On the throughput performance of cluster-based cognitive radio networks

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Abstract: This paper addresses effect of reporting channel bandwidth on cognitive radio (CR) networks. A cluster based approach is considered where the secondary base station is replaced by a fusion center and a global reporting channel is used instead of local ones. A new approach to select the fusion center based on the general centre scheme in graph theory is proposed. The minimal dominating set (MDS) clustering approach is used to minimise the set of clusters that keeps the network connected. The effect of various parameters such as cluster size and number, quality of the reporting channel and sensing time on sensing efficiency, accuracy and per node throughput are investigated. Results show cluster based cooperative sensing throughput outperforms conventional cooperative sensing especially when the reporting channel has high probability of error. Systematic ways to determine optimum number of clusters and optimum sensing time are developed.

Keywords: spectrum sensing; minimal dominating set; MDS; cluster head; throughput; fusion centre; false alarm probability; missed detection probability; cognitive radio; reporting channel.


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1 Introduction

Cognitive radio (CR) networks use sophisticated spectrum sensing techniques to make unused licensed spectrum available to unlicensed (secondary) users while avoiding interference to the licensed (primary) user. The spectrum sensing, in general, perform the following two functionalities (Akyildiz et al., 2009):

1. locating spectrum holes so that CR users can communicate in the absence of the primary network

2. monitoring the spectrum band and prepare to abandon the channel immediately as soon as the primary network resumes its operation.

This spectrum sensing cycle is followed by a transmission cycle. CRs search or monitor the spectrum bands in the spectrum sensing cycle and serve their users in the transmission cycle. Network throughput and quality of service (QoS) are strongly associated with the length of each of these cycles.

Ideally, secondary users should guarantee no interference to the primary user. Therefore, perfect spectrum sensing is very important. However, in a real communication environment, the communication channels between the primary and secondary users may be impaired due to shadowing and fading. Under deep shadowing/fading, CR users may mistake a weak primary signal for a vacant channel. Therefore, individual local sensing is prone to errors and unreliable. This can be improved by sharing the local observations among few CR users in a cooperative fashion adding diversity.

Cooperative spectrum sensing, therefore, is introduced in cognitive radio networks to improve the reliability of primary user detection by exploiting the spatial diversity among cooperative secondary radios. Cooperative sensing, however, requires secondary users to exchange their local observations with a fusion centre (secondary base
station) for decision making. Obviously, two successive channels must be considered in
cooperative sensing (Letaief and Zhang, 2009):

1. the sensing channel between the primary user and the CR terminal

2. the reporting channel between the CR terminals and the fusion centre.

The reporting channel is used by the CR terminals to report their local observations or
individual decisions to the fusion centre. This channel is also a limited bandwidth channel
and may experience shadowing and multi-path fading. Since, bandwidth limitations and
channel conditions of the reporting channel have direct influences on the cooperative
sensing performance, efficient schemes should be devised to reduce reporting bandwidth
requirements and to combat the channel conditions.

Typically, the number of cooperative users and the amount of information that each
user should send to the fusion centre determine the required bandwidth. For a large
network, the required bandwidth may exceed the capability of the reporting channels,
especially when users send their local statistics (the real time measurements) to the
fusion centre instead of individual decisions.

This issue can be improved by forming clusters to share the sensing information
locally (Zahmati et al., 2009). In this approach, the secondary users are grouped into
separate clusters. Each group then elects a cluster head that locally coordinates the
access to the shared spectrum by cluster members. In addition, cluster head acts as a
local fusion centre to forward an aggregated decision (hard decision fusion) or a linear
combination of the local observations (soft decision fusion) to the fusion centre. With
such scenario, the access to the reporting channel is significantly reduced while all CR
users are still taking part in the cooperative process.

Clustering is used widely as a hierarchical approach of topology management in
Ad-Hoc wireless networks (Younis and Fahmy, 2004; Perevalov et al., 2006; Basagni,
1999; Amis et al., 2000). Also, it has been shown in Quan et al. (2007) and Song and He
(2007) that clustering geographically close and frequently interacting nodes significantly
improves the network throughput. To demonstrate that, the per-node throughput capacity
of a general-purpose non-clustered network is shown to be $\Theta(R/\sqrt{\log(N)})$ (Gupta
and Kumar, 2000), where $R$ is the common transmission rate of each node and $N$
the total number of nodes in the network. This result indicates that the per node
throughput capacity decreases as the network size increases. On the other side, the
spatial tessellation of the network into connected clusters greatly limits the number of
hops from the source node to the destination node and significantly reduces the relay
burden carried by each node leading to a higher throughput (Quan et al., 2007).

Clustering is also useful in minimising the amount of data required to be exchanged
for cooperation by adopting the spatial reuse of the shared spectrum in terms of
TD and FD schemes (Basagni, 1999). However, with CR networks, clustering must
consider the fact that the available channel sets are changing temporally and spatially.
The need for secondary users to track the radio environment in real time to ensure
a continuous connectivity must also be considered. Furthermore, the cluster sizes and
cluster locations will change when CRs neighbourhood changes with each new operating
frequency. Consequently, clustering in CR networks is performed according to channel
topology instead of node topology. Specifically, a CR node forms a cluster on an
available channel and invites adjacent nodes to join its cluster if the same channel is
available in their channel sets (Chen et al., 2008). Another main difference compared
to Ad-Hoc networks is that the assumption of the global control channel existence may not work with CR networks due to the temporal change of channel sets that follows the existence of spectrum holes and the secondary users themselves. Therefore, the distributed cooperative approach seems to be the best solution for this problem where the need for the global control channel is eliminated since all tasks are performed at the local scale.

