Adaptive B-Spline Based Neuro-Fuzzy Control for Full Car Active Suspension System

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Abstract: The main role of a car suspension system is to improve the ride comfort and road holding. The traditional spring-damper suspension is currently being replaced by either a semi-active or active suspension system, because, passive suspension system cannot give better ride comfort and handling property. In order to improve the capability of active suspension systems, a robust Adaptive B-spline Neuro-fuzzy (ABNF) based control strategies are used to give better road handling and passenger comfort. In this paper, different orders of B-spline membership functions are used in the proposed ABNFs control strategies. The shape of B-spline membership functions can be adjusted self-adaptively by changing control points during learning process. The order of the B-spline membership functions gives a structure for choosing the shape of the fuzzy sets. The update parameters of ABNFs are trained by gradient descent based technique during the learning process. The ABNFs control techniques are successfully applied to full car suspension model which reduces the seat heave pitch and roll displacement, suspension travel and wheel displacement of the vehicle. The performance index of the proposed techniques is taken on the basis of seat, heave, pitch and roll of the vehicle. Simulation is based on the full car mathematical model by using MATLAB/SIMULINK. The simulation results show that the ABNFs control techniques give better results than passive and semi-active suspension systems.

Key words: B-spline • Fuzzy logic • Neural network • Full car • Suspension system

INTRODUCTION

In vehicle's suspension systems, the suspension system plays a very important role for the vehicle's stability or road handling and ride comfort against the road disturbances. So, the passive system cannot give better road handling and passenger’s comfort. The conventional passive suspension system comprises of combination of springs and dampers. Unlike conventional passive and semi-active suspension systems, an active vehicle suspension systems external excitation is counteracted with the generation of a control force depending on the vehicle response through an actuator driven by an external energy source. Therefore, the use active vibration control mechanism for design of advanced suspension system has attracted considerable attention in the recent years.

The main objective of an active suspension is to reduce the vibrations of the vehicle body induced by the road disturbances, to improve the vehicle stability and the passenger comfort. The primary requirements of a good vehicle suspension design are possession of good ride comfort, road handling and road holding characteristics within an acceptable suspension travel range [1-4]. The conflicting nature of these design objectives makes a necessary trade-off between them.

In the last few years, many researchers applied different linear and nonlinear control methods to the vehicle suspension models. The linear suspension control strategies range from PID to robust multivariable controllers. It is supposed that the system’s states reveal only small oscillations about a stability point so that a linear estimated model can be used. PID and direct adaptive neural control of a nonlinear half car active
suspension system is established in [5-9]. To examine the robustness of quarter car suspension system based on stochastic stability robustness presented by [10], but this technique needs large feedback gains and an appropriate phase must be chosen. Adaptive control involves the adjustment of feedback gains according to changes in the system parameters and road conditions. In [11] they combined the benefits of adaptive and robust control in the design of adaptive vehicle suspension system controller and focused on the actuator force control. The adaptive robust controller for the quarter car active suspension is designed [12]. In [13], they designed the feedback linearization control for full car suspension. A variety of simulations showed that the fuzzy logic control is proficient to give a better ride quality than the other common control approaches [14-16]. The genetic algorithm-based fuzzy PI and PD controller for the active suspension system for a quarter car model is designed by [17]. In [18], designed a hybrid neurofuzzy based sliding mode control to enhance the convenience of the passenger and stability of vehicle body.

Fuzzy logic has ability to decrease the complexity and to deal with vagueness and uncertainties of the data [19, 20]. Their arrangement permits to build up a system with fast learning abilities that can explain nonlinear structure that are described with uncertainties. Neural networks have self-learning qualities that raise the precision of the model. The combination of fuzzy logic and wavelets neural network provides the platform for resolving control problems and signal processing problems [21, 22]. There are several probable alternatives for the basis functions to propose local activation functions of neural networks, such as associated memory networks, wavelets, radial basis function [23] and B-spline functions [24]. Unlike a global activation function which is non zero for all finite inputs, a local activation function is nonzero only in a local area of the input space. However, fuzzy neural networks with local activation functions are more competent than the global activation functions in speed and memory, because, only nonzero functions and the weights are calculated [25]. In case of local activation functions, the estimators of nonlinear mapping can be expressed as a linear combination of basis functions. An appropriate choice of B-spline membership and basis functions is the B-spline [26].

