}{\partial t} = -v_x^{(1)}\omega_c, \tag{6b}$$

where $\omega_c(=eb/mc)$ is the cyclotron frequency of the plasma electron. Differentiating Eqs. (6a) and (6b) with respect to time and combining gives

$$\frac{\partial^2 v_x^{(1)}}{\partial t^2} + \omega_c^2 v_x^{(1)} = -\frac{e}{m}\omega_0 E \sin(k_0 z - \omega_0 t) \tag{7a}$$

and

$$\frac{\partial^2 v_z^{(1)}}{\partial t^2} + \omega_c^2 v_z^{(1)} = \frac{e}{m}\omega_c E \cos(k_0 z - \omega_0 t). \tag{7b}$$

It may be noted that the first order transverse and longitudinal velocities are driven by forces oscillating with the laser frequency. The solutions of Eqs. (7a) and (7b) are, respectively, given by

$$v_x^{(1)} = \frac{ca\omega_0^2}{(\omega_0^2 - \omega_c^2)} \sin(k_0 z - \omega_0 t) \tag{8a}$$

and

$$v_z^{(1)} = \frac{-\omega_0\omega_c ca}{(\omega_0^2 - \omega_c^2)} \cos(k_0 z - \omega_0 t), \tag{8b}$$

where $a(=eE/mc\omega_0)$ is the normalized radiation field amplitude. The presence of the magnetic field increases the transverse quiver velocity [Eq. (8a)] and also leads to the generation of a longitudinal velocity component [Eq. (8b)], due to the $\mathbf{v}\times\mathbf{b}$ force acting on the plasma electrons. This leads to an increase in the relativistic mass of the plasma electrons and results in modification of the refractive index.

The same procedure is used to obtain the second order electron velocity. The second order equations for the velocity components can be obtained from Eq. (4). With the help of first order velocities, the second order equations are given by

$$\frac{\partial^2 v_x^{(2)}}{\partial t^2} + \omega_c^2 v_x^{(2)} = -\frac{c^2 k_0 a^2 \omega_0^2 \omega_c(\omega_0^2 - 4\omega_c^2)}{2(\omega_0^2 - \omega_c^2)^2} \times \sin 2(k_0 z - \omega_0 t) \tag{9a}$$

and

$$\frac{\partial^2 v_z^{(2)}}{\partial t^2} + \omega_c^2 v_z^{(2)} = -\frac{c^2 k_0 a^2 \omega_0(2\omega_0^4 - 4\omega_0^2\omega_c^2 - \omega_c^4)}{2(\omega_0^2 - \omega_c^2)^2} \times \cos 2(k_0 z - \omega_0 t). \tag{9b}$$

The solutions of Eqs. (9a) and (9b) are, respectively, given by

$$v_x^{(2)} = \frac{c^2 k_0 a^2 \omega_0^2 \omega_c(\omega_0^2 - 4\omega_c^2)}{2(\omega_0^2 - \omega_c^2)^2(4\omega_0^2 - \omega_c^2)} \sin 2(k_0 z - \omega_0 t) \tag{10}$$

and

$$v_z^{(2)} = -\frac{c^2 k_0 a^2 \omega_0(2\omega_0^4 - 4\omega_0^2\omega_c^2 - \omega_c^4)}{2(\omega_0^2 - \omega_c^2)^2(4\omega_0^2 - \omega_c^2)} \cos 2(k_0 z - \omega_0 t). \tag{11}$$

The second order, high frequency, x component of velocity is generated due to uniform magnetic field and reduces to zero in its absence. However the z component of velocity is due to the magnetic vector of the radiation field as well as an external magnetic field.

