The AHP Extended Fuzzy Based Risk Management

MÁRTA TAKÁCS¹, TÓTHNÉ LAUFER EDIT²
Óbuda University
¹John von Neumann Faculty of Informatics
²Donát Bánki Faculty of Mechanical Engineering and Security Technology
Bécsi út 96/b, 1034 Budapest
HUNGARY
takacs.marta@nik.uni-obuda.hu
laufer.edit@bgk.uni-obuda.hu

Abstract: - Following a short general review of main characteristics of risk management applications, this paper will present a hierarchical, multilevel risk management model in fuzzy environment. In the last section a preliminary mathematical description is presented based on pair wise comparison matrix of the grouped risk factors which can be the basis for the AHP expanded principles in this model.

Key-Words: - risk management, Analytical Hierarchy Process, fuzzy, block diagonal matrix

1 Introduction

The economical crisis situations and the environmental and society complex processes as risk management systems in the past years indicate new mathematical model constructions to predict their effects. The health diagnostic is both a multi-parameter and multi-criteria decision making system with full of uncertainties and interactive risk factors and symptoms, and the diagnostic conclusion is nothing else, as well as a multi-criteria decision making system affected by symptoms and risk factors.

Extensive overviews of the risk modeling and some quantitative methods for risk analysis can be found in the books written by Haimes [1] and Vose [2]. The presented quantitative methods for risk analysis are based on well-known mathematical models of expert systems, quantitative optimum calculation models, statistical hypothesis and possibility theory. It can be observed that the statistical-based numerical reasoning methods need long-time experiments and they are time- and computing-demanding. The complexity of the systems increases the runtime factor, and the system parameter representation is usually not user-friend. The numerical methods and operation research models do not manage the vagueness and imprecision of the system parameters, or human-like expert knowledge represented in linguistic form. The complexity and uncertainties in those systems also raise the necessity of the soft computing based models.

The long time engineers experts’ experiences in the fields of soft computing are suited for modeling operational risks not only in the engineering sciences, but for a broad range of applications [11]. Wang introduces the term of risk engineering related to the risk of costs and schedules on a project in which there is the potential for doing better as well as worse than expected [3]. The presented engineering experiences, for example on the fuzzy applications, offer the promised alternative measuring of operational risks and risk management globally.

A fuzzy logic based decision making system contains fuzzified parameters and fuzzy logic based decision model. Especially in a fuzzy-based risk management system fuzzy sets describe the risk factors and fuzzy-based decision techniques help to incorporate inherent imprecision, uncertainties and subjectivity of available data, as well as propagate these attributes throughout the model, yielding more realistic results. Fuzzy logic modeling techniques can also be used in risk management systems to assess risk levels in the cases where the experts do not have enough reliable data to apply the statistical approaches. There are even more applications to deal with risk management and based on fuzzy environment. Fuzzy-based techniques seem to be particularly suited to modeling data which are scarce, and where the cause-effect knowledge is imprecise and observations and criteria can be expressed in linguistic terms. [4]

The structural modeling of risk and disaster managing is case-dependent, but in such kind of complex systems it is referenced to organize it in a grouped, multilevel constructed decision tree. The hierarchical model is widely applied. The system characteristics are as follows: it is a multi-parametrical, multi-criteria decision process, where the input parameters are the measured risk factors, and the multi-criteria rules of the system
behaviors are included in the decision process. The Analytical Hierarchy Process (AHP) expands this complex system with the pair wise comparison of the factors' importance and interaction [5].

In this paper there is a short general review of main characteristics of risk management applications, followed by a presentation of a hierarchical, multilevel risk management model in fuzzy environment. In the last section a preliminary mathematical description is presented based on pair wise comparison matrix and AHP expanded principles.

2 Risk and Disaster Analysis and Management

Risk management is the identification, assessment, and prioritization of risks, defined as the effects of uncertainty on objectives, whether positive or negative, followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events [6].

The techniques used in risk management have been taken from other areas of system management. Information technology, availability of resources, and other facts helped develop new risk management where the applied method to identify, measure and manage the risks thereby reducing the potential for unexpected loss or harm [7]. Generally a risk management process involves the main stages as described below.

