A New Automatic Method For Seismic Signals Segmentation

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Abstract—The solution to the problem of seismic signals segmentation constitutes a very interesting and challenging task. The main difficulty in solving this problem is attributed to the fact that both the statistical properties of seismic noise, as well as the characteristics of the recorded events are in general unknown. In this paper, by exploiting the particular nature of the signals we are treating, and by using some interesting properties that obey a difference based test statistic as well as its ingredients, we propose an approach that results in a robust and efficient automatic detection method. From a series of experiments we have conducted in both synthetic and real seismic data, the effectiveness of the proposed technique is confirmed.

I. INTRODUCTION

The successful solution of the problem of identification of the seismic events contained from continuously recorded seismic data constitutes the basic ingredient in achieving successfully the ultimate goal of picking, that is the estimation of the arrival times of the seismic waves to the recording stations. The common approach followed, in the so called off-line techniques to solve this problem is to first detect the presence of the existing events and extract segments of the record containing one event each and then apply a picking method to each one in order to estimate the corresponding arrival time. Most well known picking methods that have been proposed, follow this approach [1], [2], [3], [4], [5], [6], [7], [8].

As it is obvious, the effectiveness of this approach depends strongly on the ability of the segmentation method to obtain a proper splitting of the signal, which in turn is heavily affected by uncontrollable factors such as the magnitude and the duration of the events as well as their separation in time. Although a number of “statistical” detection methods has been proposed [9], [10], [11], [12], the majority of the techniques used in practice are based on the ratio of a Short Term Average (STA) and a Long Term Average (LTA) of some Characteristic Function (CF) of the data [13],[14], [15], [16], [17].

The general idea behind these methods is that in areas of noise the value of the ratio should remain substantially constant, while when a signal emerges, the STA term should be able to capture the change much more quickly than the LTA, thus resulting in a sudden rise of the ratio values. The decision for an arrival is then based on the comparison of the STA/LTA ratio to a - mostly empirically - predetermined threshold.

The main reason for this is the fact that the statistical properties of seismic noise are in general unknown and it is not easy to infer them, since the factor governing the properties of the noise are not evident. This constitutes the main difficulty in solving this problem.

In this paper, by exploiting the particular nature of the signals we are treating, and by using some interesting properties that obey a difference based test statistic, as well as its ingredients, we propose an approach that results in a robust and efficient detection method.

The remaining of this paper is organized as follows. In Section II the problem formulation is presented and the behavior of the proposed test statistic in the different parts of a seismogram is considered in detail. In Section III, by exploiting a useful property of the proposed statistic, an efficient solution for the automatic segmentation of a given seismic record, is proposed. In Section IV the experimental results we obtained from the application of the proposed method on both synthetic and real seismograms are presented. Finally, Section V contains our conclusions.

II. PROBLEM FORMULATION

Let us denote with $x_n$, $n = 0, 1, \cdots, T-1$, the record from a given station and let us also assume that during the recording interval occurred an unknown number, which we denote with $K$, of seismic events. If we denote with $s_n^k$, $n = 0, 1, \cdots, T_k$, the signal produced by the $k$-th event and with $n_k$ the corresponding wave arrival time, then $x_n$ can be expressed as:

$$x_n = w_n + \sum_{k=1}^{K} s_n^{k-n_k},$$

where $w_n$ is a noise process. The problem at hand is then the detection of the presence of the events in the record and the segmentation of the record into signal and noise intervals.

III. THE PROPOSED SOLUTION

In this paper, we consider the problem of seismic record segmentation as a problem of the identification of the noise intervals of the record. The proposed solution to the above mentioned problem is based on the use of a suitable test statistic, let us denote by $\lambda_n$, which indicates a totally different behavior in noise and signal parts of the given record. To this end, let us consider set $\mathcal{T} = \{0, 1, \cdots, T-1\}$, where $T$ is the duration of the record, and the subset of $\mathcal{T}$, $\mathcal{N}$, containing all the time points $n$, such that the values of $\lambda_n$ is calculated in noise intervals of the record, with $|\mathcal{N}| = N$ being its cardinality. Then, the set $\mathcal{E} = \mathcal{T} - \mathcal{N}$, will contain all the time points $n$, where the values of $\lambda_n$ are affected by the presence of a seismic event in the record, with $|\mathcal{E}| = T-N$.