Similarly the third order equation for the x component of velocity, neglecting harmonics is given by

$$\frac{\partial^2 v_x^{(3)}}{\partial t^2} + \omega_c^2 v_x^{(3)} = ca^3\left[\frac{c^2 k_0^2 \omega_0^2 \omega_c^2(5\omega_0^4 - 11\omega_0^2\omega_c^2 - 6\omega_c^4)}{4(\omega_0^2 - \omega_c^2)^3(4\omega_0^2 - \omega_c^2)} + \frac{\omega_0^4(3\omega_0^4 + 2\omega_0^2\omega_c^2 + 3\omega_c^4)}{8(\omega_0^2 - \omega_c^2)^3}\right] \times \sin(k_0 z - \omega_0 t). \tag{12}$$

Substituting first and second order quantities the x component of third order velocity is given by

$$v_x^{(3)} = -ca^3\left[\frac{c^2 k_0^2 \omega_0^2 \omega_c^2(5\omega_0^4 - 11\omega_0^2\omega_c^2 - 6\omega_c^4)}{4(\omega_0^2 - \omega_c^2)^4(4\omega_0^2 - \omega_c^2)} + \frac{\omega_0^4(3\omega_0^4 + 2\omega_0^2\omega_c^2 + 3\omega_c^4)}{8(\omega_0^2 - \omega_c^2)^4}\right] \sin(k_0 z - \omega_0 t). \tag{13}$$

Density perturbations introduced in the plasma due to interaction with the laser beam can be obtained by expanding the continuity Eq. (5) in orders of the radiation field. Thus

$$\frac{\partial n^{(1)}}{\partial t} + n^{(0)}(\nabla . \mathbf{v}^{(1)}) = 0. \tag{14}$$

Here, $n^{(0)}(=n_0)$ is the ambient plasma electron density.

It may be noted that the longitudinal velocity [Eq. (8b)] generates first order fluctuations in plasma density. Substituting the value of $v_z^{(1)}$ in Eq. (14) and using the transverse Coulomb gauge $(\nabla_\perp . \mathbf{E}=0)$ the first order electron density perturbation is given by

$$n^{(1)} = \frac{-n_0\omega_c c k_0 a}{(\omega_0^2 - \omega_c^2)} \cos(k_0 z - \omega_0 t). \tag{15}$$

This first order density perturbation arises due to the presence of the external magnetic field and reduces to zero in its absence. The second order density response [from Eq. (5)] is given by

$$\frac{\partial n^{(2)}}{\partial t} + \nabla . (n^{(0)}\mathbf{v}^{(2)} + n^{(1)}\mathbf{v}^{(1)}) = 0. \tag{16}$$

The solution to Eq. (16) gives

$$n^{(2)}=-\frac{n_0c^2k_0^2a^2\omega_0^2(\omega_0^2-4\omega_c^2)}{(\omega_0^2-\omega_c^2)^2(4\omega_0^2-\omega_c^2)}\cos 2(k_0z-\omega_0t). \quad (17)$$

The perturbed velocities and densities are used to obtain the transverse current density [Eq. (3)] as

$$J_x=J_x^{(1)}+J_x^{(3)}=-e(n^{(0)}v_x^{(1)}+n^{(0)}v_x^{(3)}+n^{(1)}v_x^{(2)}+n^{(2)}v_x^{(1)}). \quad (18)$$

The nonlinear current density is represented by the second, third, and fourth terms in Eq. (18). The presence of the external magnetic field modifies the second and fourth terms due to change in relativistic mass corrections and additional density perturbations, respectively, while the third term arises due to lowest order longitudinal electron oscillations set up by the magnetic field. These contributions of the magnetic field to the current density lead to modification of nonlinear refractive index and will therefore affect the propagation characteristics of the laser beam in plasma.

Substituting the values of first, second, and third order quantities in Eq. (18) gives the current density (linear and nonlinear) as

$$J_x=-en_0ca\left[\frac{\omega_0^2}{\omega_0^2-\omega_c^2}-a^2\left\{\frac{c^2k_0^2\omega_0^2\omega_c^2(5\omega_0^4-11\omega_0^2\omega_c^2-6\omega_c^4)}{4(\omega_0^2-\omega_c^2)^4(4\omega_0^2-\omega_c^2)}-\frac{c^2k_0^2\omega_0^2(2\omega_0^4-9\omega_0^2\omega_c^2+4\omega_c^4)}{4(\omega_0^2-\omega_c^2)^3(4\omega_0^2-\omega_c^2)}+\frac{\omega_0^4(3\omega_0^4+2\omega_0^2\omega_c^2+3\omega_c^4)}{8(\omega_0^2-\omega_c^2)^4}\right\}\right]\sin(k_0z-\omega_0t). \quad (19)$$

In deriving Eq. (19) all harmonics have been neglected.

Substitution of linear current density in Eq. (2) leads to the linear dispersion relation for a laser beam propagating in magnetized plasma as

$$c^2k_0^2=\omega_0^2-\frac{\omega_0^2\omega_p^2}{(\omega_0^2-\omega_c^2)}. \quad (20)$$

In the absence of the magnetic field ($\omega_c=0$) Eq. (20) reduces to the well known linear dispersion of a laser beam propagating in plasma.

III. WAVE DYNAMICS

Nonlinear propagation of the laser beam, in magnetized plasma can be described by substituting the current density [Eq. (19)] in the wave equation (2) as follows:

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{a}=k_{p0}^2\left[\frac{\omega_0^2}{\omega_0^2-\omega_c^2}-a^2N\right]\mathbf{a}, \quad (21)$$

where

$$\mathrm{N}=\frac{c^2k_0^2\omega_0^2\omega_c^2(5\omega_0^4-11\omega_0^2\omega_c^2-6\omega_c^4)}{4(\omega_0^2-\omega_c^2)^4(4\omega_0^2-\omega_c^2)}-\frac{c^2k_0^2\omega_0^2(2\omega_0^4-9\omega_0^2\omega_c^2+4\omega_c^4)}{4(\omega_0^2-\omega_c^2)^3(4\omega_0^2-\omega_c^2)}+\frac{\omega_0^4(3\omega_0^4+2\omega_0^2\omega_c^2+3\omega_c^4)}{8(\omega_0^2-\omega_c^2)^4}. \quad (22)$$

The first term on the right-hand side of Eq. (21) is the unperturbed linear current density for a laser beam propagating in magnetized plasma. The second term includes nonlinear perturbations due to relativistic effects, density fluctuations and coupling of radiation field with magnetic field.

Assuming the radiation amplitude to be a slowly varying function of z, the paraxial approximation of Eq. (21) leads to

$$\left(\nabla_\perp^2+2ik_0\frac{\partial}{\partial z}\right)a(r,z)=k_{p0}^2\left\{\frac{\omega_0^2}{\omega_0^2-\omega_c^2}-a^2N\right\}a(r,z). \quad (23)$$

IV. SPOT-SIZE EVOLUTION

In order to obtain the evolution of the laser spot, the source dependent expansion (SDE) method[15] is used. The laser field amplitude is expanded as a series of Laguerre-Gaussian source-dependent modes and may be written as $a(r,z)=\Sigma_m\hat{a}_mL_m(\chi)\exp[-(1-i\alpha_s)\chi/2]$, where $m=0,1,2,3,\ldots$, $\hat{a}_m(z)$ is the complex amplitude, $\chi=2r^2/r_s^2$, $r_s(z)$ is the spot size, $\alpha_s(z)(=k_0r_s^2/2R_C)$ is related to the curvature (R_C) associated with the wavefront and $L_m(\chi)$ is a Laguerre polynomial of order m. The amplitude of the lowest order mode ($m=0$) is assumed to be $\hat{a}_0=a_s\exp(i\theta_s)$, where θ_s is the phase. The evolution of real parameters a_s, $r_s(z)$, $\alpha_s(z)$, and θ_s for the lowest (Gaussian) mode is given by

$$\frac{\partial}{\partial z}(a_sr_s)=0, \quad (24a)$$

$$\frac{\partial^2r_s}{\partial z^2}=\frac{4(1+k_0r_s^2H)}{k_0^2r_s^3}, \quad (24b)$$

$$\alpha_s=\frac{k_0r_s^2}{2R_C}=\frac{k_0r_s\dot{r}_s}{2}, \quad (24c)$$

and

$$\dot{\theta}_s=-\frac{2}{k_0r_s^2}-H-G, \quad (24d)$$

where $G=(k_{p0}^2/2k_0)[-(\omega_0^2/(\omega_0^2-\omega_c^2))-(a_s^2N/2)]$ and $H=-(k_{p0}^2a_s^2N/8k_0)$. Equation (24a) shows that the total laser power is independent of z; therefore $a_s^2r_s^2=a_0^2r_0^2$. The evolution of the laser spot is determined from Eq. (24b) which may be explicitly written as

$$\frac{\partial^2r_s}{\partial z^2}=\frac{4}{k_0^2r_s^3}\left(1-\frac{k_{p0}^2a_0^2r_0^2}{8}N\right). \quad (25)$$

The first term on the right-hand side of Eq. (25) represents vacuum diffraction, the second term gives the combined effect of density perturbation, relativistic mass correction, and

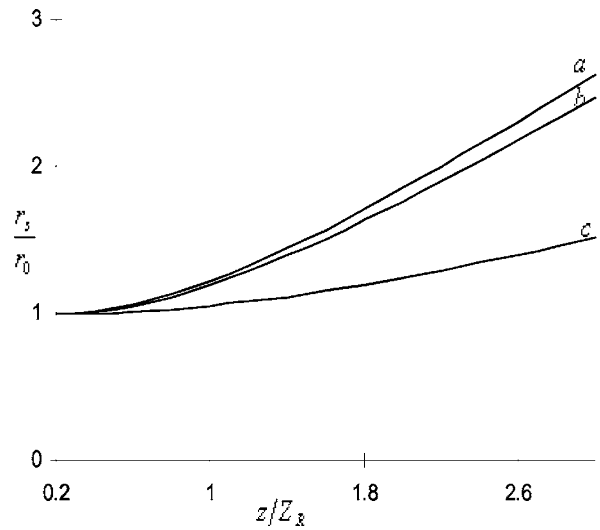

FIG. 1. Variation of r_s/r_0 with z/Z_R for (a) unmagnetized plasma, (b) $\omega_c/\omega_0=0.2$, and (c) $\omega_c/\omega_0=0.4$ with $a_0=0.271$, $\omega_0=1.88\times10^{15}$ s^{-1}, and $\omega_p/\omega_0=0.1$.

magnetic field on the evolution of the laser spot. The solution of Eq. (25) is given by

$$\frac{r_s^2}{r_0^2}=1+\left[1-\frac{P}{P_{CM}}\right]\frac{z^2}{Z_R^2}, \tag{26}$$

where $P/P_{CM}(=k_{p0}^2a_0^2r_0^2N/8)$ is the normalized power and Z_R is the Rayleigh length. It may be noted that in the absence of the magnetic ($\omega_c=0$) field, Eq. (26) reduces to the spot size evolution obtained for a laser beam propagating in an unmagnetized plasma.[15] P_{CM} defines the critical power for nonlinear self-focusing of a laser beam in magnetized plasma and its value is given by

$$P_{CM}=\frac{2\pi^2c^5m^2}{k_{p0}^2\lambda^2e^2N}. \tag{27}$$

In the absence of the magnetic field, Eq. (27) reduces to the critical power required for self-focusing of a laser beam in an unmagnetized plasma.[15]

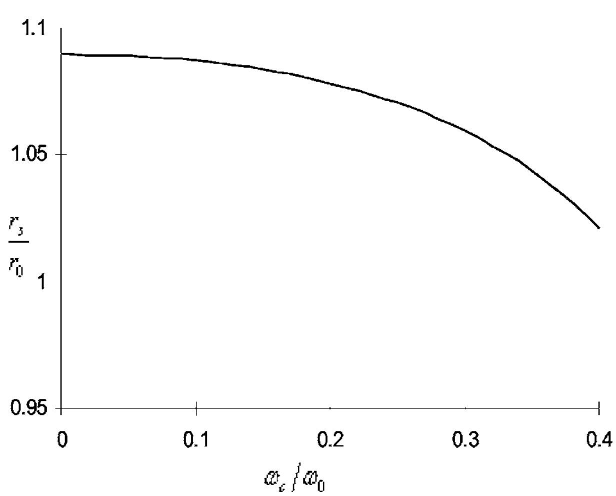

FIG. 2. Variation of r_s/r_0 with ω_c/ω_0 at $z/Z_R=0.3$ for $a_0=0.271$, $\omega_0=1.88\times10^{15}$ s^{-1}, and $\omega_p/\omega_0=0.1$.

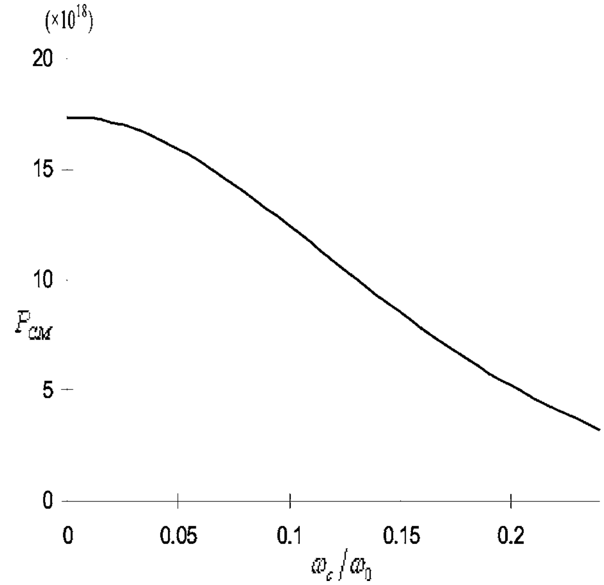

FIG. 3. Variation of P_{CM} with ω_c/ω_0 for $a_0=0.271$, $\omega_0=1.88\times10^{15}$ s^{-1}, and $\omega_p/\omega_0=0.1$.

The variation of the normalized spot-size (r_s/r_0), of a laser beam of frequency $\omega_0=1.88\times10^{15}$ s^{-1} and intensity $\approx10^{17}$ W/cm^2 ($a_0=0.271$), is plotted against the normalized propagation distance (z/Z_R) in Fig. 1 for unmagnetized [curve (a)] and magnetized [curve (b) for $\omega_c/\omega_0=0.2$ and curve (c) for $\omega_c/\omega_0=0.4$] plasma with $\omega_p/\omega_0=0.1$. The self-focusing property of the laser spot is seen to enhance due to magnetization of the plasma.

In order to study the effect of increasing magnetic fields on the laser spot, r_s/r_0 is plotted against ω_c/ω_0 in Fig. 2 at $z/Z_R=0.3$ for $a_0=0.271$. The spot-size is seen to decrease with an increase in the magnetic field. Thus the beam becomes more focused as the magnetic field is increased. The critical power (P_{CM}) required for self-focusing of the laser beam, is plotted against ω_c/ω_0 in Fig. 3. It may be noted that an increase in magnetic field leads to a significant decrease in critical power required for self-focusing of the laser beam. The parameters used are same as in Fig. 1.

V. CONCLUSION

In this paper, propagation of laser pulses through transversely magnetized plasma is studied. The $\mathbf{v}\times\mathbf{b}$ force acting on plasma electrons introduces changes in relativistic mass and causes electron density perturbations, leading to modification in the propagation characteristics of the laser beam. The present study reveals that transverse magnetization of plasma enhances the self-focusing property of the laser beam. An increase in magnetic field leads to a decrease in the laser spot size. The critical power required to self-focus the beam in the magnetized plasma has been obtained. It is seen that the critical power is reduced due to the presence of the magnetic field.

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