The first step is the identification of risks and potential risks to the system operation at all levels. Evaluation, the measure and the structural systematization of the identified risks is the next step. Measurement is defined by how serious the risks are in terms of consequence and likelihood of occurrence, and it is very important to register the possible interaction, and furthermore the level and effects of this interaction between risk factors. The registered risk factors and its levels can have a qualitative or quantitative description of their effects on the environment.

The model development can include the development of response actions to these risks, or risk level calculation methods with the chosen decision or reasoning method. Monitoring and review as the next stage is important if there is a system with a feedback, and the risk management system is open to improvement. This will ensure that the risk management process is dynamic and continuous, with correct verification and validity control. The review process includes the possibility of new additional risks and new forms of risk description. The role of the complex risk control is in the future to try to increase the damaging effects of the risk factors, and to yield to the possible global risk management system construction, based on the developed subsystems (for global environment protection, for example).

2.1 Fuzzy-based Risk Management

The uncertainties and vagueness of the complex risk management system, and the multi-criteria and multi-parametrical environment suppose, that the fuzzy descriptions of the parameters, and fuzzy logic based approximate reasoning seem to be a useful tool for the representation. Generally the risk management system in the preliminary form contains the identification of the risk factors of the investigated process, the representation of the measured risks, and the decision model. The system can be enlarged by monitoring and review in order to improve the risk measure description and decision system. The models for solving are knowledge based models, where the linguistically communicated modeling is needed, and the objective and subjective knowledge (definitional, causal, statistical, and heuristic knowledge) is included in the decision process. Considering all those conditions, fuzzy set theory helps manage complexity and uncertainties and gives a user-friend visualization of the system construction and working model.

The fuzzy based risk management models assume that the risk factors are fuzzified (because of their uncertainties or linguistic representation). Furthermore, depending on the used fuzzy calculation and goal of the risk management, the method is constructed to figure out the final result: decision, risk level, or others. Based on the human approach where the basic rules and order are known for experts, it is natural to offer the rule-base representation. In those systems the risk management and risk level calculation statements are represented in the form of if premises then conclusion rule forms, and the risk factor calculation or output decision (summarized output) is obtained using fuzzy approximate reasoning methods (Mamadani type for example). Considering the fuzzy logic and fuzzy set theory results, there are further possibilities to extend the fuzzy-based risk management models modeling the risk factors with type-2 fuzzy sets, representing the level of the uncertainties of the membership values, or using special, problem oriented types of operators in the fuzzy decision making process.

The hierarchical or multilevel construction of the decision process, the grouped structural systematization of the factors, with the possibility of gaining some subsystems, depending on their importance or other significant environment characteristics or on laying emphasis on risk management actors' is a possible way to manage the complexity of the system. Carr and Tah describe a common hierarchical-risk breakdown structure for developing a knowledge-driven risk
management, which is suitable for the fuzzy approach [8].

Starting by simple definition of the risk as the adverse consequences of an event, such events and consequences are full of uncertainty, and inherent precautionary principles, such as sufficient certainty, prevention, and desired level of protection. All of those can be represented as fuzzy sets. The strategy of the risk management may be viewed as a simplified example of a precautionary decision process based on principles of fuzzy logic decision making [9].

2.2. Grouped, Weighted Fuzzy Model

Based on the main ideas from [8] a risk management system can be built up as a hierarchical system of the risk factors (inputs), risk management actions (decision making system) and direction or directions for the next level of risk situation solving algorithm. Actually, those directions are risk factors for the action on the next level of the risk management process. To sum this up: risk factors in a complex system are grouped according to the risk event where they figure. The risk event determinates the necessary actions to calculate and/or increase the negative effects. Actions are described by the ‘if … then’ type rules.

With the output those components frame one unit in the whole risk management system, where the items are attached on the principle of the time-scheduling, significance or other criteria (Fig. 1.). Input Risk Factors (RF) grouped and assigned to the current action are described by the Fuzzy Risk Measure Sets (FRMS) such as ‘low’, ‘normal’, ‘high’, and other. Some of the risk factor groups, risk factors or management actions have a different weighted role in the system operation. The system parameters are represented with fuzzy sets, and the grouped risk factors values give intermitted results [12]. Considering some system input parameters, which determine the risk factors’ role in the decision making system, intermitted results can be weighted and forwarded to the next level of the reasoning process. There are different applications based on this system construction and fuzzy environment [12], [13].