Then, by denoting the probability distribution function (pdf) of $\lambda_n$ as $f_{\lambda_n}(z)$, and the conditional pdfs of $\lambda_n$ given each one of the above defined sets as $f_{\lambda_n}(z|\mathcal{N})$ and $f_{\lambda_n}(z|\mathcal{E})$, respectively, and using the Total Probability Theorem, we have that $f_{\lambda_n}(z)$ can be expressed by the following mixture:

$$f_{\lambda_n}(z) = p_0 f_{\lambda_n}(z|\mathcal{N}) + p_1 f_{\lambda_n}(z|\mathcal{E}),$$

where $p_0 \equiv P\{\mathcal{N}\} = \frac{N}{T}$ and $p_1 \equiv P\{\mathcal{E}\} = 1 - p_0$ are the corresponding a priori probabilities of occurrence of the sets. The latter pdf is in fact by itself a mixture of pdfs, determined by the number and the characteristics of the events contained in the record (e.g. the amplitudes of the first arrivals as well as the shapes and the durations of the events). Since we are not in place to make assumptions over these characteristics, the direct estimation of this pdf from the data record is in fact not feasible. On the other hand, since we are considering seismic records of arbitrary length and seismic events are by nature sparse signals, we can safely assume that $N \gg T/2$ (or equivalently $p_0 \gg p_1$). Under this assumption, it seems reasonable to follow a strategy that ensures that all needed estimations are based on the estimation of $f_{\lambda_n}(z|\mathcal{N})$ from the data.
Since, as mentioned above, we are unaware of either the number or the form of the recorded events, the discrimination between “signal” and “noise” can only be based on the available data record, as well as the values of the used test statistic \( \lambda_n \). This last point makes it clear that the selection of a “proper” test statistic is vital for the successful solution of the problem. Namely, we are expecting that the better the ability of the used test statistic in discriminating between segments belonging to different population is, the better, and more reliable our estimations will become. A test statistic capable of achieving this discrimination is proposed in the following subsection. Moreover, under certain assumptions, which we consider to be valid in the majority of cases, the pdf governing the behavior of the proposed \( \lambda_n \), in noise, namely \( f_{\lambda_n} (z|N) \), exhibits certain “generic” properties that are independent of particular noise models considered. As it is going to be shortly presented, these properties will become our “vehicle” to the successful identification of set \( N \), and hence, the solution of the problem at hand.

### A. The Proposed Test Statistic

In this work, the following test statistic

\[
\lambda_n = L_n^{n+} - L_n^{n-}, \quad n = 0, 1, \cdots, T - 1
\]

is proposed for the solution of the problem at hand, where \( L_n^{n+} \), \( L_n^{n-} \) are defined as follows:

\[
L_n^{n+} = \frac{1}{M} \sum_{k=n}^{n+1} y_k, \quad L_n^{n-} = \frac{1}{M} \sum_{k=n-M}^{n} y_k,
\]

and \( y_n = g(x_n), \quad n = 0, 1, \cdots, T - 1 \), is a nonlinear positive transformation of the available signal \( x_n \) (e.g., the absolute or the squared value). Let us now concentrate ourselves on the behavior of the above defined statistics in the different parts of a seismogram. To this end, let us consider that \( \eta_i, i = 1, 2, T_1 \) denote the onset time, the stopping time and the duration of the \( k \)-th, seismic event, respectively, and discriminate the following two cases regarding the specific segments of record used for the computation of the above mentioned statistics.

