2-Step Maximum Likelihood Channel Estimation for Multicode DS-CDMA with Frequency-Domain Equalization

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SUMMARY  Frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can provide better downlink bit error rate (BER) performance of direct sequence code division multiple access (DS-CDMA) than the conventional rake combining in a frequency-selective fading channel. FDE requires accurate channel estimation. In this paper, we propose a new 2-step maximum likelihood channel estimation (MLCE) for DS-CDMA with FDE in a very slow frequency-selective fading environment. The 1st step uses the conventional pilot-assisted MMSE-CE and the 2nd step carries out the MLCE using decision feedback from the 1st step. The BER performance improvement achieved by 2-step MLCE over pilot assisted MMSE-CE is confirmed by computer simulation.

key words: DS-CDMA, frequency-domain equalization, MMSE, channel estimation

1. Introduction

A very high-speed wireless access technique of e.g. 100 Mbps to 1 Gbps is required for the 4th generation (4G) mobile communication systems [1]. In the present 3rd generation (3G) systems, direct sequence code division multiple access (DS-CDMA) is adopted as the wireless access technique [2]. However, since the wireless channel for such a high speed data transmission is severely frequency-selective, the bit error rate (BER) performance of DS-CDMA with rake combining significantly degrades. The use of frequency-domain equalization (FDE) based on the minimum mean square error (MMSE) criterion can provide DS-CDMA with better BER performance than rake combining [3].

FDE requires accurate estimation of the channel transfer function. Pilot-assisted channel estimation (CE) can be used. Time-domain pilot-assisted CE was proposed for single-carrier transmission in [4]. After the channel impulse response is estimated according to the least-sum-of-squared-error (LSSE) criterion, the channel transfer function is obtained by applying fast Fourier transform (FFT). Frequency-domain pilot-assisted CE was proposed in [5], [6]. The received pilot signal is transformed into the frequency-domain pilot signal and then the pilot modulation is removed using zero forcing (ZF) or least square (LS) technique. As the pilot signal, the Chu sequence [7] has the constant amplitude in both time- and frequency-domain is used. However, the number of the Chu sequences is limited. For example, it is only 128 for the case of 256-bit period [7].

PN sequences can be used for the pilot. Using a partial sequence taken from a long PN sequence, a very large number of pilots can be generated. However, since the frequency spectrum of the partial PN sequence is not constant, the use of ZF-CE produces the noise enhancement [8]. The noise enhancement can be mitigated by using the minimum mean square error (MMSE)-CE [8]. Using MMSE-CE, the channel estimation accuracy is almost insensitive to the used pilot chip sequence. To further improve the channel estimation accuracy, the decision feedback can be introduced [9], [10]. In the decision feedback channel estimation, a pilot signal is used for the initial channel estimation. The past symbol decisions can be fed back as extra pilots to update the channel estimate for the decision on the current symbol [9]. Or, all of data symbols in a frame are detected using the initial channel estimate obtained by using pilots. Then, symbol decisions are fed back as extra pilots. The pilot and all the symbol decisions are used to estimate the channel gain. This is repeated a number of times. This is known as an iterative channel estimation [10]. The idea of decision feedback channel estimation can be applied to DS-CDMA with FDE.

In this paper, to further improve the accuracy of the MMSE-CE by feeding back the tentative symbol decisions, we propose a 2-step maximum likelihood channel estimation (MLCE) assuming a very slow frequency-selective fading environment. The 1st step uses the conventional pilot-assisted MMSE-CE and the 2nd step carries out the MLCE using decision feedback from the 1st step. We evaluate the BER performance of multicode DS-CDMA using 2-step MLCE in a frequency-selective Rayleigh fading channel by computer simulation.

2. Transmission System Model

2.1 Overall Transmission System Model

The transmission system model for multicode DS-CDMA with FDE is illustrated in Fig. 1. Throughout the paper, the chip-spaced discrete-time signal representation is used.

At the transmitter, a binary data sequence is transformed into data-modulated symbol sequence and then converted to $U$ parallel streams by serial-to-parallel (S/P) conversion. Then, each parallel stream is divided into a sequence of blocks of $N_c/SF$ symbols each. The $m$th data
symbol of the \( n \)th symbol-block \((n = 0 \sim N-1)\) in the \( n \)th stream is represented by \( d_{n,m}(m); m = 0 \sim N_c/SF-1 \), where \( SF \) is the spreading factor. \( d_{n,m}(m) \) is spread by multiplying it with an orthogonal spreading sequence \( \{c_{n}(t); t = 0 \sim SF-1\} \). The resultant \( U \) chip-blocks of \( N_c \) chips each are added and further multiplied by a common scramble sequence \( \{c_{scr}(t); t = \ldots,-1,0,1,\ldots\} \) to make the resultant multicode DS-CDMA chip-block like white-noise. The last \( N_g \) chips of each \( N_c \) chip-block is copied as a cyclic prefix and inserted into the guard interval (GI) placed at the beginning of each chip-block, as illustrated in Fig. 2. For channel estimation, one pilot chip-block is transmitted every \( N-1 \) data chip-blocks to constitute a frame of \( N \) chip-blocks, as shown in Fig. 3.

The GI-inserted chip-block is transmitted over a frequency-selective fading channel and is received at a receiver. After the removal of the GI, the received chip-block is decomposed by \( N_c \)-point FFT into \( N_c \) frequency components and then FDE is carried out. After FDE, inverse FFT (IFFT) is applied to obtain the time-domain received chip-block for de-spreading and data de-modulation.
Joint conditional probability density function

\[
\begin{align*}
    S_n(k) &= \sum_{t=0}^{N_c-1} s_n(t) \exp \left( -j2\pi k \frac{t}{N_c} \right) \\
    H(k) &= \sqrt{2P} \sum_{l=0}^{L-1} h_l \exp \left( -j2\pi k \frac{t_l}{N_c} \right) \\
    \Pi_n(k) &= \sum_{t=0}^{N_c-1} \eta_n(t) \exp \left( -j2\pi k \frac{t}{N_c} \right).
\end{align*}
\]

One-tap MMSE-FDE is carried out as

\[
\hat{R}_n(k) = W(k)R_n(k),
\]

where \( W(k) \) is the MMSE-FDE weight and is given by \([11], [12]\)

\[
W(k) = \frac{H^*(k)}{\sqrt{U_c |H(k)|^2 + 2\sigma^2}}
\]

with \( 2\sigma^2 = 2N_0N_c/T_c \) being the variance of \( \Pi_n(k) \) and \( * \) denoting the complex conjugate operation. \( H(k) \) and \( \sigma^2 \) are unknown to the receiver and need to be estimated. In Sect. 3, we describe the proposed 2-step MLCE.

\( N_0 \)-point IFFT is applied to transform the frequency-domain signal \( \{\hat{R}_n(k); k = 0 \sim N_c-1\} \) into the time-domain chip-block \( \{\hat{r}_n(t); t = 0 \sim N_c-1\} \) as

\[
\hat{r}_n(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{R}_n(k) \exp \left( j2\pi k \frac{t}{N_c} \right).
\]

Finally, de-spreading is carried out on \( \{\hat{r}_n(t)\} \), giving

\[
\hat{a}_{n,m} = \frac{1}{\sqrt{SF}} \sum_{n=mSF}^{(m+1)SF-1} \hat{r}_n(t) c_n^*(t \mod SF)c_{sc}(t),
\]

which is the decision variable for data de-modulation on \( \hat{a}_{n,m}(m) \).

### 3. 2-STEP MLCE

2-step MLCE is the channel estimation scheme to improve the estimation accuracy using all of the \( N \) transmitted chip-blocks in a frame. In Sect. 3.1, we develop a maximum likelihood channel estimation (MLCE) assuming that all of \( N \) transmitted chip-blocks are available. In Sect. 3.2, we present the 2-step MLCE combined with decision feedback.

#### 3.1 Maximum Likelihood Channel Estimation (MLCE)

Joint conditional probability density function

\[
p(\{R_n(k); n = 0 \sim N-1\} | H(k), \{S_n(k); n = 0 \sim N-1\})
\]

is given as

\[
L(k) = \log \left( p(\{|R_n(k); n = 0 \sim N-1\} | H(k), \{S_n(k); n = 0 \sim N-1\}) \right)
\]

\[
\Pi_n(k) = \sum_{t=0}^{N_c-1} \eta_n(t) \exp \left( -j2\pi k \frac{t}{N_c} \right).
\]

The log-likelihood function \( L(k) \) is obtained from Eq. (11) as

\[
L(k) = \log \left( \frac{1}{2\pi\sigma^2} \right) - \frac{1}{2\sigma^2} \sum_{n=0}^{N-1} |R_n(k) - H(k)S_n(k)|^2.
\]

We want to find the maximum likelihood channel estimate \( H_{\text{ML}}(k) \) that maximizes \( L(k) \). Solving \( \partial L(k)/\partial H(k) = 0 \) gives

\[
H_{\text{ML}}(k) = \left( \sum_{n=0}^{N-1} R_n(k)S_n^*(k) \right) / \sum_{n=0}^{N-1} |S_n(k)|^2.
\]

#### 3.2 2-Step Channel Estimation

In Eq. (13), \( \{S_n(k); n = 1 \sim N-1\} \) are unknown at the receiver. Therefore, as the 1st step, we apply the MMSE-CE [8] to the pilot chip-block \( (n = 0) \). We carry out the FDE and tentative symbol decisions on the \( (N-1) \) data chip-blocks \( (n = 1 \sim N-1) \), to generate the \( (N-1) \) transmitted chip-block replicas. Then, as the 2nd step, we perform the maximum likelihood estimation using one pilot chip-block plus \( (N-1) \) transmitted chip-block replicas. This 2-step channel estimation is called 2-step MLCE in the paper. 2-step MLCE is illustrated in Fig. 4.

#### 3.2.1 1st Step

The \( k \)th frequency component of the received pilot chip-block \( (n = 0) \) can be represented as

\[
R_0(k) = H(k)C(k) + \Pi_0(k),
\]

where \( C(k) \) is the \( k \)th frequency component of the transmitted pilot chip-block \( \{\sqrt{U_c}(t); t = 0 \sim N_c-1\} \) with \( |c(t)| = 1 \) (the pilot power is set to \( UP \) to keep it the same as the \( U \)-order code-multiplexed data chip-block power). \( C(k) \) is given by

\[
C(k) = \sqrt{U} \sum_{t=0}^{N_c-1} c(t) \exp \left( -j2\pi k \frac{t}{N_c} \right).
\]

Using MMSE-CE, the instantaneous channel gain estimate \( \hat{H}^{(1)}(k) \) is obtained as

\[
\hat{H}^{(1)}(k) = X(k)R_0(k),
\]

where

\[
X(k) = \frac{C^*(k)}{|C(k)|^2 + (P/\sigma^2)^{-1}}
\]

is the reference to remove the pilot modulation [8]. The signal power \( P \) and the noise power \( \sigma^2 \) can be estimated following to [14].