The ability of neural networks to approximate various complex nonlinear functions relating input-output data from a nonlinear system has attracted many researchers [27]. B-spline neural network can also estimate continuous functions at any arbitrary precision as long as the network is large enough [28]. B-spline neural network is illustrated by a local weight updating scheme with the advantages of low computation complexity and fast convergence speed. The B-spline has been identified as very smoothest curve in the engineering applications. This is, because of the divergence-moderate property of the B-spline wave. These characteristics make it more appropriate for the online adaptive situations. The plus point of the B-spline functions over other radial functions are the local controls of the curve shape, since the curve changes in the locality of a few control points that have been changed [29]. B-spline neural network comprises of the piecewise polynomials with a set of local basis functions to model an unknown function for which a finite set of input-output samples are available. In [30], the authors used B-spline neurofuzzy technique with different orders of B-spline functions.

In [31], the authors proved that B-spline are equivalent to certain fuzzy systems. Therefore, the B-spline basis functions may be examined as fuzzy membership functions and can be used to obtain fuzzy rules. B-splines basis functions are better to get the desired properties like, local compact support, low computational and partition of unity. These techniques are non-adaptive, so can approximate the unknown nonlinear dynamical systems. But the adaptive controllers have the ability to estimate the unknown nonlinear dynamical system, because, the updated parameters has flexibility to update their values according to the system's requirement. To estimate the nonlinearity of the unknown system, the adaptive B-spline fuzzy neural network is established. By using the online updating algorithm, both the B-spline membership functions of the fuzzy sets and weighting parameters of the adaptive fuzzy neural network is updated. This online adaptation method will improves the robustness and the convergence of the system.

In this study, adaptive B-spline Fuzzy Neural Network strategies with different orders of B-spline membership function, i.e. B-spline of order 2 and B-spline of order 3, are successfully implemented on nonlinear full car active suspension model to improve the ride comfort and road handling. The paper is divided into 6 sections. Section II describes the full car model. In Section III discusses the construction of proposed ABNF’s techniques and section IV gives the online ABFNN control algorithm. Finally, simulation results and conclusion are given in section V and VI respectively.
**Table 1: Simulation variables for full car model**

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Left Spring Constant</td>
<td>k_s1</td>
<td>15000 N/m</td>
</tr>
<tr>
<td>Front Right Spring Constant</td>
<td>k_s2</td>
<td>15000 N/m</td>
</tr>
<tr>
<td>Rear Left Spring Constant</td>
<td>k_s3</td>
<td>17000 N/m</td>
</tr>
<tr>
<td>Rear Right Spring Constant</td>
<td>k_s4</td>
<td>17000 N/m</td>
</tr>
<tr>
<td>Seat Spring Constant</td>
<td>k_s5</td>
<td>15000 N/m</td>
</tr>
<tr>
<td>Front Left Damper Constant</td>
<td>c_s1</td>
<td>2500 N/s/m</td>
</tr>
<tr>
<td>Front Right Damper Constant</td>
<td>c_s2</td>
<td>2500 N/s/m</td>
</tr>
<tr>
<td>Rear Left Damper Constant</td>
<td>c_s3</td>
<td>2500 N/s/m</td>
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<tr>
<td>Seat Spring Constant</td>
<td>c_s5</td>
<td>150 N/s/m</td>
</tr>
<tr>
<td>Front Left Tire Suspension</td>
<td>k_t1</td>
<td>250000 N/m</td>
</tr>
<tr>
<td>Front Right Tire Suspension</td>
<td>k_t2</td>
<td>250000 N/m</td>
</tr>
<tr>
<td>Rear Left Tire Suspension</td>
<td>k_t3</td>
<td>250000 N/m</td>
</tr>
<tr>
<td>Rear Right Tire Suspension</td>
<td>k_t4</td>
<td>250000 N/m</td>
</tr>
<tr>
<td>Length of Axle Suspension from C.O.G</td>
<td>a</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Length of Axle Suspension from C.O.G</td>
<td>b</td>
<td>1.4 m</td>
</tr>
<tr>
<td>Length of Unsprung mass from C.O.G</td>
<td>c</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Length of Unsprung mass from C.O.G</td>
<td>d</td>
<td>1 m</td>
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<tr>
<td>Length of Seat from C.O.G</td>
<td>e</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Length of Sear from C.O.G</td>
<td>f</td>
<td>0.25 m</td>
</tr>
<tr>
<td>Front Left Unsprung mass</td>
<td>m_1</td>
<td>25 kg</td>
</tr>
<tr>
<td>Rear Left Unsprung mass</td>
<td>m_2</td>
<td>25 kg</td>
</tr>
<tr>
<td>Rear Left Unsprung mass</td>
<td>m_3</td>
<td>45 kg</td>
</tr>
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<td>Rear Right Unsprung mass</td>
<td>m_4</td>
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<tr>
<td>Seat Mass</td>
<td>m_5</td>
<td>90 kg</td>
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<tr>
<td>Vehicle Body Mass</td>
<td>M</td>
<td>1100 kg</td>
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<tr>
<td>Moment of Inertia for Pitch</td>
<td>I_x7</td>
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</tr>
<tr>
<td>Moment of Inertia for Roll</td>
<td>I_x8</td>
<td>550 kg.m^2</td>
</tr>
<tr>
<td>Shyhook Damper Constant</td>
<td>C_sky1</td>
<td>-2500 N/s/m</td>
</tr>
</tbody>
</table>

**Full Car Model:** There are eight degrees of freedom for a full car, i.e. Φ (Roll), θ (Pitch), Ω (Heave), Z (“front left tyre”, Z_{r,1} “front right tyre”, Z_{l,1} “rear left tyre”, Z_{r,2} “rear right tyre”). It comprises only a sprung mass attached to the four unsprung masses M_{r,1}, M_{r,1}, M_{r,2}, M_{r,2} (front-right, front-left, rear-right and rear-left wheels) at each corner. The sprung mass is allowed to have pitch, heave and roll where the unsprung masses are allowed only to have heave. For simplicity all other motions are ignored for this model. This model has eight degrees of freedom and allocates the body acceleration and upright body displacement, pitch and roll motion of the vehicle body. Full car professionally ensures passenger safety and ride comfort. This model considers only one seat and this is very important to take into consideration other fixed with chassis [32].

For the Passenger Seat,

\[
M_s \ddot{Z}_s + k_s (Z_s - Z_{X_s} - Z_{Y_s} - Z_{F_s}) + c_s (\dot{Z}_s - \dot{Z}_{X_s} - \dot{Z}_{Y_s} - \dot{Z}_{F_s}) = 0
\]

For the Vehicle’s heave motion of the body,

\[
M \dddot{Z} + k_f (Z_{-X} - Z_{-Y}) + c_f (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f1} (Z_{-X} - Z_{-Y}) + c_{f1} (\dot{Z}_{-X} - \dot{Z}_{-Y}) + k_{f2} (Z_{-X} - Z_{-Y}) + c_{f2} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f3} (Z_{-X} - Z_{-Y}) + c_{f3} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) = 0
\]

For the Vehicle’s Roll motion of the body:

\[
M \dddot{\phi} + k_f (Z_{-X} - Z_{-Y}) + c_f (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f1} (Z_{-X} - Z_{-Y}) + c_{f1} (\dot{Z}_{-X} - \dot{Z}_{-Y}) + k_{f2} (Z_{-X} - Z_{-Y}) + c_{f2} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f3} (Z_{-X} - Z_{-Y}) + c_{f3} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) = 0
\]

MATERIALS AND METHODS

As full-car model has eight-degrees of freedom, then equations of motion for this model are explained by the Newton’s 2nd law of motion as:

For the Passenger Seat:

\[
M_s \dddot{Z}_s + k_s (Z_s - Z_{X_s} - Z_{Y_s} - Z_{F_s}) + c_s (\dot{Z}_s - \dot{Z}_{X_s} - \dot{Z}_{Y_s} - \dot{Z}_{F_s}) = 0
\]

For the Vehicle’s heave motion of the body:

\[
M \dddot{Z} + k_f (Z_{-X} - Z_{-Y}) + c_f (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f1} (Z_{-X} - Z_{-Y}) + c_{f1} (\dot{Z}_{-X} - \dot{Z}_{-Y}) + k_{f2} (Z_{-X} - Z_{-Y}) + c_{f2} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) + k_{f3} (Z_{-X} - Z_{-Y}) + c_{f3} (\dddot{Z}_{-X} - \dddot{Z}_{-Y}) = 0
\]

For the Vehicle’s Roll motion of the body:
For the Vehicle's Pitch motion of the body:

\[ I_{y} \ddot{\theta} = W_{f} \dot{Z}_{f} + W_{c} \dot{Z}_{c} + W_{k} \dot{Z}_{k} + W_{r} \dot{Z}_{r} + W_{c} \dot{Z}_{c} + W_{k} \dot{Z}_{k} + W_{r} \dot{Z}_{r} \]

(3)

For the Vehicle's Pitch motion of the body:

\[ M_{z} \dot{Z}_{z} = k_{f} (Z_{a} + W_{f} Z_{f}) + k_{c} (Z_{a} + W_{c} Z_{c}) + k_{k} (Z_{a} + W_{k} Z_{k}) + k_{r} (Z_{a} + W_{r} Z_{r}) \]

(4)

For the Vehicle's Pitch motion of the body:

\[ M_{z} \dot{Z}_{z} = k_{f} (Z_{a} + W_{f} Z_{f}) + k_{c} (Z_{a} + W_{c} Z_{c}) + k_{k} (Z_{a} + W_{k} Z_{k}) + k_{r} (Z_{a} + W_{r} Z_{r}) \]

(5)