Cluster-based spectrum sensing is divided into rounds of three phases: sensing, cluster setup, and transmission phase. During the sensing period, spectrum holes are located and channel sets are made available for data exchange. CR users need to synchronise their spectrum sensing phase to avoid false alarms that may be triggered by some CRs that started their spectrum sensing earlier. The synchronisation is one of the critical challenges facing the distributed system due to the absence of central coordinator and becomes more difficult with increasing number of users (Li and Rus, 2006). In the second phase, clusters are formed and cluster heads are elected based on one or more network metrics such as node ID, node degree, reporting channel gain, communication cost, load balance etc. The setup phase is followed by a transmission phase when CR terminals communicate and exchange data using the set of the sensed channels that are originally owned by the primary network. The available band is only utilised during this phase and it stays idle during sensing and clustering phases. The length of the transmission period determines how efficiently the available spectrum band can be utilised. Thus, even though the cluster set-up time is much shorter than the channel transmission time, it is not preferable to perform the cluster set-up at every sensing round so as to improve sensing efficiency. Figure 1 illustrates the sensing round structure for a cluster-based cooperative sensing.

Figure 1  Sensing round structure in cluster-based spectrum sensing

Notes: $T_M$ – channel monitoring time, $T_C$ – cluster set-up time, $T_D$ – data transmission time, $T_S$ – sensing time, $T_P$ – sensing period

Network traffic in cluster-based systems is generated mainly by intra-cluster and inter-cluster communications. Inter-cluster communications happen between the cluster heads and traffic relay gateways. A gateway is a CR user who might be in one-hop from two neighbouring cluster heads in case of overlapping clusters or in one-hop to another gateway in an adjacent cluster in the case of disjoint clusters. In both traffic types, the packets generated by a source node may reach the destination node through a single-hop
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or a multi-hop routing. For clustering to be effective, the number of cluster heads, gateways, and the links that are connecting them must be minimised while preserving the connectivity of the whole network (Bao and García-Luna-Aceves, 2003). Minimising the number of clusters reduces the overhead traffic and the network maintenance requirements. However, clustering must consider the traffic load at both cluster heads and the gateways as these nodes tend to be the bottlenecks of the entire network. Setting up an upper bound for the cluster size is essential here as it prevents the overcrowding of nodes in the clusters and avoids traffic congestion at the interconnection nodes.

In this paper, we investigate the performance of expand minimal dominating set (MDS)-based clustering algorithm and introduce an approach to select the best location for the local fusion centre that acts as a master fusion centre in a totally distributed system. The clustering algorithm aims to find the minimal number of clusters that cover all the users operating in the field while preserving network connectivity. In particular, the suggested approach aims to reduce the bandwidth required for reporting data to the fusion centre by reducing the number of reporting terminals. Since only the cluster heads report, then the objective is to minimise the number of cluster heads, i.e., the number of clusters. We also study how the sensing efficiency, sensing accuracy, and the per-node throughput are affected by the various parameters of the clustering scheme. Specifically, we investigate the influence of cluster size, number of clusters, and the sensing time on the spectrum sensing performance.

2 Network model

In this work, we assume a distributed system that does not have a centralised entity such as a base station. Instead, one of the elected cluster heads will serve as a master fusion centre to finalise the cooperative decision and broadcast it back to the cluster heads and from there to the cluster members. Following this assumption, the network has two levels of data fusion (data aggregation). The first data fusion is performed at the cluster level by each cluster head and the second one is performed at the master fusion centre.

The cluster head elected as a master fusion centre will perform both levels of data fusion. In addition, the master fusion centre synchronises all clusters within its coverage so that all the terminals start their sensing period coherently. We assume that CR terminals are deployed in a square field of area $A$. We define the cluster as a group of CR terminals that share the same channel within a circle of a radius $r_c$. Also, we assume that any terminal elected as a cluster head applies power control to communicate with the fusion centre with a different transmission range $R$, where $R > r_c$.

We assume that CR terminals are distributed uniformly as a two-dimensional Poisson point process with a density $\lambda$. A node (CR terminal) $v$ is said to be in the neighbourhood of a node $u$, if $v$ is within a distance at most $r_c$ from $u$. Each CR terminal is identified by a unique ID and assigned a channel set $S$ after each sensing period. The number of channels and their IDs in a given channel set is assumed to change from one sensing period to another. However, we assume the change in the channel set is relatively slow and the CR Terminals are static or moving slowly during the algorithm execution.

The topology of the CR network is presented by a undirected graph $G(V, E)$, where $V$ is a set of vertices in this graph that stands for CR terminals and $E$ is a set of links
between those terminals. A link \((u, v) \in E\) means that terminals \(u\) and \(v\) are sharing at least one common channel, i.e., they are connected and in one-hop distance from each other. Adjacency matrices are calculated after each simulation trial to show all the links between network nodes that are spaced one-hop of each other. The neighbourhood set of a given node \(v \in V\), represents all the nodes in a one-hop away from \(v\) that share at least one channel with \(v\).

Transmission power determines the cluster radius and largely affects the number of clusters, cluster size, and network performance. Higher transmission power means fewer hops resulting in higher network throughput. However, the higher interference resulting from higher transmission power tends to limit the network throughput. This is true in the other way as well.

We investigated the impact of transmission power on the network topology. Precisely, we investigate how the average number of isolated nodes \((N_{iso})\) is changing with cluster radius. Isolated nodes refer to those nodes that can not find any neighbouring terminal within a radius of \(r_c\) in any channel within their channel sets. These, nodes declare themselves as cluster heads and form their own clusters (single-node clusters).

Since, CR terminals are distributed as a Poisson point process with a density \(\lambda\), then, the probability that a node is unable to find any neighbouring node within a radius \(r_c\) is \(e^{-\lambda \pi r_c^2}\). If we consider a two dimensional polar coordinate \((r, \theta)\), where the node is in the origin, then the average of isolated node \(E(N_{iso})\) can be calculated as follows (see Su and Zhang, 2006),

\[
E(N_{iso}) = \lambda A \int_0^{2\pi} \int_{h_p r_c}^{\infty} e^{-\lambda \pi (h_p r_c)^2} \lambda r dr d\theta
\]

\[
E(N_{iso}) = \lambda A e^{-\lambda \pi (h_p r_c)^2}
\]

where \(h_p\) denotes the number of transmission hops. Since, our approach is to establish a connected MDS, i.e., to find the minimal number of connected clusters that covers all the deployed CR terminals, then the fraction of single-node clusters, \(E(N_{iso})\), must be very small. If \(\kappa\) (\(\kappa > 0\)), is an arbitrarily small number that represents a target percentage of single-node clusters such that,

\[
E(N_{iso}) \leq \kappa \lambda A.
\]

then by equating (2) and (3) we obtain the following lower bound of cluster radius at this specified target,

\[
r_c \geq \sqrt{\frac{-log \kappa}{\pi \lambda h_p^2}}
\]

Increasing the cluster radius will clearly increase the probability to find a neighbouring terminal for each node and increase the cluster size. However, as the cluster size increases, the probability to have network bottlenecks at the inter-cluster communication gateways increases. Also, the inter-cluster and intra-cluster interferences tend to increase with larger cluster communication range. Hence, to avoid congestion at those nodes, we introduce an upper bound for the cluster size such that the traffic load at those gateways will be upper bounded and adhere with the QoS requirements.
With low cross correlation CDMA codes, inter-cluster interference can be eliminated. We assume each cluster is assigned a unique transmitting code that is different from those codes used in neighbour clusters. Since the receiver nodes must be set to the same code as the designated transmitter, interference with other clusters is avoided. If no two nodes in a cluster are transmitting simultaneously, there will be no intra-cluster interference. Following Lin and Gerla (1997), we assume that within each cluster, the channel is slot synchronised using TDMA scheme in which each node is assigned a single time slot for transmission.

**Figure 2** Fraction of single-node clusters vs. cluster radius for 1, 2, 3-hop clusters (see online version for colours)

**Figure 3** Fraction of single-node clusters vs. cluster radius under different node densities (see online version for colours)
We conducted several trials with random deployment of 100 CR terminals in an area of 100 × 100. In each trial, the cluster radius varied and the percentage of the disconnected node is recorded. The average number of disconnected node over these trial is illustrated in Figure 2 for 1, 2, and 3-hop clustering scheme. As it can be shown, a fraction of $\kappa < 0.001$ can be achieved with a cluster radius of 15 m, 11 m, and 9 m for 1-hop, 2-hop, and 3-hop clustering scheme respectively. In Figure 3, the percentage of the disconnected nodes is plotted vs. the cluster radius with different node densities for a single-hop clustering scheme.

3 The clustering algorithm

In graph theory (Bao and Garcia-Luna-Aceves, 2003), the MDS problem is to find a subset, $C(|C| = M)$ called the dominating set such that each node belongs to one member of $C$. We refer to the dominating set members as cluster heads and the nodes that belong to one of the cluster heads as cluster members. Finding a MDS is NP-hard in general (Amis et al., 2000; Krishna et al., 1997), but sufficient approximation algorithms exist. These algorithms obtain a sub-optimum dominating set by generating a local minimum election of the dominators.

The algorithm requires a preliminary node discovery and a master channel configuration phase. The master channel is used by the cluster heads and cluster members to exchange the control data necessary for the cluster formation. We assume that at the completion of the node discovery, any node say $v_i$ knows the channel availability set of node $v_j \in v_i^1$ and the set of one-hop neighbours of the node $v_j$, $\forall v_j \in v_i^1$, where $v_i^1$ denotes the one-hop neighbourhood of node $v_i$. The node discovery phase constructs the base of the MDS algorithm which is performed periodically at the node level to determine the locally optimised cluster set. The collection of the local dominators then constructs the sub-optimal dominating set of cluster heads.

During the preliminary phase, a node listens to an advertisement message of a neighbouring cluster head on any of the channels available in its channel set. If no such message is heard in any channel, such as the case in the isolated terminals, the node, declares itself as a cluster head and forms a cluster on a randomly selected channel. Otherwise, the node replies with a cluster join request on the first listening channel where it hears an advertisement message. The advertisement message is a short message send by a cluster head to inform other nodes of its headship and asks them to join its cluster on a channel selected by the cluster head. It includes information about cluster ID, channel ID, and channel set availability. The selected channel acts as a local control channel that is used by cluster members to exchange control signals.

In MDS scheme, every node $i \in V$ is required to be covered by one member of the dominating set $C \subseteq V$. The dominating set contains $M$ subsets $C_1, C_2, \ldots, C_M$ of the base set $V = \{1, 2, \ldots, N\}$ such that $\bigcup_{j=1}^M C_j = V$. We define a binary variable $x_j$ for the subsets $C_j, j = 1, 2, \ldots, M$ as follows,

$$x_j = \begin{cases} 1 & \text{if } C_j \in C \\ 0 & \text{otherwise.} \end{cases}$$
By defining $a_{ij}$ to be 1 when a node $i \in C_j$ and 0 otherwise, we can write the problem as,

$$
\min \sum_{j \in C} x_j \quad j = 1, 2, ..., M
$$

subjected to,

$$
\forall i \in V, \sum_{j \in C} a_{ij} = 1
$$

$$
\forall i \in V, \sum_{j \in C} a_{ij} d_{ij} \leq h_p r_c
$$

$$
\forall j \in C, \sum_{i \in V} a_{ij} \leq P
$$

where $P$ is an upper bound of cluster size.

We extend the single-hop CogMesh algorithm presented in Chen et al. (2008) to a d-hop clustering scheme and generalised this algorithm for the above described system model. The MDS algorithm is performed at the node level to minimise the number of clusters in the neighbourhood of a selected node and reconfigure the whole cluster topology when a new cluster set smaller than the original one can be found. Instead of selecting a node in a random way as in Chen et al., 2008, our approach selects a node with the highest reporting channel gain from a set of neighbouring nodes that share a channel with a maximum degree to be a cluster head. The algorithm starts forming a cluster from this cluster head and all nodes within d-hop, $d = 1, 2, ..., h_p$, that shares the same channel. The new cluster head and all the assigned cluster members will be eliminated from the node neighbourhood set. Then, a node with a maximum degree on another channel will be selected as a cluster head and its assigned cluster members eliminated too from the remaining node set and so on until all the nodes are configured in the new cluster topology. The algorithm then starts the gateway nodes selection to construct the inter-cluster communications. A priority case is used in gateways selection. A node in the shortest path between any two cluster heads is given the highest priority. Once a cluster is formed, the cluster head communicates with the neighbours to select the CDMA codes. Only when the code assignment is completed data can be transmitted in the network.