3 AHP extension

Let \( X_1, X_2, ..., X_n \) be the set of elements in a decision making system. It is a natural way to use the framework of a \( A_{nn} \) square matrix to represent the pair wise comparisons of the dominance and interaction of those elements. Analytical Hierarchy Process (AHP) is a method for estimating the preference values from the pair wise comparison matrix. APH allows consideration of both qualitative and quantitative aspects of the decision, expanding the decision with the one-to-one comparison of the objectives, criteria, constraints or alternatives in the system model. The pair wise comparison in the AHP assumes that the decision-maker can compare any two elements, for example \( X_i \) and \( X_j \) at the same level of the hierarchy in the system and provide a numerical value \( a_{ij} \) for the ratio of their importance. Saaty suggests the use of a scale of 1 to 9 to describe the preference measures [5], but in different applications other possible scales are presented, too [10]. Let \( a_{ij} > 1 \) if the element \( X_i \) is preferred to \( X_j \), correspondingly, the reciprocal property \( a_{ji} = 1/a_{ij} \) for \( i = 1, 2, ..., n \) and \( j = 1, 2, ..., n \). Each set of comparisons for a level with \( n \) elements requires \( n \cdot (n-1) \) judgements, which are further used to construct a positive reciprocal matrix \( A_{nn} \) of pair wise comparisons [10]. Let the comparison matrix \( A_{nn} \) be interpreted as the matrix of the dominance measures regarding to the set of risk factors in a risk management system.

If the factors are grouped, and the groups are more or less independent, the comparison matrix has the block diagonal matrix form, and that allows to pare down the computation complexity.

**Example.** Let \( X_1, X_2, ..., X_n \) be the set of risk factors grouped in \( p \) groups, and let them contain the first factors group the factors \( X_1, X_2, X_3 \). The pair wise comparison of them is represented with the 3x3 dimensional sub-matrix \( A_{11} \). The further representations are similar to this, so the next to last group contains two factors: \( X_{n-2}, X_{n-1} \), with the 2x2 dimensional sub-matrix \( A_{p-1,p-1} \), the last group holds only one factor.

\[
A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
A_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & A_{p-1,p-1} & 0 \\
0 & 0 & 0 & A_{pp}
\end{bmatrix}
\]
It is natural that the comparison values \( a_{ii} \) are units, \( a_{ii} = 1 \) for all \( i=1,2,...,n \).

Let \( x = (x_1, x_2, ..., x_n) \) be the actual input vector of the risk factors' vector \( X = (X_1, X_2, ..., X_n) \). The influence of the pair wise dominance comparison of the factors on the actual input vector can be represented as a transformation described with the matrix operation \( A \cdot x^T \). The goal is to forward a weighted input vector to the system where the weight-multiplier \( \lambda \) holds up the information about the pair wise dominance comparison of the input factors:

\[
A \cdot x^T = \lambda \cdot x^T \quad \text{(2)}.
\]

The method to compute the \( \lambda \) multiplier may be the eigenvalue method. On practical score only real eigenvalues can be accepted. If there are not real eigenvalues in the set of solutions, the multiplier \( \lambda \) is a unit one, \( \lambda = 1 \).

If there is more than one solution with the proposed conditions, the chosen one should be the eigenvalue, which keeps the input vector in there universe, but permits the highest efficiency of the decision. The AHP should be applied before the risk level calculation or decision making process.

The open problems are:
- to find the best way to pair wise comparison of the factors, because the values are the judgments obtained from an appropriate semantic scale. In practice the decision-makers usually give some or all pair-to-pair comparison values with an uncertainty degree rather than precise ratings;
- to adjust the scale of the comparison values to keep the weighted input vector in their universe, but permitting the highest efficiency of the decision;
- to build up a fuzzy AHP model for the preliminary comparison of the risk factors in the risk management system.

## 4 Conclusion

Risk management applications are complex, multi-criteria and usually multilevel decision systems, and require to manage the uncertainties. Fuzzy environment is able to represent the ambiguous risk factors and rules in an acceptable form, where the risk factors are grouped based on theirs role in the decision making system. The system parameters' interaction is not on irrelevant moment in the modeling process, that is why the pair wise comparison matrix can be added to the risk management system model. If one builds up a fuzzy based model with the grouped risk factors on the input, a fuzzy AHP model for the multilevel, hierarchically structured risk management system can be constructed, with further open problems and possibility to fine tuning in the reasoning process.

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### References:


