An Effective Entropic Thresholding for Ultrasonic Images

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Abstract

A new entropy-based thresholding is presented in this paper. As a result, a gray level minimizing the difference between the entropy of the object and that of the background will be the desired threshold. Performance of the proposed method has been tested on some ultrasonic images. The results obtained from the method proposed by Kapur et al. are also given for comparison.

1. Introduction

Thresholding is a common technique in extracting objects from the background. In an image, if the object is apparently distinguishable from the background, the gray level histogram will be bimodal. The threshold for segmentation can be selected easily. However, in fact, the gray level histograms are not always bimodal. So many methods [4] have been proposed to solve this problem. A common idea of these methods is to base their selection on the optimization of some threshold-dependent criteria.

Since Pun [1, 2] proposed two entropy-based methods for gray level image thresholding, the concept of entropy, which derives from Shannon's information theory, has attracted more and more attention. Among these entropic methods, one proposed by Kapur, Sahoo and Wong [3] was regarded to be typical in the literature[7]. We call it KSW-method for simplicity. Abutaleb [5], making use of two-dimensional (2D) histogram, proposed a 2D entropic technique. Pal and Pal [6] proposed another 2D entropic thresholding making use of co-occurrence matrix instead of 2D histogram. Following above two 2D entropic methods, two refined methods were proposed by Brink [7] and Chang *et al.*[8], respectively.

Ultrasonic images are being widely used in computer aided diagnosis. But it is not easy to segment ultrasonic images because of noise. The gray levels of pixels were centralized within a narrow area in their histograms, i.e., ultrasonic images have low contrast. It is impossible to threshold ultrasonic images by using ordinary methods. KSW-method is not valid to do it either.

In this paper we will propose a new entropic method for gray level image thresholding. Experimental results show this method is effective to deal with ultrasonic images.

In section 2, KSW-method will be reviewed and discussed briefly. In section 3, our new method will be proposed. In section 4, some experimental results will be given. Finally a brief summary will be given in section 5.

2. Kapur , Sahoo, and Wong Method

Suppose N is the total number of pixels in an image and there are $L(0,1,\cdots,L-1)$ gray levels. Let

$$p_i = \frac{f_i}{N}, \qquad i = 0,1,\cdots,L-1, \tag{1}$$

where $f_i (i = 0,1,\cdots,L-1)$ are the observed gray level frequencies and we have

$$\sum_{i=0}^{L-1} f_i = N. \tag{2}$$

Following Shannon's information theory, Pun[1] defined the entropy of the image as

$$H = -\sum_{i=0}^{L-1} p_i \ln p_i. \tag{3}$$

If t is the threshold selected to segment the original image, the image will be divided into two parts — the object (o) and the background (B). Without the loss of generality, we assume the object covers the gray levels from 0 to t and the background covers the gray levels from $t+1$ to $l-1$, respectively. Thus we get two distributions:

$$O: \frac{p_0}{P_t}, \frac{p_1}{P_t}, \cdots, \frac{p_t}{P_t} \tag{4}$$

and

$$B: \frac{p_{t+1}}{1-P_t}, \frac{p_{t+2}}{1-P_t}, \cdots, \frac{p_{L-1}}{1-P_t}, \tag{5}$$

where

$$P_t = \sum_{i=0}^{t} p_i. \tag{6}$$

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The entropies associated with each distribution are:

$$H_O(t) = -\sum_{i=0}^{t} \frac{p_i}{P_t} \ln \frac{p_i}{P_t} \tag{7}$$

and

$$H_B(t) = -\sum_{i=t+1}^{L-1} \frac{p_i}{1-P_t} \ln \frac{p_i}{1-P_t}. \tag{8}$$

According to KSW-method, the optimal threshold T is defined as the gray level which maximizes $H_o(t) + H_B(t)$,

i.e.,

$$T = Arg \underset{t \in G}{Max}[H_O(t) + H_B(t)], \tag{9}$$

where G represents all gray levels existing in this image.

There are some problems in This method:

(1) Sometimes there is no obvious peak in the curve $H_O(t) + H_B(t) \sim t$, as shown in Fig. 2 (d). In this case, the desired threshold can not be identified easily. This problem was proposed by Kapur *et al.* in the literature[3]. They decided the threshold value by means of suitable trade-off between maximizing $H_O(t) + H_B(t)$ and having a reasonable amount of black and white pixels. However, it is not easy for computer to operate automatically.

(2) The practical meaning of $H_O(t) + H_B(t)$ is not easy to understand. In fact, it was not provided clearly in [3].