**Lemma 1:** In a d-hop cluster, the distance between any two nodes is at most $2d r_c$.

**Proof:** Let $v$ be any node belongs to cluster $C_j, j = 1, 2, ..., M$ whose cluster head is $m_j$. If $v(ID) \neq m_j(ID)$ then there must be another node $u \in C_j$ at most in a d-hop away from $v$ where $u(ID) = m_j(ID)$. Hence, $u$ is the cluster head of the cluster $C_j$ and $|v - u| \leq d r_c$. For any other node say $w \in C_j$, whose $w(ID) \neq m_j(ID)$, $|w - u| \leq d r_c$. Therefore, the distance between $v$ and $w$ is at most $2d r_c$.

**Lemma 2:** If $C$ is a connected dominating set, then any cluster head is at most $(2d + 1)r_c$ away from the nearest cluster head.

**Proof:** Let assume the contrary, and assume that the closest routing path between cluster heads $m_1 \in C_1$ and $m_2 \in C_2$ is $(2d + 2)r_c$. Let this closest path passes through the two gateways, $x$ and $y$, (see Figure 4). Since, $x \in C_1$ and $y \in C_2$, then $|m_1 - x| \leq d r_c$ and
\[ |m_2 - y| \leq d_r c \text{ and } |x - y| \geq (2d + 2)r_c - 2d_r c = 2r_c. \] This implies that each of \(x\) and \(y\) is not in a one-hop transmission range of each other and the two clusters are not connected. Since the dominating set is a connected dominating set, then, there must be another gateway \(v \in C_1\) which is in one-hop to another cluster head say, \(m_3 \in C_3\) or adjacent node say \(z \in C_3\) which is at most \(d\)-hop from \(m_3\), i.e., \(|m_3 - z| \leq d_r c\). Therefore, \(|m_1 - m_3| = |m_1 - v| + |v - z| + |z - m_3| \leq d_r c + r_c + d_r c = (2d + 1)r_c\).

\[ |m_1 - m_3| = |m_1 - v| + |v - z| + |z - m_3| \leq d_r c + r_c + d_r c = (2d + 1)r_c. \]

\(m_3\) is the nearest cluster head to \(m_1\).

**Figure 4** Maximum distance to the nearest cluster head (see online version for colours)

The master fusion centre is then elected from the minimal cluster head set. The placement of the fusion centre also needs to be optimised. The fusion centre placement problem is concerned with selecting the best location in a specified region for the network centre entity. Mainly, there are two options to solve this location problem, the centre problem and the general centre (Minieka, 1978). In graph theory, the graph centre is any vertex \(v\) whose furthest vertex is as close as possible while the general centre is any vertex \(v\) where the aggregated distance from all other vertices is as minimum as possible. Let,

\[ M^v v(i) = \max_{j \neq i} d_{ij} \]
denotes the maximum distance of any vertex from vertex \((i)\), where \(d_{ij}\), is the vertex-to-vertex distance, and,
\[
S_{vv}(i) = \sum_{j \neq i} d_{ij}
\]
denotes the aggregated distance of all vertices from vertex \(i\). Then the graph centre is any vertex \(x\) such that,
\[
M_{vv}(x) = \min_i \{M_{vv}(i)\}
\]
while the general centre is any vertex \(x\) with the smallest possible \(S_{vv}(i)\), i.e., the smallest aggregated distances from all other vertices or,
\[
S_{vv}(x) = \min_i \{S_{vv}(i)\}.
\]

Lemma 3: In the distributed cluster-based system, if all the reporting paths are subjected to identical channel conditions, then the graph centre scheme is the best solution to the location problem.

Proof: If the same transmitting power is assumed to be adopted at each cluster head, then we would like to minimise this power as much as possible to minimise the interference. The transmitting power of a given transmitter is assumed to determine a circle such that each terminal lying within the circle hears the transmitter, and any terminal lying outside is out of the transmitter reach. If we denote the strength of the transmitted signal as a function of the distance from the transmitter \(G(d)\), then (Sousa and Silvester, 1990),
\[
G(d) = \begin{cases} 
\frac{1}{d^\alpha} & , 2 < \alpha < 6 \\
0 & , 0 \leq d \leq R \\
\end{cases}
\]
Accordingly, the minimum transmitted power is determined by a lower bound which guarantees that the master fusion centre is lying within \(M-1\) circles determined by the transmitted power of \(M-1\) cluster heads. Hence, for a given allowable received power, \(P_{r_{\text{min}}}\), at a fusion centre \(y\), the minimum required transmitted power, \(P_{t_{\text{min}}} = P_{r_{\text{min}}}/G(d)\), is determined by the furthest cluster head from the fusion centre. Hence, the centre problem scheme is the best choice in such case as this scheme defines the centre by any node whose furthest node is as close as possible.

Practically, the reporting paths are subjected to different channel conditions. Therefore, assuming identical channel conditions for all the reporting paths is not realistic. Moreover, in CR networks, the priority is given to the detection accuracy which is strongly dependent on the channel gain of each path. Then for cooperative system, the shortest reporting channels may not always be the best choice especially if they come under a deep fading/shadowing. In this work, we proposed a modified general centre scheme that considers the channel gain of the \(M-1\) signal paths between the fusion centre and the \(M-1\) cluster heads. In the proposed scheme, the fusion centre is the cluster head whose aggregated channel gain has the maximum possible value. Since, the cooperative decision is made by combined received signals from different cluster heads,
we believe that the channel gain is the best parameter to be considered for this problem. Moreover, the channel gain is affected directly by both the transmitter-receiver distance and the random influence of fading and shadowing conditions.

In a fading and shadowing environment, the channel gain between a transmitter, \( i \), and a receiver, \( j \), is modelled by (Quan et al., 2007):

\[
G_{ij} = K \frac{10^{\beta/10}}{d_{ij}^\alpha}
\] (7)

where \( K \) is a constant depending on carrier frequency, antenna height, and antenna gain. \( d_{ij} \) denotes the distance between the transmitter and the receiver, \( \alpha \) denotes the path loss exponent \( 2 < \alpha < 6 \), and \( \beta \) is a zero mean Gaussian random variable with variance \( \sigma^2 \). In practice, \( 5 < \beta < 12 \) and \( 10^{\beta/10} \) represents the shadowing factor with a log-normal distribution (Rappaport, 2002; Simon and Alouini, 2005). Log-normal shadowing is usually characterised in terms of its dB-spread and indicates how the loss in dB varies about its mean value.

Clearly, the gain is affected directly by the two factors, the distances between nodes and the random influence of fading and shadowing. Thus, using the gain model in finding the best location of the fusion centre is more accurate than considering merely the topological distances. Accordingly, the master fusion centre is the cluster head \( m \) that has the maximum aggregate channel gain or,

\[
G_{hh}(m) = \max_i \{G_{hh}(i)\} \quad i = 1, 2, ..., M
\] (8)

where

\[
G_{hh}(i) = \sum_j G_{ij} \quad j = 1, 2, ..., M.
\]

4 Spectrum sensing model

In this section we present a cluster-based spectrum sensing model that implements the energy detection at each secondary radio for local spectrum measurement. Energy detector (Digham et al., 2007), which is a simple non-coherent suboptimal detector, is widely considered for local spectrum sensing in CR networks. We assume all secondary users are equipped with identical spectrum detectors. For the \( i \)th user, the objective of local sensing, is to decide between two hypothesis, \( H_0 \), for primary user absence, and, \( H_1 \), for primary user presence.

\[
H_0 : \quad y_i(t) = n_i(t) \\
H_1 : \quad y_i(t) = h_i(t)s(t) + n_i(t)
\] (9)

where \( y_i(t) \) is the signal received at the \( i \)th receiver, \( s(t) \) is the signal transmitted by the primary user, \( h_i(t) \) is the channel gain assumed to be constant during the detection interval, and \( n_i(t) \sim N(0, \sigma^2_i) \) is an additive white Gaussian noise (AWGN) with zero mean and variance, \( \sigma^2_i \). \( s(t) \) and \( n(t) \) are assumed to be statistically independent. Over a
sensing time window, $T_s$, the CRs collect a test statistics, $Y$, that measured the received signal energy, defined by,

$$Y_i = \sum_{k=0}^{u-1} |y_i(k)|^2$$

where $u$ denotes the number of collected samples over the sensing time window. $Y$ follows a central chi-square $\chi^2$ distribution under $H_0$ and non-central chi-square distribution with non-centrality parameter $2\gamma$ under $H_1$ (Urkowitz, 1967).

$$Y = \begin{cases} \chi^2_{2u} & H_0 \\ \chi^2_{2u}(2\gamma) & H_1 \end{cases}$$

(11)

where $u = T_s W$ is the time bandwidth product and $2u$ is the degree of freedom assumed to be an integer number for simplicity. $\gamma_i$ denotes the received signal-to-noise ratio (SNR) at the $i$th radio,

$$\gamma_i = \frac{E_s|h_i|^2}{\sigma_i^2}$$

(12)

where $E_s = \sum_{k=0}^{u-1} |s(k)|^2$ is the transmitted signal energy over a sequence of $u$ samples collected during each detection interval.

Based on the central limit theorem, $Y_i$, is asymptotically normally distributed if the time-bandwidth product is relatively large (Quan et al., 2008). Thus, if $n(t)$ is considered as a real-valued Gaussian variable:

$$Y_i \sim \begin{cases} \mathcal{N}(2u\sigma_i^2, 4u\sigma_i^4) & H_0 \\ \mathcal{N}(2u\sigma_i^2(\gamma_i + 1), 4u\sigma_i^4(2\gamma_i + 1)) & H_1 \end{cases}$$

(13)

For individual spectrum sensing the decision rule at each CR user is given by,

$$Y_i \begin{cases} \overset{H_1}{\geq} \\ \overset{H_0}{<} \end{cases} \epsilon$$

(14)

where $\epsilon$ is the corresponding decision threshold. Therefore, the probability of false alarm, $P_f$, and the probability of detection, $P_d$, at the $i$th secondary radio can be defined as:

$$P_f^i = P\{Y_i > \epsilon|H_0\}$$

(15)

$$P_d^i = P\{Y_i > \epsilon|H_1\}.$$ 

(16)

In a non-fading environment where $h$ is deterministic, the exact closed form of $P_f^i$ and $P_d^i$ expressions are given by (Digham et al., 2007),

$$P_f^i = \frac{\Gamma(N,\epsilon/2)}{\Gamma(u)}$$

(17)

$$P_d^i = Q_u(\sqrt{2\gamma_i}, \sqrt{\epsilon})$$

(18)
and the probability of missed detection, $P_m^i$, 

$$P_m^i = 1 - P_d^i \tag{19}$$

where $\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ are the complete and incomplete gamma functions, respectively. $Q_m(\cdot, \cdot)$ is the generalised Marcum Q-function defined as follows (Nuttal, 1975),

$$Q_m(a, b) = \int_b^{\infty} \frac{m^m}{x^{m+1}} e^{-\frac{x^2 + a^2}{2}} I_{m-1}(ax) \, dx.$$ 

$I_m(\cdot)$ denotes the modified Bessel function of $(m-1)$th order.

Under Rayleigh fading, the signal-to-noise ratio, $\gamma$ follows an exponential distribution. Therefore, assigning a constant SNR to each transmission path becomes unrealistic. Average SNR, $\gamma$, is more appropriate performance measure than instantaneous SNR in fading channel. Accordingly, the closed-form expression of $P_d$ is given by (Ghasemi and Sousa, 2007):

$$P_d^i = e^{-\frac{\epsilon^2}{2}} \sum_{k=0}^{u-2} \frac{1}{k!} \left( \frac{\epsilon^2}{2} \right)^k \left( 1 + \frac{\gamma_i}{\gamma} \right)^{u-1} + \left( \frac{1}{\gamma} \right)^{u-1} x \left( e^{-\frac{x^2 + \gamma_i^2}{2 \gamma}} - e^{-\frac{\epsilon^2}{2}} \sum_{k=0}^{u-2} \frac{1}{k!} \left( \frac{\epsilon^2}{2e^{-\frac{\gamma_i^2}{2}}} \right)^k \right). \tag{20}$$

Therefore, in spectrum sensing, $P_f$ refers to the probability of white-spaces that are misclassified as occupied channels and $P_m$ refers to the probability of harmful interference to the primary user. From the secondary users’ point of view, a lower probability of false alarm means more spectrum access opportunities. However, from the primary user side, the higher probability of detection means more protection from harmful interference. Depending on which probability is of interest, that one is fixed and the other one is optimised. In Figure 5, we plot the complementary receiver operating characteristics curves for SNR of 0, 5, 10, 15 dB and $u = 6$ samples. The sensing channel is assumed to be subjected to Rayleigh fading. Obviously, the spectrum sensing performance deteriorates at lower SNR values as illustrated in Figure 5. Therefore, the individual sensing becomes unreliable at weak primary signals which is typical in fading environment.

