Low Complexity Maximum Likelihood Estimation Of Time And Frequency Offset For DVB-T2

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Abstract—The Second generation Terrestrial Digital Video Broadcasting (DVB-T2) standard provides a specific symbol, called P1 symbol, in order to facilitate initial time and frequency synchronisation by identifying the correct start of the frame. Several schemes, exploiting this symbol, have been devised to perform this particular function. In this paper, a modified maximum likelihood estimation for the time and frequency offset is derived to significantly reduce the complexity of the maximum likelihood algorithm, without sacrificing performance. The frequency offset is derived to significantly reduce the complexity of the maximum likelihood algorithm. In this paper, a modified maximum likelihood estimation for the time and frequency offset is derived to significantly reduce the complexity of the maximum likelihood algorithm, without sacrificing performance. The proposed scheme is robust against continuous wave (CW) interference. Moreover, post FFT time synchronisation is no longer required.

Index Terms—maximum likelihood, synchronization, Digital video broadcasting.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) system is widely used nowadays due to its robustness against the effects of frequency selective fading. However, OFDM systems are sensitive to time and frequency offsets and synchronisation errors [1]. To facilitate the estimation of these offsets, the DVB-T2 standard provides a specific symbol, called the P1 symbol. This is an OFDM symbol with two cyclic prefixes instead of one in a normal OFDM symbol. Not only does it transmit some basic transmission parameters, but also it provides for initial synchronisation because it is located at the start of the T2 frame. In other words, the start of the frame may be determined by correctly detecting this symbol.

The P1 symbol in time domain consists of three parts [2]. The first, and main part is a 1K OFDM symbol called the A-part. The other two parts are called the B-part and C-part, respectively, are represent the cyclic prefix of the A-part. The B-part consists of the last 482 samples of the A-part, shifted in frequency by \( f_{SH} \) and the C-part, shifted in frequency by \( f_{SH} \) as shown in Fig. 1.

A correlation based synchronisation (CBS) algorithm is presented in the DVB-T2 implementation guidelines [3]. However, this technique does not provide an accurate time and frequency estimation due to its randomness [4]. Another coarse time synchronisation method is proposed in [4] by shifting the output of the upper branch of the correlator by \( N_b \) instead if \( N_a \), yielding a more accurate estimation for the time offset than (CBS). A time domain synchronisation is proposed in [5] by correlating the received signal with the time domain P1 symbol. This method does not require a post FFT, since it gives a good estimation for the time offset. However, it is more complex than the previous methods. An estimation scheme based on maximum likelihood (ML) estimation was presented in [6]. This method results in very accurate estimation for the time offset. However, the complexity of this algorithm is prohibitive. For this reason, a suboptimal algorithm, called pseudo ML (PML) scheme was presented in [6] to reduce the complexity of the ML algorithm. In this paper, we propose a low complexity ML algorithm. The proposed algorithm is much less complex than the ML, and significantly less complex than PML schemes presented in [6]. The performance, on the other hand, is very similar to the PML scheme. The rest of the paper is organized as follows: The ML scheme is described in section II. In section III, the proposed algorithm is introduced. The basic architecture is developed in section IV. Simulation results are presented in section V. Finally, the conclusion is presented in section VI.

II. MAXIMUM LIKELIHOOD ALGORITHM

Being an OFDM system, DVB-T2 suffers from symbol time offset (STO), carrier frequency offset (CFO) and noise coming from the channel. Hence, we derive our synchronisation scheme assuming an additive white Gaussian noise (AWGN) channel that adds a time delay offset of \( \theta \) samples, and a carrier frequency offset (normalized with respect to the bandwidth \( 1/T \)) of \( \phi \). Assuming that the transmitted signal is \( S_n \), the received signal, in an AWGN channel \( r_n \) may be written as

\[
r_n = S_n - \theta e^{j2\pi \phi n} + w_n
\]

where \( w_n \) is the noise term. The structure of the P1 symbol is divided into four regions [7], [8], namely:

- \( R_C = \{ \theta, \ldots, \theta + N_c - 1 \} \)
- \( R_{A-C} = \{ \theta + N_c, \ldots, \theta + 2N_c - 1 \} \)
- \( R_{A-B} = \{ \theta + 2N_c, \ldots, \theta + 2N_c + N_b - 1 \} \)
- \( R_B = \{ \theta + 2N_c + N_b, \ldots, \theta + 2N_c + 2N_b - 1 \} \)

The log-likelihood function (LLF) is defined [9], [10] as

\[
\Lambda (\theta, \phi) = \log (f (r_n))
\]

where \( f (\cdot) \) denotes the probability density function (PDF) of the variables in its argument. Correlating the four regions results in [6]
\[
\Lambda(\theta, \phi) = \log \left( \prod_{n \in R_c} f(r_n, r_{n+N_c}) \cdot \prod_{n \in R_b} f(r_n, r_{n+N_b}) \right)
\]

where \(f(a, b)\) denotes the joint density function between \(a\) and \(b\). The product term, \(\prod_n f(r_n)\), in Eqn. 3 is independent of \(\theta\) and \(\phi\). Since ML estimation is done by maximising \(\Lambda(\theta, \phi)\), this term may be omitted.