3. Our New Method

We propose a new entropy-based criterion: the optimal threshold T is defined as the gray level which minimizes $|H_O(t) - H_B(t)|$, i.e.,

$$T = Arg \underset{t \in G}{Min} |H_O(t) - H_B(t)|. \tag{10}$$

It seems that there is just a little difference between (9) and (10). But some basic ideas are embodied within our criterion:

(1) In general we should not make a judgment which consider one of the object and the background more, the other less, unless we have enough prior knowledge about the image.

(2) In our opinion, the entropy is regarded as an interesting statistic of a distribution. The entropy of a distribution is not decided by the value or the quantity of each sample in this distribution, but by the overall probability structure of this distribution. Hence, in many images, the entropy of the object and that of the background may be similar, although the gray levels and the portion of the object are different from those of the background.

In practice, we can get several t^*, which locally minimizing $|H_O(t) - H_B(t)|$. What's more, the value of $|H_O(t) - H_B(t)|$ at some of these t^* are similar to the global minimum. If we threshold the image at these t^*, some meaningful results can be got, as shown in Fig. 1.

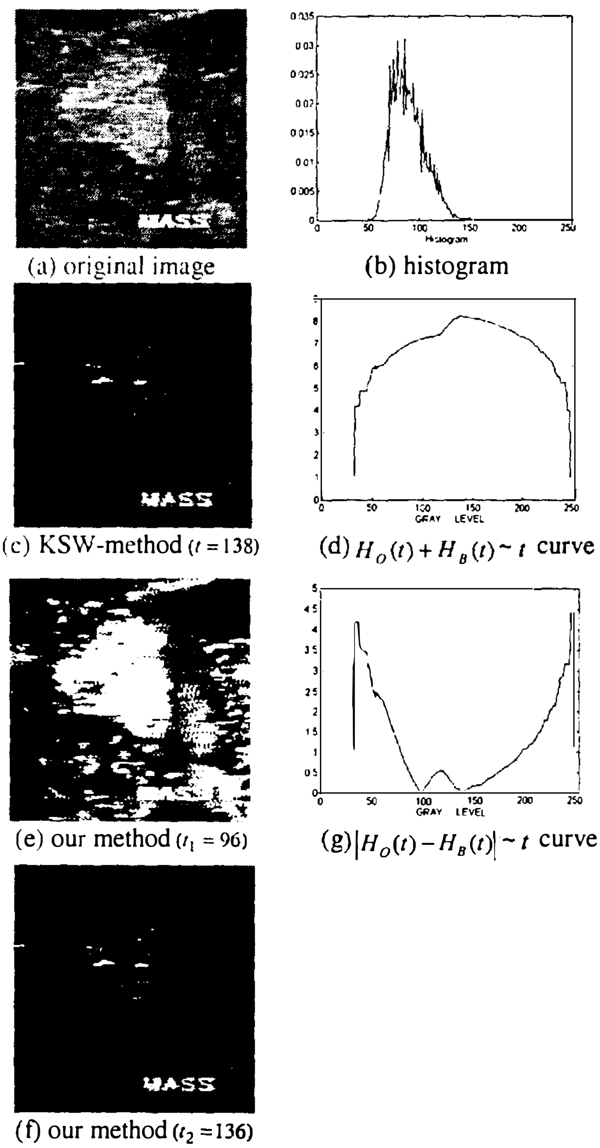
4. Experimental results

Our method has been applied to B-scan ultrasonic image thresholding. The results are shown in Fig. 1 and Fig. 2. For comparison, the results obtained from KSW-method are also given.

Furthermore, we test the performance of both methods on an artificial image for comparison generally, as shown in Fig. 3.

(1) An ultrasonic image for a tumor in liver (Fig. 1):

As show in (a), the upper left area with brighter gray levels represents a tumor in liver. By using our method, the tumor and the word (mass) can be separated from the background respectively, as shown in (e) and (f). However, only (c) can be got by using KSW-method.



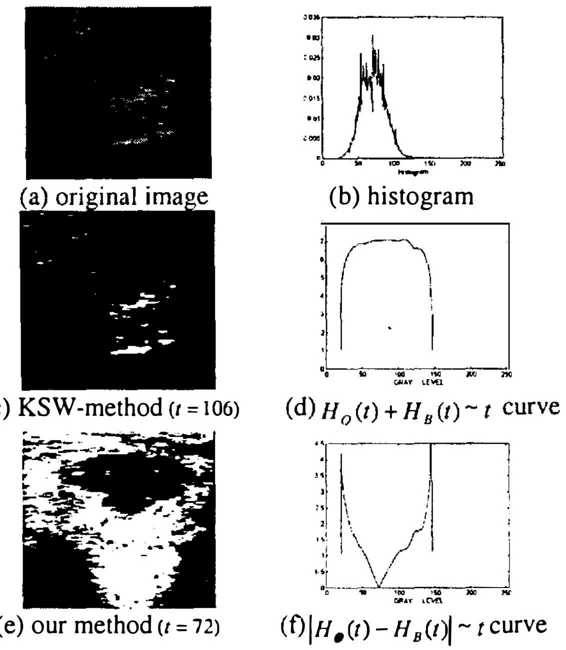
(a) original image (b) histogram

(c) KSW-method ($t = 138$) (d) $H_O(t) + H_B(t) \sim t$ curve

(e) our method ($t_1 = 96$) (g) $|H_O(t) - H_B(t)| \sim t$ curve

(f) our method ($t_2 = 136$)

Figure 1. Ultrasonic image for a tumor in liver



(a) original image (b) histogram
(c) KSW-method ($t = 106$) (d) $H_O(t) + H_B(t) \sim t$ curve
(e) our method ($t = 72$) (f) $|H_O(t) - H_B(t)| \sim t$ curve

Figure 2. Ultrasonic image for an abnormal case in breast

(2) An ultrasonic image for an abnormal case in breast (Fig. 2):

As shown in (a), the upper area with darker gray levels represents an abnormal case in breast, which is the object to extract. Obviously, it can be got by using our method, as shown in (e). The threshold is not easy to select by using KSW-method, as shown in (d). The result with the threshold at t decided by (9), is shown in (c). Apparently, it is not desired.

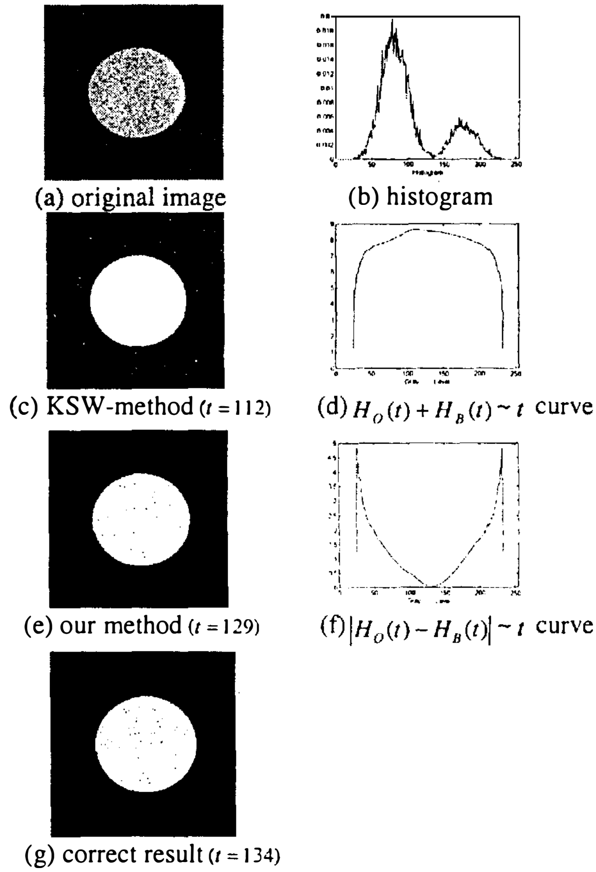
(3) An artificial image (Fig. 3):

The original image (a), degraded by Gaussian noise, has the histogram of bimodal (b). The correct result, with the threshold at the bottom of the valley, is shown in (g). The result obtained from KSW-method and our method are shown in (c) and (e), respectively.

5. Summary

A new entropic method for gray level image thresholding is described. The idea is to find a threshold which minimizes the difference between the entropy of the object and that of the background. Some experimental results show this method is effective to threshold ultrasonic images.

We should point out that our method only use one order entropy based on one-dimensional histogram, without regard to local statistics. This method can be extended to two order entropy based on two-dimensional histogram, with local statistics in consideration. The extended method will show better effectiveness in gray level image thresholding.



(a) original image (b) histogram
(c) KSW-method ($t = 112$) (d) $H_O(t) + H_B(t) \sim t$ curve
(e) our method ($t = 129$) (f) $|H_O(t) - H_B(t)| \sim t$ curve
(g) correct result ($t = 134$)

Figure 3. Artificial image

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