In the following section, we will use the approximate-form expression for $P_f$ and $P_d$ defined by:

$$P_f^i = Q \left[ \frac{\epsilon - E(y_i|H_0)}{\sqrt{Var(y_i|H_0)}} \right] \tag{21}$$

$$P_d^i = Q \left[ \frac{\epsilon - E(y_i|H_1)}{\sqrt{Var(y_i|H_1)}} \right] \tag{22}$$

where $E(\cdot)$ and $Var(\cdot)$ are the mean value and the variance of the test statistics respectively.
5 Cluster-based spectrum sensing

To improve the spectrum sensing, the MDS-based clustering scheme introduced in Section 3 is used to allow the secondary users in each cluster to cooperatively share their local observations. In each cluster, the cluster head makes the cluster decision and forwards it to the master fusion centre where the final decision is made. The sensing information is shared only at the local scale within each cluster and there is no need for each CR terminal to send its own decision to the master fusion centre. As mentioned previously, the scheme performs two levels of data fusion, one at the cluster heads and another one at the master fusion centre.

For a cluster $C_j$, with a cluster size of $N_{cj}$, the individual local sensing statistics of cluster members $Y_i, i = 1, 2, ..., N_{cj}$, are linearly combined at the cluster head.

$$Y_j = \sum_{i=1}^{N_{cj}} Y_i \quad \forall C_j, j = 1, 2, ..., M. \quad (23)$$

Since $Y_i$s are normal random variables, according to (23), their linear combination is also normally distributed with the following mean values and variances:

$$Y_j \sim \begin{cases} \mathcal{N} \left( u \sum_{i=1}^{N_{cj}} \sigma_i^2, 2u \sum_{i=1}^{N_{cj}} \sigma_i^4 \right) & H_0 \\ \mathcal{N} \left( u \sum_{i=1}^{N_{cj}} (\gamma_i + 1)\sigma_i^2, 2u \sum_{i=1}^{N_{cj}} (2\gamma_i + 1)\sigma_i^4 \right) & H_1. \end{cases}$$

The cluster head then makes the cluster decision by comparing the linear combination with a pre-fixed threshold, $\epsilon_c$. We assume identical thresholds are used for all clusters. According to (21) and (22), the probability of false alarm and the probability of detection computed at each cluster head can be defined by:
\[
P_{fc}^j = Q \left[ \frac{\epsilon_c - u \sum_{i=1}^{N_{cj}} \sigma_i^2}{\sqrt{2u \sum_{i=1}^{N_{cj}} \sigma_i^4}} \right] \quad (24)
\]
\[
P_{dc}^j = Q \left[ \frac{\epsilon_c - u \sum_{i=1}^{N_{cj}} (\gamma_i + 1)\sigma_i^2}{\sqrt{2u \sum_{i=1}^{N_{cj}} (2\gamma_i + 1)\sigma_i^4}} \right]. \quad (25)
\]

Equations (24) and (25), clearly illustrate that the spectrum sensing performance of each cluster is largely effected by the number of CR terminals in each cluster (cluster-size), the threshold \( \epsilon_c \), and the number of samples collected by the detector during the sensing time. Since, the priority is given to interference avoidance to the primary network, a high detection probability must be guaranteed. Therefore, if we set a lower bound for the detection probability say, \( P_{dc}^j \) at the cluster level, then the objective is to minimise the probability of false alarm as much as possible so as to increase the spectrum access opportunities and thereafter the network throughput. By combining equations (12), (24), and (25):

\[
P_{fc}^j = Q \left[ \frac{(P_{dc}^j)^{-1} \sqrt{2u \sum_{i=1}^{N_{cj}} (1 + 2\gamma_i)\sigma_i^4 + uE_s \sum_{i=1}^{N_{cj}} |h_i|^2}}{\sqrt{2u \sum_{i=1}^{N_{cj}} \sigma_i^4}} \right]. \quad (26)
\]

Under the bandwidth constraints of the reporting channel, we allow each cluster head to send only 1-bit decision \( \{0\} \) for \( H_0 \) and \( \{1\} \) for \( H_1 \) to the master fusion centre rather than their decision statistics. The fusion centre then makes the final decision according to the fusion rule implemented. Various fusion functions can be used. Data fusion rules are often implemented as ‘\( k \)’ out of ‘\( n \)’ logical function (Chair and Varshney, 1986). Thus, for \( M \) cluster decisions \( B_j, j = 1, 2, \ldots, M \), the final decision \( D \), is:

\[
D = \begin{cases} 
1 & \sum_{j=1}^{N_{cj}} B_j \geq k \\
0 & \sum_{j=1}^{N_{cj}} B_j < k.
\end{cases}
\quad (27)
\]

The common fusion functions such as OR-rule and AND-rule are special case of the ‘\( k \)’ out of ‘\( n \)’ rule. For the OR-rule, a decision of \( \{0\} \) for \( H_0 \) is only made when all the \( M \) cluster decisions demonstrate the absence of the primary user. Such kind of rule is perfect for interference avoidance to the primary user since the secondary user will only be allowed to access the spectrum if all the cluster heads reported the binary decision \( \{0\} \) to the fusion centre. Therefore, the false alarm probability and the detection probability can be defined as:

\[
Q_f = 1 - \prod_{j=1}^{M} (1 - P_{fc}^j) \quad (28)
\]
\[
Q_d = 1 - \prod_{j=1}^{M} (1 - P_{dc}^j). \quad (29)
\]
A more general formulation of the data fusion problem is introduced in Chair and Varshney (1986) and Thomopoulos et al. (1987).

Under deep fading/shadowing, a reported decision of \{0\} for primary network absence may be received at the fusion centre as \{1\}, or a reported decision of \{1\} for active primary network may be received at the fusion centre as \{0\}. In the first case, a false alarm will be triggered, and a missed detection risk may be encountered in the second case. If \(P_{f_c}^j\) denotes the probability of receiving \{1\} at the master fusion centre when the \(j\)th cluster head reports \{0\} and \(P_{mC}^j\) denotes the probability of receiving \{0\} at the master fusion centre when the \(j\)th cluster head reports \{1\}, then under OR-rule, the probabilities of false alarm, \(Q_f\), and missed detection, \(Q_m\), become (Sun et al., 2007),

\[
Q_f = 1 - \prod_{j=1}^{M} [(1 - P_{f_c}^j)(1 - \tilde{P}_{f_c}^j) + P_{f_c}^j \tilde{P}_{mC}^j]
\]

\[
Q_m = \prod_{j=1}^{M} [P_{mC}^j (1 - \tilde{P}_{f_c}^j) + (1 - P_{mC}^j) \tilde{P}_{f_c}^j].
\]

Let \(P_e^j = \tilde{P}_{f_c}^j = \tilde{P}_{mC}^j\), where \(P_e^j\) denotes the probability of the reporting errors for cluster head \(m_j\), then equations (30) and (31), can be rewritten as,

\[
Q_f = 1 - \prod_{j=1}^{M} [(1 - P_{f_c}^j)(1 - P_e^j) + P_{f_c}^j P_e^j]
\]

\[
Q_m = \prod_{j=1}^{M} [P_{mC}^j (1 - P_e^j) + (1 - P_{mC}^j) P_e^j].
\]

6 Throughput performance

To meet the throughput requirements, the secondary users network needs to increase the sensing efficiency by decreasing the false alarm probability and increasing the spectrum access opportunities. Since CR users can not transmit and sense at the same time, periodic sensing rounds are required where sensing and transmitting processes are alternating in a periodic manner in successive frames. The frame time, \(T\), consists mainly of the sensing time \(T_s\) and the transmission time \(T_D\). We will ignore the clustering time as it very small compared to the sensing and transmission time (Heinzelman et al., 2002; Hussain et al., 2009). For interference avoidance, the observation time needs to be long enough to achieve sufficient detection accuracy. But, for a fixed frame size \(T = T_s + T_D\), increasing sensing time inevitably decreases the transmission time and consequently decreases sensing efficiency. On the other hand, a longer transmission time enhances the sensing efficiency but causes higher interferences to primary user as the detection accuracy decreases due to the lack of sufficient sensing.
information (Lee and Akyildiz, 2008). Let \( \eta \) denotes the spectrum sensing efficiency, then:

\[
\eta = \frac{T_D}{T} = \frac{T_D}{T_D + T_S}.
\]

(34)

The throughput of secondary network comes as a result of the following two network operation cases (Liang and Zeng, 2008): (1) CR terminals communications and data exchange in the absence of the primary user with probability of \((1 - P_f)P(H_0)\), (2) CR terminals communications when the primary user is mis detected by the secondary network with a probability of \((1 - P_d)P(H_1)\) where \(P(H_0)\) and \(P(H_1)\), are the probabilities of primary user absence and presence respectively. The second case of throughput is less likely to happen since the detection probability has to be high to avoid the harmful interference to the primary user. It is highly likely to have \(P_d > 0.9\) in the conservative systems where the priority is given to primary user protection. In addition, the interference generated by the primary signal significantly limits the secondary network throughput.

To determine the probability of the the primary user absence \(P(H_0)\), the licensed channel usage (primary user activity) needs to be modelled. The ON/OFF model will be considered in our approach. In this model, the primary user activity is modelled as independent and identical distributed random processes where the ON and OFF states represents the busy and idle periods of the licensed channel (Ghasemi and Sousa, 2007; Lee and Akyildiz, 2008). The duration of the idle and busy periods are assumed to be exponentially distributed with a mean \(\tau_{OFF}\) and \(\tau_{ON}\) respectively. Therefore, the probability of primary user absence can be defined as:

\[
P(H_0) = \frac{\tau_{OFF}}{\tau_{OFF} + \tau_{ON}}.
\]

(35)

As a performance measure, we will use the per node throughput capacity to show how the per-node throughput varies with the cluster parameters such as number of clusters, cluster size, and sensing time. The per node throughput represents the average number of bits per second that can be transmitted from the source node to the destination node. In Quan et al. (2007), the per node throughput capacity for a cluster-based wireless sensor network, \(T_n\), is obtained as \(\Theta(MR/N)\) for a single-hop clustering and \(\Theta(R\sqrt{M(\log N - \log M)}/N^3)\) for multi-hop clustering. However, in the CR networks, the sensing parameters such as sensing efficiency and primary user activity must be considered for throughput calculations. In this work, we extend the per-node throughput given by Quan et al. (2007) to include the spectrum sensing parameters that largely effect the network performance. Accordingly, the per-node throughput is defined by:

\[
T = \eta T_n \frac{\tau_{OFF}}{\tau_{OFF} + \tau_{ON}}(1 - Q_f).
\]

(36)

It can be shown that the per node throughput capacity depends on the sensing efficiency, the number of clusters, the probability of false alarm, and the total nodes in the network. Since, the cluster size plays a key role in the probability of false alarm, its effect on throughput performance is investigated beside the other parameters in the next section.
7 Performance evaluation

In this section, we present computer simulations to demonstrate the performance of the proposed cluster-based system. We assumed a CR network of 100 terminals deployed in an area of 100 × 100. The additive noise is assumed as a zero-mean real-valued Gaussian process. The sensing frame, \( T \), is assumed to be fixed and assigned a value of 20 ms. The signal paths between the cluster members and the cluster head are assumed perfect channels as they are considered short distances. We implement the OR-rule in this work. This rule is proved to be better than other data fusion rules especially when the priority is given to the interference avoidance to the primary user. We assume that local reporting channels are always available for cluster heads to communicate with the selected fusion centre. The probability of error is used to introduce the effect of channel fading of the reporting paths between the cluster heads and the master fusion centre. The same power is assumed to be adopted by each terminal for intra-cluster communications with the power control capability which is required for the communication between cluster heads and fusion centre.

To demonstrate the effect of the cluster size \( N_c \), on the overall performance of the clustered network, the complementary receiver operating characteristics (ROC) curves are plotted in Figure 6. The curves show the probability of missed detection vs. the probability of false alarm for a cluster size of 5, 10, and 15 nodes. The reporting channel conditions is realised with probability of error, \( P_e \), equals to 0.0001. Clearly, the network performance is improved by increasing the number of nodes in each cluster. However, the cluster size must be upper bounded to prevent network bottlenecks resulting from overcrowded clusters.

Figure 6 Cluster-based cooperative sensing performance with various cluster size, \( u = 12 \), \( \gamma = 10 \text{ dB} \) (see online version for colours)

![Figure 6](image_url)

Figure 7 shows that the spectrum sensing accuracy is improved by collecting more energy samples at each CR detector. The ROC curves are plotted for number of samples, \( u \), equals 5, 10, 15, 20 samples with \( P_e = 0.0001 \). Obviously, the detection probability is greatly increased and consequently, the missed detection probability is decreased when the number of samples increases. However, a longer sensing time is required to collect...
more samples at the CR detector. Since, the sensing efficiency is reduced with the longer sensing time, a trade off between the sensing accuracy and sensing efficiency must always be considered.

Figure 7  Cluster-based cooperative sensing performance vs. number of collected energy samples, $\tau = 10$ dB (see online version for colours)

To investigate the throughput performance of the proposed system, a target detection probability of 0.9 is used with a mean value of 0.7 and 0.3 for the idle and the active periods respectively. Figure 8 shows that probability of false alarm, $Q_f$, initially decreases rapidly when the number of cooperative clusters increases. However, it increases later with more cluster heads involved in the cooperative process. In fact, after the initial drop, $Q_f$ increases with a rate that greatly depends on the probability of error. Since OR-rule is increasing function in terms of $m$, the initial drop occurs as a result of the term $Q^{-1}(1 - \sqrt{1 - P_{dc}})$ in equation (32). However, when the probability of false alarm, $P_f$, becomes very small compared to the probability of errors, $P_e$, equation (32) can be reduced to $Q_f = 1 - \prod_{j=1}^{M} (1 - P_{je})$ and that explains why $Q_f$ becomes mainly dependent on $P_e$. An optimal number of cluster heads that gives a minimum probability of false alarm is obtained which varies according to the reporting channel probability of error.

In Figure 9, the normalised per-node throughput is plotted vs. the number of clusters for reporting channel probability of error, $P_e$ equals 0.1, 0.01, 0.001, and 0.0001. The perfect reporting channel ($P_e = 0$) is also plotted for comparison. The figure reveals that at a higher probability of error the throughput performance deteriorates with more users added to the cooperative decision. This can be easily understood from Figure 8 where the probability of false alarm increases more rapidly at higher probability of error.

To compare the cluster-based system with the conventional system, the per-node throughput performance is plotted in Figure 10 for $P_e = 0.1$. As shown, the cluster-based system outperforms the non-cluster system especially when more users are engaged in the cooperative process. Thus, it is not necessary to involve all the deployed users in the cooperative decision as many false information received at the fusion centre may lead to wrong final decisions. For the given parameters, a maximum achievable throughput
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obtained with six cooperative clusters. Since with MDS approach, a minimal cluster set is always achievable. Therefore, by adjusting the cluster radius, the number of clusters in the minimal cluster set can be adjusted to match the requirement of both the reporting channel bandwidth and the optimal achievable per-node throughput. Figure 11 shows that a minimal dominating set of six clusters is achieved with cluster radius of 20 m and 12 m for one-hop and two-hop clustering scheme respectively.

Figure 8 False alarm vs. cluster size, \( u = 12, \gamma = 10 \) dB (see online version for colours)

Figure 9 The normalised per-node throughput vs. number of cooperative clusters, \( u = 12, \gamma = 10 \) dB (see online version for colours)
Figure 10  Per-node throughput performance of clustered and non-clustered network vs. number of cooperative terminals, $a = 12$, $\tau = 10$ dB (see online version for colours)

In Figure 12, the normalised per-node throughput is plotted vs. the sensing time for reporting channel probability of error, $Pe$ equals to 0.1, 0.01, 0.001, and 0.0001. The figure reveals a maximum throughput is achieved at a sensing time of about 0.7–0.9 ms for three reporting channels conditions. Initially, the throughput increases sharply, but after a maximum value, it starts to decrease when the sensing time keeps increasing.

Finally, the normalised per node throughput is plotted in Figure 13 for both single-hop and multi-hop clustering with $Pe = 0.01$. The figure shows that a higher throughput capacity is achieved with single-hop clustering. The reason can be explained by the fact that a multi-hop clustering tends to group more nodes within a single cluster, hence, the per-node throughput will be less compared with the single-hop assuming that the same one-hop transmission range adopted in both types of clustering.
8 Conclusions

We investigated a distributed spectrum sensing scheme that eliminates the need for a base station and replace it with a local master fusion centre. The MDS approach is used as a clustering scheme and the general centre problem scheme is used as the base to select the master fusion centre. A lower bound of the cluster radius that keeps the number of isolated nodes under an upper limit is determined in this work. The influence of the cluster size, number of cluster, sensing time, and the probability of reporting channel errors on the per-node throughput capacity is investigated. The results obtained reveals that under bad channel conditions, it is not necessary to include all the cooperative user for the best performance. Instead, an optimal number of clusters that gives a minimum probability of false alarm and consequently a maximum per-node throughput is obtained. When this optimal number matches the MDS, the reporting channel bandwidth requirements is also achieved. A maximum per-node throughput is also obtained that corresponds to an optimal sensing time for different values of error probabilities.
References


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Notes

1 The notation $y = \Theta(g(x))$ is used to signify that there exist positive constants $\ell_1$ and $\ell_2$ such that $\ell_1 g(x) \leq y \leq \ell_2 g(x)$.  