Heuristics and Pedigrees for Drawing Directed Graphs*

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Abstract We describe here a collection of heuristics for producing "nice"- drawings of directed graphs, and a simple dual-mode software tool for testing and evaluating them.

In *playing* mode, the heuristics are applied in random sequence over a set of drawings, in the manner of an asynchronous team (A-team). As new drawings are added to the set, others are deleted based on a multi-valued aesthetic evaluation function.

By inspecting the "pedigree" of the best solutions found in *playing* mode, the user can obtain insights into the best order in which the heuristics should be applied. Then the user can test these insights in the *working* mode, where the heuristics are applied in a fixed sequence.

Some of the heuristics that we describe here are similar to the steps of Sugiyama's D-ABDUCTOR graph-drawing package; and indeed we can obtain results similar to those of D-ABDUCTOR, by applying our heuristics in the proper sequence.

Keywords: graph drawing, aesthetic criteria, visualization, multivalued optimization, A-team.

1 Introduction

An automatic graph drawing module is a necessary component of many information systems. Experience shows that abstract relations are better understood when displayed in graphical form; and most abstract relations can be naturally represented by directed graphs. Many general-purpose graph drawing packages and utilities can be found in the literature, such as GIOTTO [1], "Graph Browser" [14], DAG [7] and DRAG [23], ISI Grapher [12], EDGE [11], and D-ABDUCTOR [16], to cite just a few.

In order to be effective, however, the drawing of a graph must obey certain aesthetic and clarity criteria, that take into account the way the human eye and brain process visual input. The criteria that define a "nice"-graph may depend on the application, but usually include minimizing the number of line crossings and

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the size of the drawing, uniformizing the spacing of nodes and lengths of edges, etc. In the case of directed graphs, it is also desirable to have most edges oriented in a fixed general direction (say top to bottom, or left to right), so as to minimize the need for explicit arrowheads. Unfortunately, for almost any combination of criteria that one would like to use in practice, the problem of producing the "nicest possible"- drawing of an arbitrary graph is very difficult, often NP-hard.[5]. Therefore, practical graph-drawing systems such as the ones mentioned above are based on heuristic procedures, whose results are hopefully nice but not always the nicest possible.

The package D-ABDUCTOR by Sugiyama *et al*.[16, 17, 18] is a typical example. Its method consists of four steps executed sequentially. In each step a different algorithm is used to optimize some aspect of the drawing: the *y*-coordinates of the vertices, the horizontal order of the vertices in each *y*-layer, the *x*-position of each vertex in each layer (preserving their order), and so on. The result is not necessarily optimal, because these aspects are not independent: the best assignment of *y*-coordinates depends on the *x*-coordinates, and vice-versa. Nevertheless the D-ABDUCTOR system is quite popular, because it is fast and usually produces fairly nice results.

The main contribution of this article is a set of heuristics (described in section 4) which we have found effective in drawing directed graphs (as defined in section 2). Some of the heuristics are similar to the main steps of D-ABDUCTOR; indeed, we have obtained results comparable to those of D-ABDUCTOR by applying them in a particular sequence; these experiments are detailed in section [6]

We also describe in section [5] a simple software testbed that assists the user in evaluating and combining those heuristics. The software operates in two modes, *playing* and *working.* In *working* mode in which the user directly specifies a sequence of heuristics to be applied. In *playing* mode the heuristics are applied at random to a fixed-size pool of solutions, in the manner of the so-called *asynchronous team* or *A-team* model [19, 2, 3] with a multi-valued goal function [4, 13]

Theory predicts, and experience confirms, that an A-team is an inefficient way to combine heuristics. In our case, we found that a fixed sequence of heuristics, properly chosen, produced better drawings than any A-team, and was enormously faster (by three orders of magnitude, even on small graphs). This, incidentally, is the approach embodied in D-ABDUCTOR, and the reason for its popularity.

However, an A-team allows heuristics to cooperate and iterate in arbitrary ways, with a bias towards combinations that produce good solutions. It is therefore helpful for understanding how heuristics interact with each other, and for discovering general rules-of-thumb for combining them in effective ways. Thus, the "pedigree"- of the solutions generated in *playing* mode often suggests effective "programs"- for use in *working* mode. To support this claim, we describe our experience with our graph-drawing heuristics, described in section [6].

2 Terminology

2.1 Graphs

A *graph* $G = (V,E)$ consists of a finite set V of *vertices*, and a set E of *edges*, which are ordered pairs of distinct vertices.

If $e=(u,v)$ is an edge of G, we say that u and v are the *endpoints* of e, that u is the *origin*, v is the *destination*, and that u and v are *adjacent* or *neighbors* of each other. The *neighborhood* of a vertex x is the set of all of its neighbors.

2.2 Graph Drawings

Following Sugiyama's approach, we will restrict our attention to graph drawings where each vertex is placed on a point of the integer grid, and each edge is drawn as an *y*-monotone polygonal line whose corners lie on such points.

More precisely, we define a *polyline* as a sequence of distinct points $p_0,p_1,...,p_n$ with integer coordinates $p_i = (x_i,y_i)$, such that the ordinates $y_0, y_1,..., y_n$ are consecutive integers, increasing or decreasing. By definition, the *links* of the polyline are the open straight-line segments $p_i\, p_{i+1}$, implicitly oriented from p_i to p_{i+1}. The points p_0 and p_n are the *origin* and *destination* of the polyline, while p_1, p_2, $...p_{n-1}$ are its *joints*. A joint will be called a *bend* if the two adjacent segments are not collinear.

By convention, in our illustrations the *x*-coordinates increase from left ot right, and the *y*-coordinates increase *from top to bottom*. We therefore say that a link or polyline is *descending* if $y_{i+1} < y_i$, and *ascending* if $y_i < y_{i+1}$. In keeping with tradition, we consider descending edges to be more "natural"- than ascending ones.

We define a *drawing* of a graph $G=(V,E)$ as an assignment of each vertex v to a distinct point v^* of the integer grid Z^2, and of each edge $e=(u,v)$ to a valid polyline e^*, whose endpoints are u^* and v^*. The joints

of these polylines must be all distinct, and must not coincide with any of the vertex points *v**.

A drawing of a graph *G=(V,E)* can be interpreted as another graph *G'=(V',E')*, where the set *V'* consists of all the points *v** together with the joints of all polylines *e**; and the set *E'* consists of the pairs (p_i,p_{i+1}) for all segments $p_i\ p_{i+1}$ in those polylines. To avoid confusion, we will refer to the elements of *V'* and *E'* as *nodes* and *links*, reserving the terms *vertices* and *edges* for the elements of *G*.

In the illustrations of this article, the nodes of *G'* that correspond to vertices of *G* are drawn as numbered ovals; all other nodes of *G'* (the joints) are left unmarked. To reduce clutter and make the drawings easier to read, we do not draw arrowheads on the edges; instead, we use solid lines for descending edges and dotted lines for ascending ones.

The set of all points of the integer grid with the same *y* coordinate is called *layer y of the grid*, and the set of all nodes of *G'* assigned to those points is *layer y of the drawing*. Note that the lowest-numbered layer is at the top in our figures. It follows from all the definitions that each link of the graph *G'* connects two nodes from adjacent layers. Also, every joint node on layer *y* has exactly one neighbor in layer *y+1* and one in layer *y-1*.

2.3 Edge crossings

We define an *edge crossing* as a point of intersection between the two polylines *e** and *f** that represent two distinct edges *e*, *f* of *G*. Recall that polylines do not include their endpoints, links are open and span at most two layers, and the joints in a drawing are all distinct. It follows that the edge crossings correspond to pairs of links $(a,b) \in E' \times E'$ that intersect in their interiors.

Note that these conditions on the graph *G'* guarantee that the topology of the original graph *G* can be recovered without ambiguity from a drawing of *G'*, provided the vertices are drawn as sufficiently small dots and the links as sufficiently narrow straight lines.

3 Aesthetic criteria

In order to compare drawings and algorithms, or to program a computer to automatically search good drawings, we need some well-defined measure of the quality of a drawing. Unfortunately, this notion is subjective and dependent on the application being considered.

An *aesthetic indicator* or *score* is a numerical function of the drawing that measures or counts some type of aesthetic defect which is generally undesirable (for the given application) and ought to be minimized. The following quantities, in particular, fit this definition is most contexts:

1. The number of edge-edge crossings.
2. The number of ascending edges.
3. The number of bends on edges.
4. The total length of all edges.

There are obvious conflicts between the above indicators. For instance, minimizing the number of crossings may require opening up the drawing and/or representing edges by longer paths. Typically, there is no single drawing of *G* that optimizes two independent aesthetic scores; so the problem of finding the "nicest drawing"- does not have a well-defined solution, much less an algorithm.

3.1 Penalty functions

A popular solution to this difficulty is to combine all the relevant aesthetic indicators into a single combined score, by weighted averaging, or some other mathematical function. Then the goal can be formally defined as finding a drawing *G'* with minimum score.

However, this approach merely pushes the difficulty to the user, who must select the weights or scoring function best suited to the application. Unfortunately the user generally doesn't know a priori what is the relative importance of each indicator (e.g., whether one extra crossing will look uglier than three extra bends). In fact, the best set of weights often depends on the graph itself. Typically the user and/or the implementer have to adjust the weights by trial and error, which is extremely time-consuming and rarely converges to the best settings.

3.2 Multi-valued comparison

Another solution, that does not require parameter tuning, is the *multi-valued scoring* approach of [4, 13]. In this approach, the merit of a solution *A* is measured not by a single score but by a *score vector* $q(A) = \langle q_1(A),q_2(A),...,q_m(A)\rangle$, where each q_i is an aesthetic indicator; and no assumptions are made about the relative importance of the components. The score vectors thus define only a *partial* ordering of the solutions: we say that a solution *A* is *better than* a solution *B* (written $A \subset B$) if $q_i(A) \le q_i(B)$ for all *i*, and $q_k(A) < q_k(B)$ for at least one index *k*. Obviously, if

these conditions hold, then *A* will get a lower mark than *B* under any combined score that is a monotone function of the individual score components.