\( C_1 \): \( n \in N \). From the above definition it is clear that for each \( n \in N \), RVs \( L_n^{n+} \) and \( L_n^{n-} \), and consequently RV \( \lambda_n \) which is defined by their difference, are computed by using samples belonging to noise segments of the record. As we have already mentioned, our main “tool” for the identification of set \( N \) is the behavior of the test statistic \( \lambda_n \) in this set, i.e. for \( n \in N \). In order to assess this behavior in a statistical manner, certain assumptions have to be made concerning its ingredients, \( L_n^{n+} \) and \( L_n^{n-} \), \( n \in N \). More specifically, in this work the following statements are considered to be valid, although we cannot prove them without imposing some assumptions on the noise process:

- \( S_1 \): RVs \( L_n^{n-} \) and \( L_n^{n+} \) are considered to be identically distributed (i.d.), according to some (unknown) pdf, which we denote by \( f_{\lambda_n}(z) \).
- \( S_2 \): RVs \( L_n^{n-} \) and \( L_n^{n+} \) are exchangeable [18], that is their joint pdf, \( f_{L_n^{n-}, L_n^{n+}}(x, y) \) is bivariate symmetric, i.e. \( f_{L_n^{n-}, L_n^{n+}}(x, y) = f_{L_n^{n-}, L_n^{n+}}(y, x) \).

We must stress at this point that in the case of an i.i.d. noise process, both \( S_1 \) and \( S_2 \) are satisfied. Note though that this is a sufficient and not a necessary condition. We proceed now to state an interesting property satisfied by the distribution of the difference of two i.d. RVs whose joint pdf function exhibits the symmetry mentioned in \( S_2 \).

**Proposition 1**: If the joint pdf of i.d. RVs \( X, Y \), is bivariate symmetric, then the pdf of their difference, \( Z = X - Y \) is even symmetric.

A direct consequence of Proposition 1 is that the median value of the test statistic defined in Equ. (3), for \( n \in N \) equals 0, or equivalently, \( P(\lambda_n > 0|N) = 1/2 \). As we are going to see in the next subsection, these properties constitute very useful tools for the successful identification of set \( N \).

\( C_2 \): \( n \in \mathcal{E} \). Contrary to \( C_1 \), in this case the behavior of the statistics depends on uncontrollable factors such as the magnitude and the shape of the events, as well as their separation in time and as such, cannot be assessed in a statistical manner. It can be explained only intuitively, by taking into account the particular statistics used, as well as the general characteristics of the seismic signals, namely the fact that the energy of the record is higher in signal intervals than its energy in noise intervals, the “abrupt” change occurring in the beginning of an event, and the “generic” amplitude envelope of these particular signals, i.e. their fading nature.

Specifically, let us first concentrate on the sequences \( L_n^{n+} \) and \( L_n^{n-} \), calculated for a given record. These sequences behave as smooth positive “envelopes” of \( y_n \), following conceptually its shape in intervals containing events, and attaining low values, around a constant level, in intervals of noise. Based on this, and due to the increased energy of the record in signal intervals, compared to the noise ones, these sequences take higher values during the occurrence of an event, than they take in intervals of noise, making these statistics an appropriate segmentation “tool”.

The behavior of \( \lambda_n \) on the other hand, is different in different parts of a signal interval, and as such, cannot be described “uniformly” for the whole interval. Based on the above mentioned general characteristics of the seismic signals, the presence of the \( k \)-th event in the record is reflected in the \( \lambda_n \) sequence by the presence of a narrow peak (of approximately \( 2M \) samples in the beginning of such intervals (centered on the onset time of the event), were \( \lambda_n \) takes large values (depending on the signal-to-noise ratio), followed by a large interval (of approximately \( T_k \) samples), of values that are biased towards a negative level (due to the fading of the event amplitude). Note also that \( \lambda_n \), being a difference statistic, is sensitive to the amplitude of the recorded events, which is a highly desirable feature for the task at hand. By taking these into account, it becomes apparent that the behavior of \( \lambda_n \) in signal intervals is highly different from its behavior in noise intervals, concerning both the amplitude of the values it attains, as well as the lack of the assumed symmetry it possesses in the latter intervals. To put this into a statistical perspective, there are two main features that differentiate \( f_{\lambda_n}(z) \) from \( f_{\lambda_n}(z|N) \), namely its “heavy” tails, as a result of the “extreme” values of the statistic in signal intervals, and the lack of the symmetry property possessed by the latter pdf (expressed by Proposition 1), again as a result of the highly non-symmetric behavior of \( \lambda_n \) in the signal intervals. The aforementioned behavior of the used statistics in a synthetic data record, is displayed in Fig. 1.