For the Vehicle's Pitch motion of the body:

\[ M_{z} \dot{Z}_{z} = k_{f} (Z_{a} + W_{f} Z_{f}) + k_{c} (Z_{a} + W_{c} Z_{c}) + k_{k} (Z_{a} + W_{k} Z_{k}) + k_{r} (Z_{a} + W_{r} Z_{r}) \]

(6)

For the Vehicle's Pitch motion of the body:

\[ M_{z} \dot{Z}_{z} = k_{f} (Z_{a} + W_{f} Z_{f}) + k_{c} (Z_{a} + W_{c} Z_{c}) + k_{k} (Z_{a} + W_{k} Z_{k}) + k_{r} (Z_{a} + W_{r} Z_{r}) \]

(7)

For the Vehicle's Pitch motion of the body:

\[ M_{z} \dot{Z}_{z} = k_{f} (Z_{a} + W_{f} Z_{f}) + k_{c} (Z_{a} + W_{c} Z_{c}) + k_{k} (Z_{a} + W_{k} Z_{k}) + k_{r} (Z_{a} + W_{r} Z_{r}) \]

(8)

The simulation parameters for full car model are shown in Table 1.

The general class of nonlinear MIMO system is described by [3,21]:

\[ y = A(x) + B(x)u + G(x)z \]

(9)

\[ y = \sum_{i=1}^{P} \sum_{j=1}^{S} B_{ij}(x)u_{j} + \sum_{i=1}^{P} \sum_{j=1}^{S} G_{ij}(x)z_{j} \]

Where \( x = [y_{1}, y_{2}, ..., y_{p}, y_{p+1}, ..., y_{p+s}]^{T} \in \mathbb{R}^{P} \) is the overall state vector, which is assumed available and \( \mathbb{R}^{P+1} = u = [u_{1}, u_{2}, ..., u_{p}]^{T} \in \mathbb{R}^{P} \) is the control input vector, \( y = [y_{1}, y_{2}, ..., y_{p+s}]^{T} \in \mathbb{R}^{P} \) is the output vector and \( z = [z_{1}, z_{2}, ..., z_{s}]^{T} \in \mathbb{R}^{s} \) is the disturbance vector. \( A_{i}(x), i = 1, ..., p \) are continuous nonlinear functions, \( B_{i}(x), i = 1, ..., p, j = 1, ..., s \) are continuous nonlinear control functions and \( G_{ij}(x), i = 1, ..., p, j = 1, ..., s \) are continuous nonlinear disturbance functions.

Let

\[ A = [A_{1}(x) A_{2}(x) ... A_{p}(x)]^{T} \]

(10)

The control matrix is:

\[ B(x) = \begin{bmatrix} b_{11}(x) & ... & b_{1s}(x) \\ ... & ... & ... \\ b_{p1}(x) & ... & b_{ps}(x) \end{bmatrix} \]

(11)

The disturbance matrix is:

\[ G(x) = \begin{bmatrix} g_{11}(x) & ... & g_{1s}(x) \\ ... & ... & ... \\ g_{p1}(x) & ... & g_{ps}(x) \end{bmatrix} \]

(12)

Also, the generic nonlinear car model is,

\[ y = f(x) + B(x).u + G(x).z \]

(13)

Where \( f(x) \in \mathbb{R}^{(16 \times 1)}, b(x) \in \mathbb{R}^{(16 \times 4)}, g(x) \in \mathbb{R}^{(16 \times 4)} \), state vector \( x \in \mathbb{R}^{(16 \times 1)}, u \in \mathbb{R}^{(4 \times 1)} \) and \( y \in \mathbb{R}^{(4 \times 1)} \). The above matrices can be shown in state-space form, with state vector x that is also represented in row matrix form.

\[ f(x) = [A_{1}(x) A_{2}(x) ... A_{16}(x)] \]

\[ x = [x_{1}, x_{2}, x_{3}, ..., x_{16}]^{T} \]

\[ A_{i}(x) \text{ to } A_{i}(x) \text{ are velocity states and } A_{i}(x) \text{ to } A_{i}(x) \text{ are acceleration states of four tires, seat, heave, pitch and roll} \]

[16]. The disturbance inputs for each tire individually are represented in the form of ‘z’ matrix.

\[ z = [z_{1}, z_{2}, z_{3}, ..., z_{4}]^{T} \]

\[ z_{i} \text{ are } ‘n’ \text{ disturbances applied to full car model. } u_{n} \text{ are } ‘n’ \text{ controllers output to full car model, to regulate the car model disturbances. } y_{n} \text{ are } ‘n’ \text{ states of car. } r_{n} \text{ are } ‘n’ \text{ desired outputs of the controller.} \]

**Abfnn for Full Car Suspension Control:** In this section, the structure of the fuzzy B-spline neural network is described. Firstly, the fuzzy B-spline membership function
and B-spline basis function is explained. Secondly, B-spline fuzzy neural network is described and finally the update parameters of the B-spline fuzzy neural network are given.