In the multi-valued approach, a solution *A* is considered "good" — if it is minimal under the '$\subset$' relation among all known solutions; that is, there is no other known solution *B* with $B \subset A$. It follows that if *A* and *B* are known good solutions, then either they have exactly the same score vector, or they are incomparable: there is some component q_i by which *A* is better than *B*, and some other component q_j by which the opposite is true. In the graph-drawing problem, for instance, *A* may have less crossings but more ascending edges than *B*.

By the same token, an heuristic is considered "useful"- if it can reduce one of the scores q_i, even at the cost of increasing the others.

Finally, in drawing programs that explore various alternatives, the goal is not a single solution that minimizes an arbitrary combined score, but rather a set of solutions that are all minimal under '$\subset$ ', and hence mutually equivalent or incomparable.

In general, the result set will contain not only a solution A_i that minimizes criterion q_i, for each *i*, but also many "compromise"- solutions. For example, the result set may have solutions with score vectors *(0,5,5)*, *(5,0,5)*, *(5,5,0)*, and *(4,4,4)*. Note that the last one is better than each of the other three under *some* criterion, but is not optimal under any criterion (or even under any weighted average of the criteria).

In multi-valued optimization, the user still has the problem of selecting one solution among the ones returned. However since the non-minimal solutions have been eliminated, — the output set generally contains many good—if not optimal — solutions, so even a random choice may be acceptable. Moreover, the user can choose based on subjective criteria, without having to express them as a mathematical formula.

4 Heuristics for drawing directed graphs

We now describe a small set of heuristics that have been found quite effective in drawing directed graphs. First, however, we need to define some basic operations.

4.1 General-purpose graph-drawing tools

The following procedures are used by several of the heuristics to modify the current drawing *G'*. Actually these procedures assume only that *G'* is a *sub-drawing* of *G* — a subgraph of some valid drawing of *G*.

- *shiftnodes*($p_1,...,p_m;x_1,...,x_m$): given a set of nodes $p_1,...,p_m$ of *G'*, this procedure tries to move them to new abscissas $x_1,...,x_m$, without changing their *y*-coordinates.

This operation may entail shifting some other nodes of *G'* horizontally, in order to make space for the nodes p_i. Also, each node p_i may be placed at some abscissa that is not exactly at x_i, but only close to it. (This happens, in particular, if the operation is asked to shift two nodes in the same layer to the same abscissa x_i.)

- *addedge(u,v)*: given two vertices *u* and *v* of *G* (not *G'*), this procedure adds a polyline to the graph *G'*, from node *u** to node *v** — which must lie on different layers.

The pair *e=(u,v)* must be an edge of *G*, but its polyline *e** must be missing in *G'*. The inserted polyline will resemble an integral sign ∫, with vertical stem. More precisely, the polyline will be a single link, if *u** and *v** are in adjacent layers. Otherwise, if *u** and *v** are *k* layers apart, it consists of a string of *k-2* vertical links (at an abscissa × roughly midway between the abscissas of *u** and *v**), preceded and followed by two additional links incident to *u** and *v**.

Note that some existing nodes of *G'* in the intervening layers may have to be shifted horizontally, in order to make space for the joints of the new polyline.

- *movevertices* ($v_1,...,v_m$; $p_1,...,p_m$). this procedure tries to reassign the vertices $v_1,...,v_m$ of *G* (not *G'*) to points $p_1,...,p_m$ of the integer grid. This reassignment must not cause two neighboring vertices to lie on the same layer of *G'*.

Let $p_i = (x_i,y_i)$ for each *i*. First, the procedure deletes all the polylines of *G'* that represent edges of *G* incident to $v_1,...,v_m$. Then it relocates node v^*_i to any vacant spot on layer y_i. and it performs *shiftnodes* (v^*,x). Finally, the procedure performs *addedge(e)* for every edge *e* of *G* incident to *v*.

Note that the vertex v_i will always end up in layer y_i, but its abscissa may be somewhat removed from x_i, depending on the crowding conditions around p_i.

These operations are not entirely deterministic: coin flips are used in several cases to break ties, or just to

ensure a minimum of "genetic diversity"- in the drawings generated by the heuristics.

4.2 The heuristics

Each heuristic operates on a *current drawing G'* of the graph *G*. By definition, an heuristic *succeeds* when it modifies *G'* — even if the modified drawing is worse than the original in all scores. Otherwise the heuristics *fails*.

Some of these heuristics use a *reference layer y*, which is either the maximum or minimum *y*-coordinate of the vertices, or is the *median layer* — the *y* coordinate which separates the vertices of *G* (but not necessarily the nodes of *G'*) as evenly as possible.

- *NewRandom*: The heuristic *NewRandom* creates a random drawing for the graph *G*. The heuristic assigns each vertex to a different layer in the range *[0..n-1]*, at the same abscissa x=0, and then uses *addedge* to connect them with polylines.

- *AdjustY*: This heuristic tries to displace each vertex v_i of *G* to an "optimum"- layer y'_i

The heuristic begins by setting y_i to the initial ordinate of vertex v_i, for all *i*. Then, for each vertex v_i in turn, the heuristic first determines the widest possible interval of ordinates $Y_i = [lo(Y_i)..hi(Y_i)]$ where that vertex could be placed without creating new ascending edges. Note that this interval always contains the current ordinate y_i. The heuristic then sets y_i to some ordinate y_i' in the interval Y_i, avoiding any layers that contain neighbors of v_i in *G*.

When computing the interval Y_i, the heuristic assumes that any vertex v_j examined previously has already been moved to its chosen layers y_j. Specifically, $lo(Y_i)$ is one more than the maximum y_j for all in-neighbors v_j of v_i that are currently placed above v_i *(i.e. have* $y_j < y_i$*)*. The bound $hi(Y_i)$ is defined symmetrically.

After computing all ordinates y_i, the heuristic uses *movevertices* to place each vertex *vi* at the point $p_i = (x_i,y_i)$, where where x_i is its original abscissa.

There are several variants of this heuristic, distinguished by the order in which vertices are processed, and the criterion used to choose the new layers y_i':

- *AdjustYPack*: processes the vertices in order of increasing distance from a reference layer *y*, and choose y_i' as the ordinate in the interval $[lo(Y_i)..hi(Y_i)]$ that is closer to the median *y*. The effect of this heuristic is to "squeeze"- all the vertices towards the reference layer *y*.

- *AdjustYFlip*: Processes the vertices in random order, and chooses the new layer *yi'* at random in the range *[lo(Yi)..hi(Yi)]*. (However the procedure only chooses $y'_i < y_i$ if v_i has some neighbor v_j with $y_j < y_i$; and similarly for $y'_i > y_i$.).

- *AdjustYMean*: Processes the vertices in random order, and selects each y_i as a weighted average of the ordinates y_j the neighors of v_i in *G*. (Actually, each neighbor ordinate is modified by an offset δ_i that is *-1* for out-neighbors, and *+1* for in-neighbors.) Each term has weight 1 if the corresponding polyline in *G'* is descending, and 2 if it is ascending.

- *AdjustYMedian*: Similar to *AdjustYMean*, except that the weighted median is used instead of the weighted mean, and ties are broken by a sum-of-distances criterion. This variant has the effect of displacing the vertex v_i vertically, as far as possible, in the direction where most of its neighbors lie.

In all these variants, a random displacement in *[-1..+1]* may be applied to the chosen y_i', taking care not to leave the interval Y_i or placing two adjacent nodes in the same layer.

Note that, as the vertices are inserted into the chosen layers, the *x* coordinates of all nodes may be substantially disturbed.

These heuristics will not increase the number of ascending edges, and will generally reduce the overall height and total edge length of the graph. However, they may increase its width, and create new crossings and bends.

- *FixEDirs*: this heuristic checks whether every edge *(u,v)* of *G* is drawn as a descending polyline. Whenever it finds an ascending edge (with *v* higher than *u*), the heuristic tries move *v* from its current layer to a new layer just below *u*; or to move *u* to the layer just above *v*. In the first case, in order to avoid new ascending

edges, the heuristic will also displace other nodes connected to *v*. More precisely, if it decides to displace a vertex *v* from its current layer *y* to a layer *y'>y*, the heuristic will also move all out-neighbors of *v* currently in layers *[y+1..y']* to the layer *y'+1* or beyond; and so on recursively. The symmetrical correction applies when *u* is moved up.

In either case, the correction is performed only if it does not create any new ascending edges; that is, only if there is no directed path from *v* to *u* that consists entirely of descending edges.

If the graph *G* is acyclic, repeated applications of this heuristic will eventually remove all ascending edges.

- *AdjustX*: For each layer *y* of *G'*, and each *pi* of *G'* in that layer, the heuristic computes a new abscissa *xi*, taking into account the abscissas of the neighboring nodes in layers *y+1* and *y-1*. The heuristic then uses *shiftnodes* to place each p_i at the corresponding x_i.

The new abscissas x_i for each layer are computed in two stages. First, the heuristic tries to improve the left-to-right order of the nodes in the layer. To that end, it tests each node the layer in turn, checking whether moving that node to some other position in the left-to-right order would reduce the number of link crossings. (Note that this heuristic may not find the optimum order). In a second stage, the abscissas of the nodes are recomputed, while preserving their order, so as to minimize a weighted sum of the squares of the link lengths. Note that this criterion tends to place each node near the barycenter of its neighbors.

In the second stage, extra penalty terms are used (a) to discourage bends, (b) to encourage vertical links in general, and (c) to discourage vertical links incident to nodes with even in- or out-degree (so as to encourage more symmetric drawings). In spite of this complexity in the cost function, the fixed ordering of the nodes allows the optimum abscissas to be computed in quadratic time, by dynamic programming.

- *SmoothX*: this heuristic is similar in purpose to *AdjustX*, but its method is simpler, and more similar to the barycentric adjustment of Sugiyama and Missue [17, 18]. For each layer *y* of *G'*, the heuristic computes a new tentative abscissa *x'* for each node *w* of *G'* in that layer, by taking a weighted mean of the abscissas of the neighboring vertices of *G*. More precisely, if *w* is a vertex of *G*, then the average is taken over all neighbors of *w* in *G*. Otherwise, *w* is a joint in a polyline that represents some edge *(u,v)* of *G* (not *G'*); in that case the "neighbors"- of *w* are by definition *u* and *v*.

Again we have several variants of the *AdjustX* and *SmoothX* heuristic, differing by the order in which layers are processed:

- *AdjustXDown*, *SmoothXDown*: processes the layers sequentially, in order of increasing *y*-coordinate.
- *AdjustXUp*, *SmoothXUp*: ditto, in order of decreasing *y*-coordinate.
- *AdjustXOut*, *SmoothXOut*: ditto, in order of increasing distance from the median layer *y*.

5 Exploring the heuristics

The heuristics described above interact in complex ways, and sometimes have antagonical effects. For instance, when the *AdjustY* heuristic is applied for the first time, it usually has the effect of moving all vertices to new layers, and re-routing all edges, with drastic effects on the horizontal order and placement of the nodes. Therefore, any previous applications of *AdjustX* before the first *AdjustY* are basically wasted effort.

As described in the introduction, our graph drawing tool provides a *playing* mode in which the heuristics are applied in random order to a fixed-size pool of drawings (of the same graph *G*). Each successful application of a heuristic generates a new drawing, which is added to the pool, while some other drawing is discarded to make room for it. This arrangement follows the *asynchronous team* or *A-team* approach, a "meta-heuristic"- for combinatorial optimization that was introduced by Talukdar, Souza and others [2, 20, 3, 21]. Basically, an A-team is an organization of autonomous agents in which each agent communicates with the others in an asynchronous way, using a shared memory, and producing a cyclic data flow. It can also be seen as a simplified and abstracted form of the *genetic algorithm* [9,24] — in this case, without crossover since we did not implement a heuristic to

combine two or more drawings.

In ouwr team, the probability of a drawing A to be discarded from the pool is proportional to its rank ρA in the '$\subset$' partial order (section 3). The *rank* is defined as the maximum r such that the pool contains a chain of drawings $A_0 \subset A_1 \subset ... \subset A_r$, with $A_r = A$. Thus, instead of obtaining a single drawing that happens to be optimum by one particular criterion, we obtain a collection of drawings that span the gamut of $\subset$-minimal solutions.

The team starts with the heuristic NewRandom, which writes initial drawings into the memory. After that, the other agents run reading drawings from the memory and producing and writing new ones into it. This processing continues until a stop criterion be reached, for example, a time limit or an expected quality of the produced drawings.

At the end of the playing-mode run, the tool writes the drawings that remain in the pool, and their histories. The history of a drawing shows the sequence of heuristic applications that produced it, and, for each heuristic, its cost (running time) and the score of the resulting drawing.

A playing-mode run produces also a set of plots like the one in figure 1, showing the evolution of each score during the simulation, as a function of time. Each horizontal gray line represents a drawing that was generated some heuristic, and extends horizontally from the time of its creation to the time it was deleted from the pool. Its ordinate is its score.

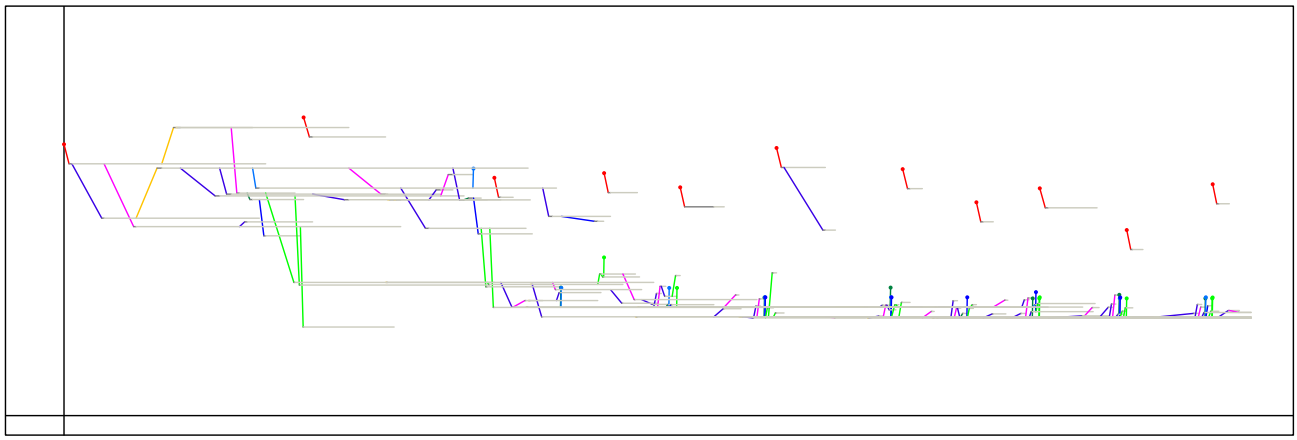

Figure 1: A scoreplot produced by the tool in *playing* mode.

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Each diagonal line represents the execution of some heuristic, indicated by the line's color (not visible in the reproduction). The line usually bridges two gray lines, representing the input drawing (leftmost endpoint) and the output drawing (rightmost endpoint). However, heuristics that take no input, like *NewRandom*, are shown starting from a dot; and heuristics that fail are shown ending in one. (In these two cases, the y-position of the dot is not significant).

These results are very important since they provide an insight how the agents do work and cooperate in the team. Once these pieces of information have been collected we can use them in the *working* mode of our tool.

After the *playing* mode there are many questions to arise which may include: What happens if I increase this the pedigree with the execution of a particular heuristic? Is a particular subsequence in the pedigree necessary? Can I reduce a subset of the pedigree sequence and get the same result? Are there any better drawings with same heuristics?…

To answer such questions the user must get first some insight about the heuristics inspecting the pedigrees of the good and bad solutions and than try some different recipes in the *working* mode. In this mode the user can tell exactly which order, intensity and number of times a set of heuristics will be run. The same recipes are used in both modes, however in playing mode the order is disregarded. A good start is the recipe found in the pedigrees of good solutions. Further information may be collected from the pedigrees of destructed solutions since they are samples of bad recipes, specially when several times a sequence is repeated just before destruction, in this case we say

that such sequence is a disease.

Therefore, the *playing* mode is an initial or training stage that provides us a starting "cake recipe" of some ways to use the heuristics (and most of the time how to do not use the heuristics). One can see that the history of the pedigree of good solutions are good since they have survived the destruction policy during the early processing. Experience shows that a pedigree which is good for one graph does not apply to another graph.

An approach containing only the *playing* mode was applied to drawing straight line general graphs [10] obtaining satisfying results at the expense of expending several days to draw a few graphs. In most cases it takes about an hour to test the validity of an idea in *playing* mode.

Thus, after the user gets some insight about how heuristics interact he will never use the *playing* mode again. In the next section the best drawings were obtained and refined in the *working* mode. The insight were acquired after a series of trial and error in the *playing* mode sometimes getting a set of mediocre results.

6 Results

One necessary limitation of heuristic methods is that they can only be evaluated empirically, by comparing their results on "typical"- problems (which of course depend on the application).

Here we show some results obtained by *playing* and *working* mode recipes on selected graphs. The tests were performed with teams consisting of recipes of the heuristics described in section 4, applied in round-robin fashion (*playing* mode) to a pool of 10 solutions. It follows some results obtained by the application of the same recipes found in the pedigree of good solutions and some recipes found by intuition. The machine used was a SPARCStation 4 100 Mhz.

Figure 2 displays drawings of a C-syntax graph used by Sugiyama [15]; the output of his algorithm is shown in figure 2

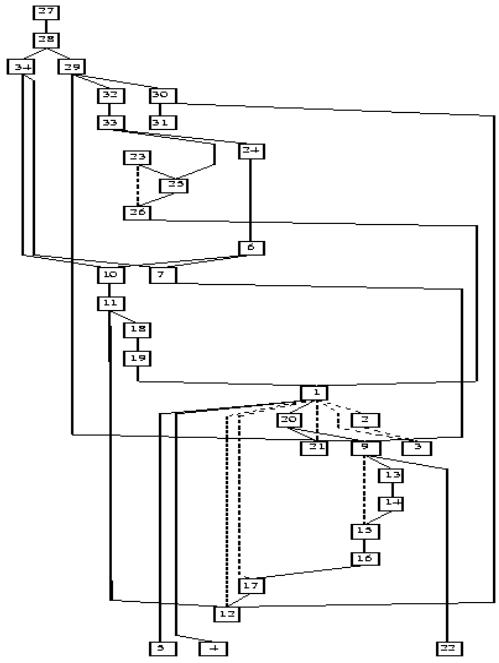
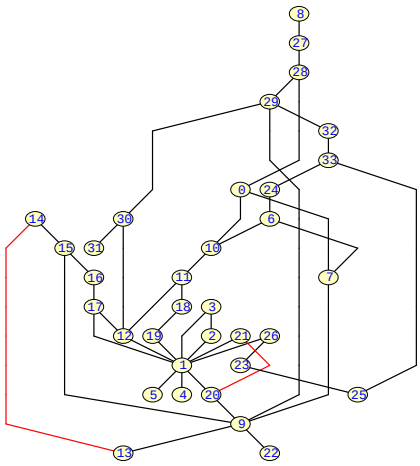


Figure 2: Drawings of the C-syntax graph.

.Figures[1] 3(a) and 3(b) show the best drawings for two A-team recipes obtained in *playing* mode.

(a)



[1] We added vertex 8 adjacent with vextex 27 in our drawings.

(b)

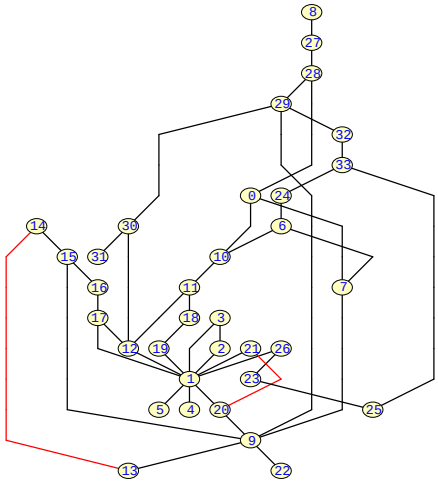
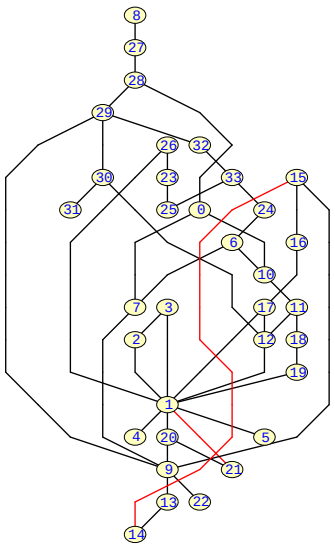


Figure 3: Drawings of the C-syntax graph in *playing* mode.

Figures 4(a) and 4(b) show drawings obtained in *working* mode after refining the pedigree of the drawing of figures 3(a) and 3(b), respectively. The time to refine the pedigree recipes of both drawing was about a couple of minutes and the score of each solution are (24 crossings, 2 ascending arcs, 30 bends, 150.98 edge length and cost = 236.953ms) and (14 crossings, 4 ascending edges, 19 bends, 116.64 edge length and cost = 702.526ms), respectively.

(a)



(b)

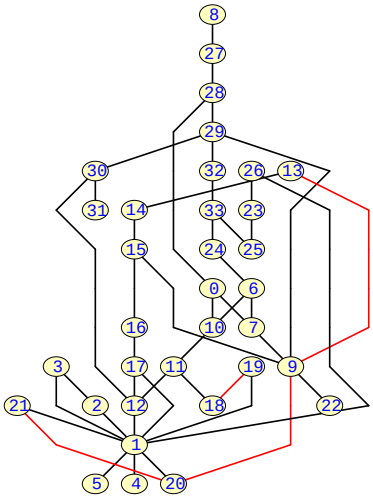
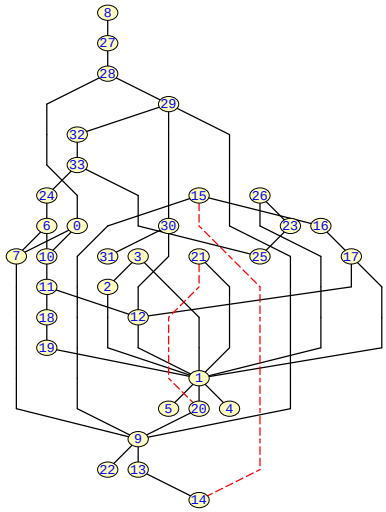


Figure 4: Drawings of the C-syntax graph in *working* mode.

Figures 5(a) and 5(b) show drawings obtained in *working* mode of C-syntax Graph. The drawings consist of third and fourth refinement of a cooked pedigree found by intuition. The refining time took about 10 minutes and the score of each solution are (32 crossings, 2 ascending arcs, 32 bends, 174.68 edge length and cost = 1208.389ms) and (24 crossings, 3 ascending edges, 29 bends, 153.89 edge length and cost = 573.809ms), respectively.

(a)



(b)

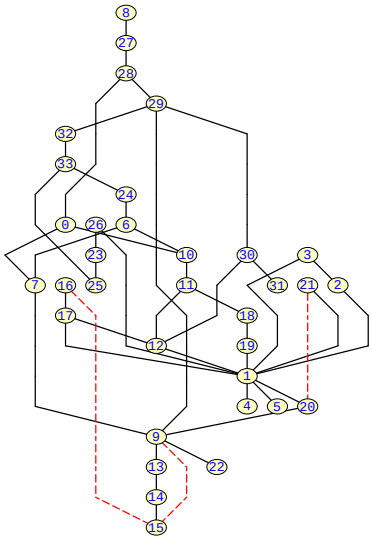
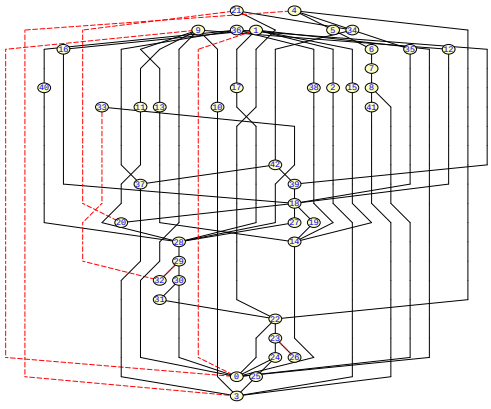


Figure 5: Drawings of the C-syntax graph, “cooking” in *working* mode.

Figures 6(a) and 6(b) show drawings obtained in *working* mode of Forrester's World Dynamics Graph [18]. The drawings consist of first and fourth refinement of a cooked pedigree found by intuition. The refining time took about 20 minutes and the score of each solution are (165 crossings, 7 ascending arcs, 80 bends, 702.92 edge length and cost = 3989.507ms) and (106 crossings, 7 ascending edges, 69 bends, 469.97 edge length and cost = 2127.546ms), respectively.

(a)



(b)

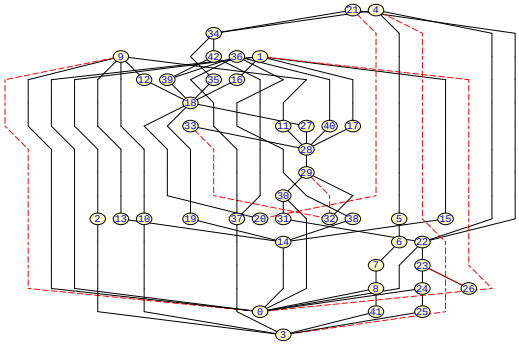


Figure 6: Drawings of Forrester's World Dynamics Graph, “cooking” in *working* mode.

Figure 7 shows a drawing obtained in *working* mode of Unix Systems Family Tree [18]. The drawings consist of the first refinement of a cooked pedigree found by intuition. The refining time took less than a minute and minute and the score of the solution is(19 crossings, 0 ascending edges, 8 bends, 92.90 edge length and cost = 453.649ms).

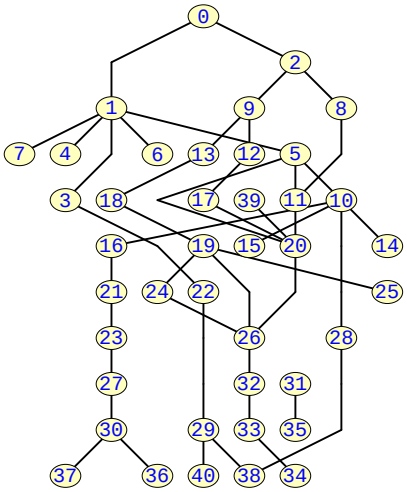


Figure 7: Drawing of the Unix Systems Family Tree, “cooking” in *working* mode.

To illustrate a pedigree in the *working* mode we give the pedigree recipe and score of drawing of figure 4(a) in figure 8. The fields in each line are <time, heuristic, [/ intensity], [@ internal repetition number]> and the last line contains the score of the pedigree: number of crossings, number of ascending edges, number of bends, sum of the length of all edges and

running time in a SPARCStation 4 100 Mhz.

```
102.076ms NewRandom
15.733ms AdjustYPack
7.095ms FixEdgeDir@5
3.750ms AdjustYPack/0.25
6.584ms FixEdgeDir@5
4.347ms AdjustYPack/0.25
40.945ms FixEdgeDir@5
6.623ms AdjustYPack/0.25
4.407ms AdjustYMean/0.10@3
6.779ms SmoothXUp/0.10
6.860ms SmoothXDn/0.10
7.106ms SmoothXUp/0.10
7.016ms SmoothXDn/0.10
17.632ms SmoothXOt/0.10@4
score = { 24 xng, 2 asc, 30 bnd,150.98 len } cost = 236.953ms
```

Figure 8: The pedigree recipe and score of drawing of figure 4(a) in *working* mode.

7 Conclusion

The heuristics we described, when suitably composed, can produce useful drawings of complex graphs. The best sequence of heuristics obviously depends on the graph. Like a training photographer that switch the camera to *automatic* to learn how to adjust the machine and later turn to *manual* to take better pictures, our tool provides two modes a *playing* mode for beginners and a *working* mode for experts. We have found instructive to observe the behavior of the heuristics when combined at random, as in an A-team (the *playing* mode of our tool). However, in this mode very little control is left to the user. After the user acquire insight about how the heuristics interact it may turn the tool in *working* mode.

For instance, even a cursory inspection of figure 1 shows that the *NewRandom* heuristic (which is rather expensive, due to the large area of the graphs it produces), quickly becomes ineffective. On hindsight, it is obvious that a freshly generated random drawing is much worse than drawings that have gone through many heuristics and have survived many rounds of selective deletion. Thus the random drawing is almost certain to rank last among the solutions in the pool, and hence is almost certain to be deleted within a few generations, before it has a chance to be processed. (Incidentally, this observation illustrates the basic inefficiency of the A-team model).

Moreover, a closer look at the pedigrees of the solutions found in the pool at the end of the run produced other valuable (and, in retrospect, equally obvious) insights, like the observation (section 5) that *AdjustX* and *SmoothX* are useful only after all instances of *AdjusY* and *FixEDirs* — as in Sugiyama's method. This observation also hints that a more intelligent edge insertion procedure (say, one that finds the path with least crossings) might allow the layers to be finely adjusted at later stages, without undoing the work of *AdjustX* and *SmoothX*.

Therefore, the A-team approach is not enough by itself. It demands much time to produce good drawings and allows some bad combinations of the heuristics, which are not useful. However, the history of the best drawings (the pedigree solutions) produced tells us how to run the heuristics in a effective way, leading us to the *working* mode of our tool.

An approach containing only the *playing* mode was applied to drawing straight line general graphs [10] obtaining satisfying results, therefore, we believe that the tool proposed in this article may be applied for other classes of graphs with other aesthetic criteria.

8 Aknowledgements

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