### B. Identification of set \( N \)

In this section, we treat the problem of segmentation of the seismic record. The main idea is to first locate all the intervals of the record that can be considered as “candidates” for containing seismic events, and then to remove iteratively in a systematic manner each of these intervals, up to the point where the remainder of the record can be considered as “pure” noise. In order to achieve our goal, we are going to exploit the differences between \( f_{\lambda_n}(z) \) and \( f_{\lambda_n}(z|N) \), which as presented above, are attributed to the presence of events in the record. To this end, we are going to define appropriate measures for quantifying these differences, and a suitable two-step procedure that will take us from set \( T \) (the whole record), to the desired set \( N \). Specifically, in the first step we exploit the appropriateness of statistic \( L_n^{n+} \) as a segmentation tool and we obtain a “gross” segmentation of the record, by thresholding the \( L_n^{n+} \) sequence, using a very conservative threshold, thus making sure that all the signal intervals are selected. In the second step, we identify the desired set by solving a well defined optimization problem, based on the aforementioned
characteristics of $f_{\lambda_n}(z|\mathcal{N})$. Let us now analyze in detail each one of the above steps, starting from step one, that is the “gross” segmentation of the record. Let us consider the following sequence of intervals 

$$\tilde{E}_1, \tilde{E}_2, \ldots, \tilde{E}_L,$$  

resulting from the sequence $L^M_{m_n^+}$, $n = 0, 1, \ldots, T - 1$ by sequentially performing the following actions:

- $A_1$: find the intervals where the values of $L^M_{m_n^+}$ are greater than $m_L$, and
- $A_2$: sort the resulting intervals according to the variance of $\lambda_n$ in each one of them, in descending order,

where $m_L$ denotes the median value of sequence $L^M_{m_n^+}$, $n = 0, 1, \ldots, T - 1$, which by taking into account the sparsity of the seismic events in the data record constitutes a good estimator of the median of this statistic under the assumption of noise $(n \in \mathcal{N})$, $m_{L|\mathcal{N}}$.

Since the selected threshold is indeed a very conservative one, we anticipate that the union of all intervals formed from the application of action $A_1$, will contain all the segments of the signal intervals of the record (i.e. all $\tilde{E}_k$), as well as a great number of other segments, containing only noise, due to the random fluctuations of $L^M_{m_n^+}$ around $m_L$, in the noise intervals. Based on the fact that in intervals containing seismic events $\lambda_n$ takes greater values than in intervals of noise, by sorting the intervals according to $A_2$, the signal intervals are anticipated to be in the beginning of the resulting sequence, followed by the ones that contain only noise.

Let us now proceed to the second step of the procedure, namely the identification of the desired set. To this end, let us define the following sequence of sets:

$$\tilde{N}_l = \tilde{N}_{l-1} \setminus \tilde{E}_l, \quad \tilde{N}_0 = T, \quad l = 1, \ldots, L,$$  

where ‘\setminus’ denotes set subtraction, formed by iteratively discarding the “candidate” signal intervals obtained in the first step of the procedure, from the record. Note now that this sequence of sets can be used for the definition of the following sequence of pdfs:

$$f_{\lambda_n}(z|\tilde{N}_l) = \frac{|\tilde{N}_{l-1}|}{|\tilde{N}_l|} f_{\lambda_n}(z|\tilde{N}_{l-1}) - \frac{|\tilde{E}_l|}{|\tilde{N}_l|} f_{\lambda_n}(z|\tilde{E}_l),$$  

with $f_{\lambda_n}(z|\tilde{N}_0) \equiv f_{\lambda_n}(z)$, and $|\tilde{N}_{l-1}| = |\tilde{N}_{l-1}| - |\tilde{E}_l|$, by definition.

If there exists a member $\tilde{N}_l$ of the set sequence that is a good approximation of the desired set $\mathcal{N}$, then the corresponding pdf $f_{\lambda_n}(z|\tilde{N}_l)$ can be considered as a good approximation $f_{\lambda_n}(z|\mathcal{N})$. Thus, our goal now is to identify this member of the set sequence defined in Eqn (11). In order to solve this problem we resort to the above analysis concerning the assumed characteristics of $f_{\lambda_n}(z|\mathcal{N})$. Specifically, we define the following cost function (CF):

$$C(\tilde{N}_l) = E[\lambda_n^0|\tilde{N}_l] \sup_x \int_{-\infty}^{0} f_{\lambda_n}(z|\tilde{N}_l) dz - \int_{0}^{x} f_{\lambda_n}(z|\tilde{N}_l) dz,$$  

in order to assess the degree by which the estimated pdf exhibits the aforementioned characteristics, namely the lack of “extreme” values, measured by its variance, and the symmetry property imposed by Proposition 1, measured by the second term of the CF. In order to complete our analysis, the following proposition (that we give without proof), stating a very interesting statistical property of the difference of two i.i.d. RVs is of crucial importance, as we shortly see.

Proposition 2 : Let $X, Y$ be two i.i.d. RVs, and let $m$ denote their median value. Let also $Z = X - Y$. Then $P(Z > 0 | X > m) = \frac{m}{2}$. Let us now give a more descriptive outline of the procedure followed. To this end, let us assume that the first $K$ elements of the sequence defined in Eqn. (5), are intervals that correspond to seismic events, while the rest $L - K$ correspond to noise, meaning that the solution we are seeking in this case, is $l^* = K$, or equivalently $\tilde{N}_K = \tilde{N}_L$. In the $l$-th iteration of the procedure, we consider the values of set $\{\lambda_n|n \in \tilde{N}_l\}$ as a sample drawn from the distribution governed by the pdf $f_{\lambda_n}(z|\mathcal{N})$ and obtain an estimation of the latter, by a detailed histogram of the sample. Since the first $K$ intervals of the sequence ($\tilde{E}_l, l = 1, \ldots, K$) are signal intervals, for reasons made clear above, sets $\{\lambda_n|n \in \tilde{N}_l\}$, $l = 1, \ldots, K$, will gradually contain fewer “extreme” values and will increasingly exhibit the aforementioned even symmetry, up to set $\{\lambda_n|n \in \tilde{N}_K\}$, which represents the “best” approximation of set $\{\lambda_n|n \in \mathcal{N}\}$ in the given example. This will result in a rapidly decreasing sequence of $C(\tilde{N}_l)$ values, up to the $K$-th iteration. From this point on, although the removed intervals ($\tilde{E}_{l+1}, \tilde{E}_l, \tilde{E}_L$) are noise ones, due to the particular way followed for their identification (recall that from $A_1$, for all $n \in \tilde{E}_l$, $L_{m_n^+} > m_L$ holds), and based on the result of Proposition 2, the values of $\lambda_n$ contained in them are expected to be heavily biased towards a positive level. As a consequence, sets $\{\lambda_n|n \in \tilde{N}_l\}$, resulting from $\{\lambda_n|n \in \tilde{N}_K\}$ by iteratively removing sets $\{\lambda_n|n \in \tilde{E}_l\}$, $l = K + 1, \ldots, L$, will exhibit the even symmetry to an increasingly lesser degree, thus leading to an increasing sequence of $C(\tilde{N}_l)$ values, for $l = K + 1, \ldots, L$. This behavior of $C(\tilde{N}_l)$ is depicted in Fig 1.c.