\[
\Lambda(\theta, \phi) = \frac{\theta+N_c-1}{\theta+2N_c+N_b-1} \sum_{n=0}^{\theta+N_c-1} \log \frac{f(r_n, r_{n+N_c})}{f(r_n, f(r_{n+N_c})}
\]

The ML estimation is obtained by maximizing the LLF, i.e.

\[
\hat{\theta}_{ML} = \arg \max_{\theta} \{ \Lambda(\theta, \phi) \}
\]

After some algebraic manipulation, the LLF may be expressed as:

\[
\Lambda(\theta, \phi) = -\rho [\Phi_c(\theta) + \Phi_b(\theta)] + |\gamma_c(\theta)| \cos \left( 2\pi \frac{\theta}{N_a} + \angle \gamma_c(\theta) \right) + |\gamma_b(\theta)| \cos \left( 2\pi \frac{\theta}{N_b} + \angle \gamma_b(\theta) \right)
\]

where \(\rho = \frac{SNR}{SNR+T}\), and

\[
\gamma_c(\theta) = \sum_{n=0}^{\theta+N_c-1} r_n r_{n+N_c} e^{-j2\pi \frac{\theta}{N_a}}
\]

\[
\gamma_b(\theta) = \sum_{n=0}^{\theta+2N_c+N_b-1} r_n r_{n+N_c} e^{-j2\pi \frac{\theta}{N_b}}
\]

\[
\Phi_c(\theta) = \frac{1}{2} \sum_{n=\theta}^{\theta+N_c-1} |r_n|^2 + |r_n+N_c|^2
\]

\[
\Phi_b(\theta) = \frac{1}{2} \sum_{n=\theta+2N_c}^{\theta+2N_c+N_b-1} |r_n|^2 + |r_n+N_b|^2
\]

III. COMPLEXITY REDUCTION

Observing \(\Phi_c\) and \(\Phi_b\) it can be seen that

\[
|r_n|^2 + |r_{n+N_c}|^2 = |S_{n-\theta}|^2 + |S_{n-\theta+N_c}|^2
\]

Similarly

\[
|r_n|^2 + |r_{n+N_b}|^2 = |S_{n-\theta}|^2 + |S_{n-\theta+N_b}|^2
\]

It follows that \(\Phi_c\) and \(\Phi_b\) represent magnitudes. Thus, they do not affect the estimation, since they are constant with respect to \(\theta\) and \(\phi\).

Additionally, we notice that

\[
\angle \gamma_c(\theta) \leq \angle r_n r_{n+N_c}^* S_{n-\theta}^* e^{-j2\pi \phi(n+N_c)}
\]

\[
\leq \angle |S_{n-\theta}|^2 e^{j2\pi f_{su}(n-\theta)T} e^{-j2\pi \phi \frac{N_c}{N_a}}
\]

\[
\leq \angle e^{j2\pi f_{su}(n-\theta)T} e^{-j2\pi \phi \frac{N_c}{N_a}}
\]

At \(n = \hat{\theta}_{ML} \approx \theta\), \(e^{j2\pi f_{su}(n-\theta)T} \approx 1\), which implies that

\[
\angle \gamma_c(\hat{\theta}_{ML}) \simeq \angle e^{-j2\pi \phi \frac{N_c}{N_a}}
\]

It follows that the term

\[
2\pi \frac{N_c}{N_a} + \angle \gamma_c(\hat{\theta}_{ML}) \approx 0
\]

Similarly

\[
2\pi \frac{N_b}{N_a} + \angle \gamma_b(\hat{\theta}_{ML}) \approx 0
\]

Thus, the cosine term in Eqn. 6 will be approximately 1. Thus, the LLF may be expressed, in simplified form as follows:

\[
\Lambda(\theta, \phi) \simeq |\gamma_c(\theta)| + |\gamma_b(\theta)|
\]

In addition, we also notice that \(|\gamma_c(\theta)| \simeq |\gamma_b(\theta)|\) because the correlation of the C-part is approximately equal to the correlation of the B-part, albeit shifted by \(N_a - (N_c - N_b)\) as shown in Fig. 2.

Hence, the need to calculate the two parts separately is alleviated. Therefore, the LLF may be simplified to the following form:

\[
\Lambda(\theta, \phi) \simeq |\gamma_b(\theta)|
\]

It follows that the estimation of the modified maximum likelihood (MML) STO \(\hat{\theta}_{MML}\) is reduced to:

\[
\hat{\theta}_{MML} = \arg \max_\theta \{ |\gamma_b(\theta)| \}
\]

Finally, we may estimate the normalized fraction part of the CFO \(\hat{\phi}_{MML}\) from the CBS correlator as mentioned in the DVB-T2 implementation guideline [3], as done in [6].

Fig. 2. The output of the correlator for C-part and B-part

Output of the correlator
IV. PROPOSED ARCHITECTURE

In this section, the time synchronisation estimator is briefly described. Then we present the overall estimator, which includes the fractional part of CFO estimation. Finally, the complexity of the various algorithms is compared.

Eqns. 18 and 19 suggest the time synchronisation scheme block diagram shown in Fig. 3. It consists of a time delay, by $N_b$ samples, and two complex multipliers. This is followed by a running average filter (RAF), with a window size of $N_b$, and the LLF maximum search block. The scheme is robust against continuous wave (CW) interference and false detection problems.

The output of the correlator has a triangular shape, as shown in Fig. 4, which simplifies searching for the maximum point. The peak of the correlator output is itself the STO, shifted by $2N_a$. Thus, after obtaining the correct STO, the CBS correlator is compensated by this value to get the fractional part of the CFO.

In order to accelerate the search for the point which yields the maximum value, the search range is successively divided into eight regions. The midpoint of each range is substituted in LLF with mid-point for each range; 4) Note the range which corresponds to the highest value; 5) End for loop. 6) Search linearly to get the maximum point. 7) $\hat{\theta}_{MML} = $ Maximum point - $2N_a$. 8) Substitute by the $\hat{\theta}_{MML}$ in CBS to get $\hat{\phi}_{MML}$.

The block diagram of the whole estimator is shown in Fig.5

A. Complexity Comparison

The number of complex multipliers (CM) is the main metric for complexity measurement, due to the complex architecture of the complex multiplier. Thus, the complexity of the algorithms under consideration are compared with respect to the number of the complex multiplications required.

The CBS algorithm and also coarse synchronisation algorithm presented in [4] requires one CM for the multiplication by the frequency shift $f_{SH}$ and two CMs for the for the correlation on the two branches and another two CMs for the RAFs and one CM for multiplication of the output of the two branches, total of six CMs. For the ML algorithm the total number of CMs equal 140 CM [6]. For the PML, the number of the CMs equal 12 CM. The proposed scheme requires only seven CMs arranged as follows: one CM for the multiplication by the frequency shift $f_{SH}$ and two CMs for the for the correlation on the two branches and three CMs for the RAFs and one CM for multiplication of the output of the two branches. The complexity comparison of the various architectures is summarized in Table. II.

V. SIMULATION RESULTS

The proposed scheme was simulated on MATLAB, along with the other schemes, using the DVB-T2 test data published by the DVB-T2 Verification and Validation (V&V) working
group. An AWGN channel was used to test the timing synchronisation of the various algorithms for low SNRs. The algorithm is tested against continuous wave interference by adding a continuous wave with frequency equal to $f_{CW} = 0.3f_s/2$ as mentioned in [3], [11]. The mean square error is measured versus the SNR as shown in Fig. 6.

The results, indicated in Fig. 6, demonstrate that the CBS always exhibits an error, even for high SNRs. This may be attributed to the fact that the output of the CBS is a range, not a point. Here, we are simply selecting the midpoint of this range, which is not an accurate point due to its randomness. Additional processing would be required to accurately determine the point of the maximum. The coarse synchronisation presented in [4] results in large errors for low SNRs. However, for high SNRs, the error diminishes. The ML algorithm provides the best performance, at the cost of high complexity. The proposed algorithm and the PML give approximately the same results. Moreover, the proposed algorithm matches the performance of the ML algorithm for SNRs above -8dB, despite the considerable reduction in complexity.

The proposed scheme has also been compared with the CBS to measure the error in the detection of the fractional part of the CFO ($\hat{\phi_{MML}}$). The result, shown in Fig. 7, indicates superior performance to that of the CBS. In general, the accuracy of the estimation process of the fractional part of CFO depends on the quality of the estimate of the STO used for compensation. The proposed scheme determines the STO much more accurately. Consequently, the resultant estimate of the fractional part of CFO is also more accurate, as is evident from Fig. 7.

VI. CONCLUSION

In this paper, a robust, but comparatively simple scheme was devised for initial time and frequency synchronisation in DVB-T2 reception, using the P1 symbol. This scheme attains a performance very close to that of the maximum likelihood estimation [9], with only about 60% of the complexity of the pseudo maximum likelihood scheme described in [6]. Compared to the scheme described in [3], the performance is superior, while the complexity is only slightly larger. The simplification was largely effected by close observation, and subsequent simplification of the mathematical behaviour of the relevant estimation functions. The proposed scheme was shown to be robust against continuous wave (CW) interference, while eliminating the need for post FFT time synchronisation. The scheme was augmented with a fast search which also reduces the search delay by around two orders of magnitude.

REFERENCES