**Abnfs for Full Car Suspension Control:** In this section, the structure of the fuzzy B-spline neural network is described. As B-spline has localizability properties, fast convergence and low complexity calculation, so, it can approximate the nonlinear functions of the dynamical system. Firstly, the fuzzy B-spline membership function and B-spline basis function of order 2 and order 3 are explained. Secondly, B-spline fuzzy Neural Network is described and finally the update parameters of the B-spline fuzzy neural networks are given.

**Fuzzy B-spline Membership Function of Order 2:**

B-spline are the parametric surfaces that give flexibility for the visualization and modeling of the numerous types of data [34]. All the B-spline fuzzy membership functions of fuzzy logic rules are defined here as of order 2 and calculated by the algorithm explained below. The fuzzy B-spline membership function is given by

\[ \mu_{ij}(x) = \sum_{q=0}^{k-1} c_{ij} N_{q,2}(x_i) \]  

(14)

Where \( \mu_{ij}(x) \) is the membership function of the fuzzy variable \( x_i \) to the \( c_j \) fuzzy set, \( q = 0,1,2,...,k-1 \), are control points, is the number of control points. The general form of the B-spline membership function is can be expressed as:

\[ N_{q,k}(x_i) = \frac{x_i - t_k}{t_{q+k-1} - t_k} N_{q,k-1}(x_i) + \frac{t_{q+k} - x_i}{t_{q+k} - t_k} N_{q,k-1}(x_i) \]  

(15)

The B-spline basis function for order order (k=2), i.e. \( N_{q,2} \) is can be written as:

\[ N_{q,2}(x_i) = \frac{x_i - t_q}{t_{q+1} - t_q} N_{q,1}(x_i) + \frac{t_{q+2} - x_i}{t_{q+2} - t_q} N_{q+1,1}(x_i) \]  

(16)

Where \( t_q \) and \( t_{q+2} \) are the nodes defined in the interval of \( x_i \). On substituting (16), (17) and (18) into (14), it becomes

\[ \mu_{ij}(x_i) = \sum_{q=0}^{k-1} c_{ij} N_{q,2}(x_i) \]

Where \( t_{q+k-1} \) is the new value $m$ of the closest control point \( c_j \) from the targeted value \( s(x) \).

Generally, the B-spline basis functions with higher order, i.e. more than 1 are not a normal fuzzy set, because, the maximum value of these basis function does not reach at 1. This can be resolved by multiplying a positive number with B-spline basis function, so that maximum value of basis function reaches at 1. i.e.

\[ a = 1 / \sup_{x_i \in X_i} N_{q,2}(x_i) \]

Or,

\[ a N_{q,2}(x_i) = a_{q,2}(x_i) \]  

(20)

Fig. 2 shows the shapes of B-spline membership function.

The modeling potentials of the network is depends on the distribution of the control points, shape and size of the B-spline membership function. Where the knots and the order determine the shape and the smoothness of the B-spline membership function. The fuzzy B-spline membership function shape can be regulated by changing the value of the control points \( c_q \). All the controls points are equally allocated over the fuzzy set and \( (x_{ik}, s(x_{ik})) \) is the kth targeted value. Thus the values of control points \( c_q \) can be adjusted as

\[ a_{q,2}(x_i) = \begin{cases} 1 & \text{if } \text{round}(x_{ik}) = q \\ 0 & \text{otherwise} \end{cases} \]  

(21)

Where \( \text{round}(x_{ik}) \) is the new value $m$ of the closest control point \( c_m \) from the targeted value \( s(x_{ik}) \). Now from (16) and (19), it gives

\[ c_m = t_q(x_{ik}) = s(x_{ik}) \]  

(22)
The output of the network depends on the B-spline basis function, because, the B-spline basis function uses the product operator and their smoothness is affected by the output of the network. For example, the Fig. 2 depicted that the control points of fuzzy set and the kth target datum change the shape of the B-spline membership function.

**Fuzzy B-Spline Membership Function of Order 3:** The fuzzy B-spline membership function of order 3 is given by

\[
t_{ij}(x_i) = \sum_{q=0}^{s-1} c_q N_{q,3}(x_i)
\]

Where \( t_{ij}(x_i) \) is the membership function of the fuzzy variable \( x_i \) to the jth fuzzy set, \( c_q \) are control points, \( q=0, 1, 2, \ldots, s-1 \), ‘s’ is the number of control points and \( N_{q,3} \) is the B-spline basis function of order 3. The B-spline basis function \( N_{q,3} \) is expressed as:

\[
N_{q,3}(x_i) = \frac{x_{i+1} - t_{q+2}}{t_{q+3} - t_{q+1}} N_{q,2}(x_i) + \frac{t_{q+3} - x_{i+1}}{t_{q+3} - t_{q+1}} N_{q+1,2}(x_i)
\]

Where \( t_q \), \( t_{q+1} \), \( t_{q+2} \) and \( t_{q+3} \) are the nodes defined in the interval of \( x_i \).

The normalized third order basis function can be obtained by the (20). The control points for order 3 are changing same as explained in order 2 B-spline membership function. Fig. 3 shows the shape of B-spline membership function of order 3.

**Fuzzy B-Spline Neural Network Algorithm:** A fuzzy logic system normally accumulates its data in the shape of a fuzzy algorithm [35], which consists of a fuzzy linguistic rules relating to the input and output of the network. Then the ith rule has the form:

If \( x_1 \) is \( A_{1i} \) and \( x_2 \) is \( A_{2i} \) and
... \( x_m \) is \( A_{mi} \) Then \( u \) is \( y_i \)
Fig. 4: Structure of the ABNF

The output of the system can be expressed as:

$$u = \frac{\sum_{i=1}^{m} \mu_i w_i}{\sum_{i=1}^{m} \mu_i}$$  \hspace{1cm} (27)

The structure for the fuzzy B-spline neural network is depicted in Fig. 4. It comprises of four layers:

**Layer I**: This layer is the input layer, i.e. introduces the inputs $x_1, x_2, ..., x_m$. This layer accepts the input values and transmits it to the next layer.

**Layer II**: In this layer the fuzzification process is performed and neurons represent fuzzy sets used in the antecedents’ part of the linguistic fuzzy rules. The outputs of this layer are the values of the membership functions, i.e. $\eta_i$. The membership of $i$th input variable to $j$th fuzzy set is defined by B-spline function as

$$\mu_{ij}(x_i)=\sum_{q=0}^{x_i} c_q N_{q,2}(x)$$  \hspace{1cm} (28)

Where $\mu_{ij}$ shows the membership function of the $i$th input to the $j$th fuzzy set. Where $i=1,2,...,m$ and $j=1,2$.

**Layer III**: This layer is the fuzzy inference layer. In this layer each node represents a fuzzy rule. In order to compute the firing strength of each rule and $\min$ operation is used to estimate the output value of the layer, i.e.

$$\eta_i(x_i)=\nu_i \eta_i(x_i)$$  \hspace{1cm} (29)

where $\nu_i$ is the meet operation, $\eta_i(x_i)$ are the degrees of the membership function of the layer II and $\mu_i(x_i)$ are the input values for the next layer.

**Layer IV**: This layer is the output layer. In this layer, the defuzzification process is made to calculate the output of the entire network, i.e. It computes the overall output of system. Therefore, the output for the fuzzy B-spline neural network can be expressed as:

$$u = \frac{\sum_{i=1}^{m} \mu_i w_{pi}}{\sum_{i=1}^{m} \mu_i}$$  \hspace{1cm} (30)

Where ‘$u$’ is the output for the entire network. The training of the network starts after estimating the output value of the fuzzy B-spline neural network and $w_{pi}$ are the weights between the neurons of III and IV layers and $p=1,2,...,n$, ‘$n$’ is the number of classes.

**Parameters Update Rules**: The fuzzy B-spline neural network learning is to minimize a given function or input and output values by adjusting network parameters. The gradient descent method is used to adjust the values of weights $w_{pi}$ and the control points $c_{qj}$ of B-spline function. To minimize the error between the actual output value of the system and the desired value. For this purpose, gradient descent method is can be expressed as:

$$J = \frac{1}{2} \epsilon^2$$

Then

$$J = \frac{1}{2} (y_d - y_i)^2$$  \hspace{1cm} (31)

Where $y_d$ is the desired output of fuzzy neural network, $y_i$ is the actual output of system. The updated amount for the $w_{pi}$ and $\eta_{ih}$ can be obtained as:

$$w_{pi}(t+1) = w_{pi}(t) - \gamma \frac{\partial J}{\partial w_{pi}}$$  \hspace{1cm} (32)

$$\eta_{ih}(t+1) = \eta_{ih}(t) - \gamma \frac{\partial J}{\partial \eta_{ih}}$$  \hspace{1cm} (33)
Where $\gamma$ is the learning rate.

By using chain rule the values of $\frac{\partial J}{\partial w_{pi}}$ and $\frac{\partial J}{\partial \eta_{ij}}$ can be expressed as:

$$\frac{\partial J}{\partial w_{pi}} = \frac{\partial J}{\partial y_i} \frac{\partial y_i}{\partial u} \frac{\partial u}{\partial w_{pi}}$$

and

$$\frac{\partial J}{\partial \eta_{ij}} = \frac{\partial J}{\partial y_i} \frac{\partial y_i}{\partial u} \frac{\partial u}{\partial \eta_{ij}} \frac{\partial \eta_{ij}}{\partial \eta_{ij}}$$

(34)

(35)

By taking the derivative of the above equations, it gives

$$\frac{\partial J}{\partial w_{pi}} = -(y_d - y_i) \xi \sum_{i=1}^{m} \frac{\mu_i}{\left( \sum_{i=1}^{m} \mu_i \right)^2}$$

(36)

$$\frac{\partial J}{\partial \eta_{ij}} = -(y_d - y_i) \xi \sum_{j} w_{pi}^{-u} \mu_j$$

(37)

Where the quantity $\frac{\partial J}{\partial u}$ is approximated by a constant $\zeta$ by Frechet derivative (Lin et al. 2000) or and (Khaldi et al. 1993).

Therefore, updated values of $w_{pi}$ and $\eta_{ij}$ can be obtained by substituting (36) and (37) in (32) and (33) as:

$$w_{pi}(t+1) = w_{pi}(t) + \kappa (y_d - y_i) \xi \sum_{i=1}^{m} \frac{\mu_i}{\left( \sum_{i=1}^{m} \mu_i \right)^2}$$

(38)

$$\eta_{ij}(t+1) = \eta_{ij}(t) + \kappa (y_d - y_i) \xi \sum_{j} w_{pi}^{-u} \mu_j$$

(39)

Where $\kappa = \gamma \zeta$. Therefore, (38) and (39) gives the required updated values for the parameters $w_{pi}$, respectively. Fig. 5 depicts the closed loop diagram of the feedback system.

**Online Abnf Adaption Algorithm:** Fig. 5 shows the closed loop control structure for the proposed ABNFs control. This structure is employed to converge and update the parameters of the ABNFs control. The calculations at any instant of time can be described in the following steps.

**Step 1:** Set the input-output, $y_d$ and $y$ of the system.

**Step 2:** Update the parameters of the proposed ABNFs scheme, i.e. $\eta_{pi}$ and $w_{pi}$ by using (32) and (33) equations.

**Step 3:** Calculate the output of proposed ABNFs control strategies.

**Step 4:** Finally, output of the proposed controller added with disturbances is given to the system.

**Step 5:** Repeat steps (2-4) until solution converges.

**Simulation Result:** In this study, it is assumed that the vehicle is moving with uniform velocity unless the road disturbances create the undesired oscillations in the vehicle body. To check the performance of the proposed ABNFs control strategies, a road profile containing bumps, pothole and white noise has the form, i.e.

$$z(t) = \begin{cases} 
-15 & 1 \leq t \leq 2, 3 \leq 4 \\
15 & 7 \leq t \leq 8, 9 \leq 10 \\
 a_p(1-cos8\pi t) & 13 \leq t \leq 14, 15 \leq 16 \\
\sum_{i=1}^{N} A_i \sin(O_i \psi_i - \theta_i) & 19 \leq t \leq 22, 23 \leq 26 \\
0 & \text{otherwise}
\end{cases}$$

(40)

Where $-0.15$ m is the pothole and $0.15$ m is the bump. This type of road profile shows the low frequency high amplitude response. The term ‘$a(1-cos8\pi t)$’ shows the bell shaped bumps and ‘$a_p=15$ m’, is the amplitude of the bumps on the road. Now the term $\sum_{i=1}^{N} A_i \sin(O_i \psi_i - \theta_i)$ shows the road profile like white noise. Where value of $A_i$ is the amplitude of the road, $O_i$ is the number of waves and $\psi_i$ is the phase angle ($\psi_i=i(1)N$) ranging from ‘0’ to $2\pi$. Where
This road profile is very helpful for observing the heave, pitch and roll of the vehicle. This road profile is shown in Fig. 6.

In order to fulfill the aim of the active suspension system, the proposed strategy ABNFs is successfully implemented on full car suspension system to improve the vehicle’s stability and passenger’s comfort. The comfort is examined by the vertical displacement and acceleration felt by the passenger. The controller goal is to minimize the vertical displacement of the vehicle body with reference to the open-loop, so, as to avoid the suspension travel hitting the rattle space limits. The lifetime of elements of the vehicle is conserved by keeping away from hitting the rattle space limits i.e. to stay away from \( v = |\nabla| \) which is the allowable peak to peak displacement of the system.

Where

\[ v = x_6 - x_{[1,2,3,4]} \]  

(41)

and \( \nabla \) is the maximum limit for bumps amplitudes. The suspension travel can physically be expressed as

\[ v \leq |\nabla|, \forall t \geq 0 \]  

(42)

The controller performance is good when it reduces the vehicle vibrations under road disturbances. In this section, the simulation results of displacement of the heave, pitch, roll and seat (with and without controller) for the above mentioned road profile is given. These results are compared with passive suspension and semi-active suspension systems. The simulation time of this proposed work is up to 30 seconds.

Fig. 7 and Fig. 8 show that the response of seat is improved as compared to passive suspension and semi-active suspension system. In passive suspension and semi-active suspension, the maximum value for seat displacement is 0.1796 m and 0.088 m while, for the ABNF-1 and ABNF-2 control, the maximum value for seat displacements are 0.028 m and 0.031 m respectively. Here, the ABFN-1 and ABFN-2 increase the passenger comfort by 85% and 82% as compared to passive suspension system and 69% and 65% as compared to semi-active suspension system. The response of seat with controller \((w_c)\) is better than seat without controller \((w_{oc})\). Also, the settling time of ABNFs controllers is reduced and steady state response is improved as compared to passive and semi-active suspension systems. The seat with controller increased the passenger comfortability by 10% and 8% as compared to the seat without controllers. The ABFN-1 ameliorates the ride comfort than ABNF-2 control strategy.

Fig. 9 to Fig. 11 shows that the response of heave, pitch and roll is improved as compared to passive suspension and semi-active suspension system. In passive suspension and semi-active suspension, the maximum value of displacement for heave are 0.105 m and 0.072 m while, for the ABFN-1 and ABFN-2 controls, the maximum value of displacement for heave is 0.022 m and 0.025 m, respectively. Here, the ABFN-1 and ABFN-2 increase the passenger comfort by 80% and 77% as compared to passive suspension system and 68% and 65% as compared to semi-active suspension system. The seat with controller increased the passenger comfortability by 10% and 8% as compared to the seat without controllers. The ABFN-1 ameliorates the ride comfort than ABNF-2 control strategy.
In passive suspension and semi-active suspension, the maximum value of displacement for roll is 0.029 m and 0.017 m while, for the ABFN-1 and ABFN-2 controls, the maximum value of displacement for roll are 0.009 m and 0.01 m, respectively. Here, the ABFN-1 and ABFN-2 increase the passenger comfort by 69% and 66% as compared to passive suspension system and 48% and 44% as compared to semi-active suspension system. This increased the vehicle stability, ride comfort and passenger comfortability. Also, the settling time of ABFNN controller is reduced and steady state response is improved as compared to passive suspension. The proposed active suspension shows better performance and robustness. [33-35] From the above discussion it is observed that the ABFN-1 enhance the vehicle stability and passenger comfortability than ABFN-2 control strategy.

Fig. 12 and Fig. 13 show the suspension travel for each front and rear tyres. The results show that the ABFN-1 and ABFN-2 minimize the suspension travels by 68% and 64% than passive suspension system and 47% and 44% than semi-active suspension system. It is observed that the ABFN-1 based active suspension system provides better road handling and passenger comfort than ABFN-2 control strategy.

The performance index used for evaluation of the active control is given by,

$$P_I = \frac{1}{2} \int_0^T (Z_p^T Q Z_p)dt$$  \hspace{1cm} (43)

where, ‘Zp’ is the vector for displacement or acceleration, ‘Q’ is the identity matrix. The Root
Mean Square (RMS) value for displacement and acceleration of heave, pitch, roll and seat has been calculated by,

$$Z_{\text{disp.}}^{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T [x_6(t)]^2 dt}$$  \hspace{1cm} (44)

$$Z_{\text{acc.}}^{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T [x_8(t)]^2 dt}$$  \hspace{1cm} (45)

Table 2 shows the performance index of ABNF-1 and ABNF-2 control strategies as compared to passive and semi-active suspension systems. The performance of ABNF-1 and ABNF-2 based active suspension is checked with the said road profile.

CONCLUSION

In the paper, an ABNF control strategies with different orders of B-spline membership functions, i.e. order 2 (ABNF-1) and order 3 (ABNF-2) are successfully implemented to the full car active suspension. The objective was to improve the passenger comfort and the road handling of the vehicle against the road disturbances. The proposed ABNFs active suspension system has ability to achieve low steady-state error and fast error convergence than passive and semi-active suspension systems. Therefore, ABNF-1 and ABNF-2 based active suspension system show that it improves the vehicle stability and passenger comfort. The simulations results depict that heave, pitch, roll, seat displacement and suspension travel are reduced and road handling is improved for the active suspension systems as compared to passive and semi-active suspension systems. Also, the experimental results show that the ABNF-1 based active suspension a system performs better than the ABNF-2 based active suspension system.

REFERENCE