Based on the above, we are now in place to define the desired solution as the member of the sequence of Eqn. (6) that leads to the minimization of $C(\tilde{N}_l)$ over $l$, i.e.: 

$$l^* = \arg \min_{1 \leq l \leq L} C(\tilde{N}_l),$$  

resulting in the following estimation of the desired set: $\tilde{N}_l^* \approx \mathcal{N}$. Note that in the example of Fig 1.c, $C(\tilde{N}_l)$ attains its minimum for $l = 3$, which is the correct number of events in this case. The result of the segmentation is indicated in Fig 1.a by the red line. Having completed the presentation of the proposed technique, in the next section we are going to apply it in a number of experiments.

IV. EXPERIMENTAL RESULTS

In this section we evaluate the performance of the proposed method by applying it in both synthetic as well as real data sets.
A. Experiment I

In the first experiment, we applied the proposed method to a synthetic data set containing a large number of synthetic records, and we measured the percentage of successfully detected events as a function of their Signal-to-Noise Ratio (SNR). The synthetic seismic signals were modeled as low-pass filtered Gaussian noise, multiplied by a half-Gaussian window for the effect of amplitude shaping, and a constant gain, controlling the SNR. In order to construct a data record, first the noise process $w_n$ was created and then a value for the number of events $K$ was selected randomly in a range $[K_{\min}, K_{\max}]$. Then the synthetic signals $x_n^s$ were created as described above. Finally, the onset times were obtained by random selection in the interval $[1, T]$ and the resulting “recorded” signal $x_n$ was calculated by using Equ. (1). In this experiment, the noise process used was the superposition of an AR(1) process, with its pole equal to 0.7 (thus resulting in correlated data), and a white Gaussian process. An example of a synthetic record created using this setup, containing three (3) events with SNRs 5.5 and 7, respectively is shown in Fig. 1. Finally, for the given experiment, the following selection of the various parameters used by the method was made: window size $M = 200$, record duration $T \approx 30000$, $K_{\min} = 5$, and $K_{\max} = 10$. Also, the positive transformation used was the squared value of the samples, i.e., $y_n = g(x_n) = x_n^2$. The measured detection percentage for values of SNR ranging from -2 to 10 dB is shown in Fig 2. The curve depicted in this figure is a clear demonstration of the high performance and the robustness of the proposed method, resulting in acceptable percentages even in the extremely unfavorable cases of negative SNRs (taking also into consideration the noise process used). The detection percentage reaches values close to 1 even for very low positive SNR values close to 2. Of high importance is also the fact that the number of “false alarms”, (i.e. noise intervals that were falsely detected as signal ones) was very low in this experiment, averaging approximately 0.9 false alarms per record.

B. Experiment II

In this experiment we evaluate the performance of the proposed method by applying it in real seismic data. The real data set, was comprised by 300 pre-cut records of continuously recorded seismic data, during a period of high seismicity. The “true” number of events, counted by a human analyst, contained in the above mentioned records were 2312, with different amplitudes and durations. By using $M = 200$ (2 sec), the proposed detector was applied to the above data set and succeeded in identifying 2098 events, with 96 false alarms. This results in a successful identification in approximately 91% of the cases, thus reinforcing the findings of Experiment I and confirming its appropriateness for the problem at hand. This is also evident in Fig. 3 where the results of the solution to the detection problem for a record real data is shown.

V. CONCLUSIONS

In this paper, the problem of identification of the seismic events contained from continuously recorded seismic data was examined.

Fig. 2. Successful detections as a function of SNR, for the experiment using the synthetic data set.

Fig. 3. Application of the proposed method to a real seismic record. The detected events are indicated by the red line.

The use of a difference test statistic, as well as a two step procedure for solving the problem at hand, were proposed. The effectiveness of the proposed technique was confirmed from its application in solving the detection problem on a series of experiments, where synthetic and real seismic records were used.

VI. REFERENCES