# Using Wavelet Transform Reassignment Techniques for ECG Characterisation

D Clifton[2], PS Addison[1, 2], MK Stiles[4], I Romero Legarreta[1], N Grubb[4], JN Watson[2], GR Clegg[3], CE Robertson[3]

[1] Faculty of Engineering , Napier University, Edinburgh, Scotland. [2] CardioDigital Ltd, Edinburgh, Scotland. [3]Department Of Accident and Emergency Medicine, The Royal Infirmary, Edinburgh. [4]Department Of Cardiology, The Royal Infirmary, Edinburgh, Scotland

## Abstract

*Reassignment, a technique which sharpens the time frequency information within the time-frequency plane, can be applied to the wavelet transform to produce a more succinct, highly focused, representation. In this paper this technique is applied to interrogate and represent heart rate variation recorded by means of temporary reprogramming of a volunteer's pacemakers. The work has shown that the reassignment technique is capable of extracting subtle features, and in particular, providing enhanced temporal isolation, from components that were not readily observable in the wavelet scalogram alone.*

## 1. Introduction

Advances in digital computational devices and algorithms have led to the feasibility for implementing real-time analysis of medical signals. Such automated objective assessment is desirable to overcome limitations and inaccuracies of subjective assessments. In particular, the analysis of ECG signals is an important task and is the focus of much current research.

The wavelet transform (WT) has been found particularly useful for signal analysis because of its ability to localise in both time and frequency. The WT has been successfully applied to the digital processing of ECG signals [1, 2]. Reassignment techniques can be applied as a secondary processing technique to the WT to produce a more succinct, highly focused, representation of the required information. Reassignment is a technique which sharpens the time frequency information within the time-frequency plane. It works by shifting the components from the geometrical centre of the time-frequency analysis window to the centre of gravity of their complex energy density distribution. In this way, the energy density components are relocated to positions in the time-frequency plane closer to the signal features causing them. Various authors have followed the original ideas of Kodera et al [3], who developed the reassignment method for the short-time Fourier transform, applying them to other time-frequency representations [4, 5]. A few recent papers have begun to apply time-frequency reassignment methodology to a variety of real signals, including: speech signals using the STFT [6]. An example of the application of the reassignment technique to medical signals is for the extraction of respiration rate from the photoplethysmogram [7]. As an example of this technique figure 1 shows data from a trial in which a volunteer's photoplethysmographic waveform was recorded whilst the volunteer was asked to vary their respiration rate from 15 bpm (0.25 Hz) with a sudden transition to 10 bpm (0.17 Hz) and then back to 15 bpm. The volunteer was asked to actuate a switch in time with their breath which was used to generate the breath timing marker shown at the bottom edge of the scalogram in figure 1. The Features related to respiration are clearly shown in the scalogram as areas of increased energy. In the reassigned scalogram these features become highly focused at the respiration rates with the rate transition clearly represented. Note that small fluctuations in the respiration rate, which are not readily observed in the scalogram, have been resolved in the reassigned scalogram.

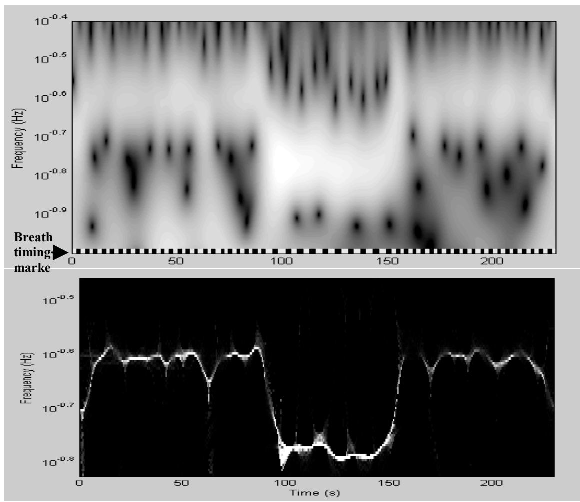


Figure 1. Shows respiration rate features from a photoplethysmogram trace as a wavelet scalogram (top) and reassigned scalogram (bottom).

In this paper we demonstrate the application of wavelet reassignment technique to feature extraction and characterisation of paced ECG traces.

## 2. The wavelet transform and scalogram reassignment

A full and detailed description of the wavelet transform and its properties can be found in [8]. However, it is worth recapping on a few basic properties of the transform. The wavelet transform of a continuous time signal, *x(t)*, is defined as:

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi^*(\frac{t-b}{a})dt$$

(1)

where $\psi^*(t)$ is the complex conjugate of the wavelet function $\psi(t)$, $a$ is the dilation parameter of the wavelet and $b$ is the location parameter of the wavelet. The contribution to the signal energy at the specific $a$ scale and $b$ location is given by the two-dimensional wavelet energy density function known as the scalogram:

$$E(a,b) = | T(a,b) |^2 \quad (2)$$

and the total energy in the signal may be found from its energy density as represented in wavelet space as follows:

$$E = \frac{1}{C_g} \int_{-\infty}^{+\infty}\int_{0}^{\infty} \frac{1}{a^2} | T(a,b) |^2 \, dadb \qquad \left[ = \int_{-\infty}^{+\infty} x(t)^2 dt \right] \quad (3)$$

where $Cg$ is the admissibility constant. As with the Fourier transform, the original signal may be reconstructed using an inverse transform:

$$x(t) = \frac{1}{C_g} \int_{-\infty}^{+\infty}\int_{0}^{\infty} \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})T(a,b)\frac{dadb}{a^2} \quad (4)$$

In practice a fine discretisiation of the CWT is computed where usually the $b$ location is discretised at the sampling interval and the $a$ scale is discretised logarithmically. Finally, as the wavelet transform given by equation 1 is a convolution of the signal with a wavelet function, we can use the convolution theorem to express the integral as a product in Fourier space, i.e.

$$T(a,b) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{x}(\omega)\hat{\psi}^*_{a,b}(\omega)\, d\omega$$

where

$$\hat{\psi}^*_{a,b}(\omega) = \sqrt{a}\,\hat{\psi}^*(a\omega)e^{i\omega b}$$

is the Fourier spectrum of the analysing wavelet at scale $a$ and location $b$. That is:

$$T(a,b) = \sqrt{a}\int_{-\infty}^{\infty} \hat{x}(f)\hat{\psi}^*(af)e^{i\omega b}\, d\omega$$

In this way, the Inverse Fast Fourier Transform (IFFT) algorithm can be employed in practice to quickly compute the wavelet transform.

Now, following the ideas of Kodera et al [3] for the reassignment of the Gabor transform one can obtain a reassignment recipe for the wavelet transform If the wavelet function is written in terms of its modulus and phase as

$$\psi(t) = |\psi(t)|e^{i\phi_\psi(t)}$$

where $\phi_\psi(t)$ is the argument of $\psi(t)$. In a similar way we may write the wavelet transform of a signal as

$$T(a,b) = |T(a,b)|e^{i\Omega(a,b)} \quad (5)$$

Hence the inverse wavelet transform of equation (4) can be written as

$$x(t) = \frac{1}{C_g}\int_{-\infty}^{+\infty}\int_{0}^{\infty} |T(a,b)|e^{i\Omega(a,b)} \frac{1}{\sqrt{a}}\left|\psi\left(\frac{t-b}{a}\right)\right| e^{i\phi_\psi\left(\frac{t-b}{a}\right)} \frac{dadb}{a^2}$$

The stationary phase argument for the evaluation of oscillatory integrals states that most of the contribution to the value of the integral

$$I = \int_{-\infty}^{+\infty} a(t)\, e^{if(t)} dt$$

comes from the neighbourhood of the point $t_s$ where the phase is stationary, i.e. $f'(t_s)=0$ (where we assume that $a(t)$ changes very slowly compared to the oscillations of $e^{if(t)}$). Using this argument we can say that for each $t$ the value of the wavelet transform integral comes mostly from the contributions of points ($a$,$b$) close to the stationary points ($a_0$,$b_0$) which satisfy:

$$\begin{cases} \partial_b\Omega(a,b) = \frac{1}{a}\phi'_\psi\left(\frac{t-b}{a}\right) \\ \partial_a\Omega(a,b) = \frac{t-b}{a^2}\phi'_\psi\left(\frac{t-b}{a}\right) \end{cases} \quad (6)$$

We can simplify these equations if we employ the standard Morlet wavelet defined by:

$$\psi(t) = \frac{1}{\sqrt[4]{\pi}} e^{i\omega_o t} e^{-\frac{t^2}{2}}$$

For which there is a simple expression for the derivative of the phase given by: $\phi_{\psi}^{'} = \omega_0$ where $\omega_o$ is the central frequency of the mother wavelet. We can obtain the reassignment operators from equations 6 to give:

$$\begin{cases} \widetilde{b}(a,b) = b + \dfrac{a^2 \, \partial_a \Omega(a,b)}{\omega_0} \\ \widetilde{a}(a,b) = \dfrac{\omega_0}{\partial_b \Omega(a,b)} \end{cases} \qquad (7)$$

The reassignment procedure is then performed by placing the the original wavelet coefficient at the new location $\left(\widetilde{a}, \widetilde{b}\right)$ using equations 7. We know that the calculation of the wavelet transform may be performed as an inverse Fourier transform because

$$\hat{T}(a,.)(\omega) = \hat{x}(\omega)\sqrt{a}\hat{\psi}^*(a\omega)$$

In a similar way we can calculate the partial derivatives of the wavelet transform using the following relationships:

$$\partial_b(\hat{T})(a,.)(\omega) = \hat{x}(\omega)\sqrt{a}\hat{\psi}^*(a\omega)(i\omega)$$

where $\partial_b(\hat{T})(a,.)(\omega)$ is the Fourier transform of the derivative of the wavelet transform with respect to *b*. For $\partial_a(T)$

$$\partial_a(\hat{T})(a,.)(\omega) = \hat{x}(\omega)\left(\frac{\sqrt{a}}{2a}\hat{\psi}^*(a\omega) + \sqrt{a}\omega\hat{\psi}^{*'}(a\omega)\right)$$

Representing the wavelet transform in terms of its modulus and phase so that the partial derivative of the wavelet transform with respect to *b* becomes:

$$\partial_a T(a,b) = \partial_a |T(a,b)| e^{i\Omega(a,b)} + |T(a,b)| e^{i\Omega(a,b)} i \partial_a \Omega(a,b)$$

from this we can see that

$$\partial_a \Omega(a,b) = \Im\left[\frac{\partial_a T(a,b)}{T(a,b)}\right]$$

In the same way we can derive similar expression for $\partial_b \Omega(a,b)$. Hence the partial derivatives of the phase may be obtained from the partial derivatives of the wavelet transform. By applying these techniques a simple test case can be developed as shown in figure 2.

.

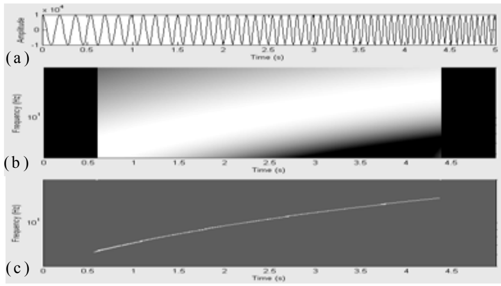

Figure 2. Reassignment of chirp signal showing (a) time signal, (b) wavelet scalogram, and (c) reassigned scalogram.

In this example a wavelet scalogram and its reassigned scalogram plot have been computed from a numerically generated chirp signal. The representation show the energy densities collapsing, after reassignment, to a thin line curving through energy density space. An obvious point which is perhaps worth making explicit is that reassignment conserves energy in the time frequency plane as energy is simply moved around the plane and neither destroyed nor increased. Hence the signal, scalogram and reassigned scalogram both have the same energy.

## 3. ECG signals

The ECG signals shown in figures 3 to 5 are signals collected during pacing trials. In these trials, volunteers with permanent pacemakers had their heart rate varied by means of temporary reprogramming of their pacemaker. The aim of this approximation of physiological heart rate variation was to characterise the wavelet transform scalogram features of the resting ECG and the rate dependence of these features during atrial pacing.

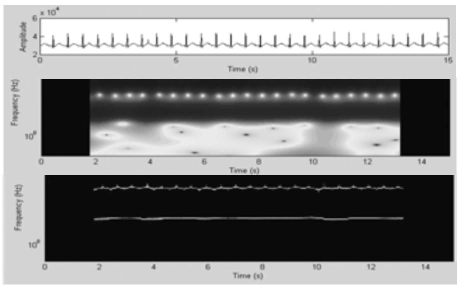

Figure 3. Showing ECG signal (top), wavelet plot (middle) and reassigned plot (bottom) for constant rate pacing at 110 bpm

In figure 3, a constant rate of pacing at 110 bpm was recorded. From the reassigned plot at the bottom of this figure, the individual features associated worth components of the QRS complex (top line) and the Twave (bottom line) are shown with clear separation in the reassigned plot.

In figure 4 an upwards transition in beat rate from 70 to 90 bpm is shown in the form of a conventional ECG time-trace (top), wavelet scalogram (middle), and reassigned scalogram (bottom). The transition in rate is readily observable from the reassigned scalogram. In figure 5 a downward transition in the heart rate from 110 to 90 bpm is illustrated. In this case the reassigned scalogram shows some instability in the settling time for the beat rate just after the transition. This instability is not readily observed from either the time domain trace or the wavelet scalogram. Note also that in the case of all the reassigned scalogram erroneous features that appear in the original scalogram have been suppressed to produce a more succinct, highly focused, representation of the required transition information.

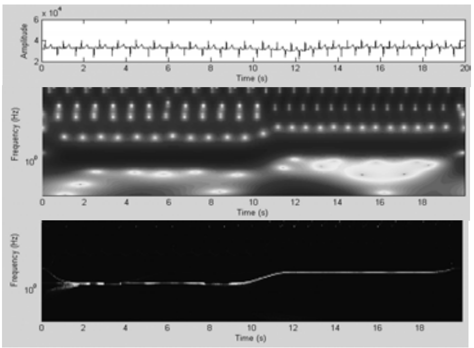

Figure 4: Showing ECG signal (top), wavelet plot (middle) and reassigned plot (bottom) for paced heart rate transition from 70 to 90 bpm.

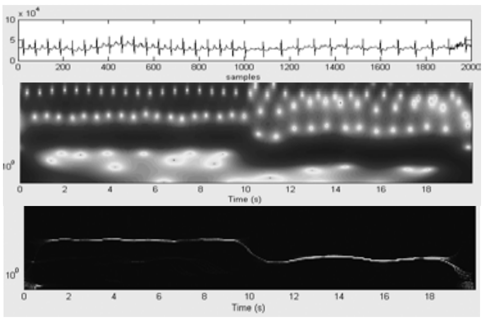

Figure 5: Showing ECG signal (top), wavelet plot (middle) and reassigned plot (bottom) for paced heart rate transition from 110 to 90 bpm.

## 4. Concluding remarks

In this paper we have applied the reassignment method to the detection of both stationary and non-stationary features in paced ECG signal. The examples shown have demonstrated the extraction of subtle features that were otherwise hidden if using the scalogram directly. It can be concluded that the complete Morlet wavelet with reassignment methods appears to provide a useful tool for the enhanced temporal isolation of signal components. It is envisaged that the method will offer a potential feature detection tool for a variety of medical signal analysis tasks, including the ECG where high temporal focus is required.

## References


[1] Addison PS, Watson JN, Clegg G, Holzer M, Sterz F, Robertson C. Evaluating Arrhythmias in ECG Signals using Wavelet Transforms. IEEE Engineering in Medicine and Biology 2000; 19(4): 383-392.

[2] Addison PS, Watson JN, Clegg G, Holzer M, Sterz F, Robertson C. A Novel Wavelet Transform based Analysis Reveals Hidden Structure in Ventricular Fibrillation, Resuscitation 2000;.43(2): 121-127.

[3] Kodera K, Gendrin R, de Villedary C. Analysis of time-varying signals with small BT values. *IEEE Transactions on Acoustics, Speech, and Signal Processing* 1978; ASSP-26(1): 64-76.

[4] Auger F, Flandrin P. Improving the readability of time-frequency and time-scale representations by the reassignment method. *IEEE Transactions on Signal Processing* 1995; 43(5): 1068-1089.

[5] Chassande-Mottin E., Flandrin P., Reassigned scalograms and singularities. *Proc. 3rd Euro Congress Math., Barcelona,* 2000: 583-590

[6] Plante F, Meyer G, Ainsworth A. Improvement of speech spectrogram accuracy by the method of reassignment. *IEEE Transactions on Speech and Audio Processing* 1998; 6(3): 282-287.

[7] Addison PS, Watson JN. Secondary Wavelet Feature Decoupling (SWFD) and its use in Detecting Patient Respiration from the Photoplethysmogram, *25th Annual International Conference of the IEEE Engineering in medicine and Biology Society,* Cancun, Mexico, 2003: 17-21.

[8] Addison P.S., 2002, *The Illustrated Wavelet Transform Handbook: Applications in Science, Engineering, Medicine and Finance*. The Institute of Physics Publishing.



Address for correspondence:

Dr David Clifton
CardioDigital Ltd
Elvingston Science Centre
Gladsmuir, East Lothian, Scotland EH33 1EH, UK
Email:d.clifton@cardiodigital.com