The instantaneous channel gain estimate \( \{\hat{H}^{(1)}(k); k = \)
$0 \sim N_c - 1$ obtained from the pilot chip-block is noisy. The noise can be suppressed by applying delay time-domain windowing technique [15], [16]: $\tilde{H}^{(1)}(k); k = 0 \sim N_c - 1$ is transformed by $N_c$-point IFFT into the instantaneous channel impulse response $[\tilde{h}^{(1)}(\tau); \tau = 0 \sim N_c - 1]$ as

$$\tilde{h}^{(1)}(\tau) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} H^{(1)}(k) \exp \left(j 2\pi \frac{k \tau}{N_c} \right).$$

The actual channel impulse response is present only within the GI length, while the noise is spread over an entire delay-time range. Replacing $\tilde{h}^{(1)}(\tau)$ with zero’s for $N_g \leq \tau \leq N_c - 1$ and applying $N_c$-point FFT, the improved channel gain estimate $\{\tilde{H}^{(1)}(k); k = 0 \sim N_c - 1\}$ is obtained as

$$\tilde{H}^{(1)}(k) = \sum_{\tau=0}^{N_c-1} \tilde{h}^{(1)}(\tau) \exp \left(-j 2\pi k \frac{\tau}{N_c} \right) \sum_{k=0}^{N_c-1} A(k - k') \tilde{H}^{(1)}(k'),$$

where

$$A(n) = \frac{1}{N_c} \sin \left(\pi N_g \frac{n}{N_c} \right) \times \exp \left(-j \pi (N_g - 1) \frac{n}{N_c} \right).$$

The MMSE-FDE weight is computed using Eq.(8) with replacing $H(k)$ by $\tilde{H}^{(1)}(k)$. After FDE, $\{\tilde{R}_n(k); n = 1 \sim N_c - 1\}$ is transformed by $N_c$-point IFFT into the time-domain chip-block, followed by de-spreading and tentative symbol decision.

The tentatively detected symbol sequence $\{\tilde{d}^{(1)}_{m,u}; m = 0 \sim N_c/\text{SF}-1\}$, $u = 0 \sim U - 1$, is spread to obtain the transmitted chip-block replica $\{\tilde{s}^{(1)}_n(t); t = 0 \sim N_c - 1\}$:

$$\tilde{s}^{(1)}_n(t) = \left\{ \sum_{u=0}^{U-1} \tilde{d}^{(1)}_{m,u} \left[ \frac{t}{\text{SF}} \right] \right\} c_{\text{scf}}(t).$$

Applying $N_c$-point FFT to $\{\tilde{s}^{(1)}_n(t)\}$, the $k$th frequency component of the transmitted chip-block replica is obtained as

$$\tilde{S}^{(1)}_n(k) = \sum_{t=0}^{N_c-1} \tilde{s}^{(1)}_n(t) \exp \left(-j 2\pi k \frac{t}{N_c} \right).$$

3.2.2 2nd Step

$\{S_n(k)\}$ is replaced by $\{\tilde{S}^{(1)}_n(k)\}$ for $n \neq 0$. $\tilde{H}^{(2)}(k)$ is obtained, from Eq. (13), as

$$\tilde{H}^{(2)}(k) = \frac{R_0(k) C^* + \sum_{n=1}^{N_c-1} R_n(k) \left| \tilde{S}^{(1)}_n(k) \right|^2}{|C(k)|^2 + \sum_{n=1}^{N_c-1} \left| \tilde{S}^{(1)}_n(k) \right|^2}.$$

By applying delay time-domain windowing technique to $\{\tilde{H}^{(2)}(k); k = 0 \sim N_c - 1\}$ as in the 1st step, the improved channel gain estimate $\{\tilde{H}^{(2)}(k); k = 0 \sim N_c - 1\}$ is obtained.

4. Computer Simulation

The simulation condition is shown in Table 1. We assume 16QAM data modulation, an FFT block size of $N_c = 256$ chips and a GI of $N_g = 32$ chips. One pilot chip block is transmitted every 15 data chip-blocks (i.e., $N = 16$). We assume the spreading factor $\text{SF} = 16$ and an $L = 16$-path frequency-selective block Rayleigh fading channel having exponential power delay profile with decay factor $\alpha$.

In computer simulation, we also measured the BER performance using pilot-assisted MMSE-CE with decision feedback [8] and that with ideal CE for comparison.

The simulated BER performance of multicode DS-CDMA with MMSE-FDE is plotted in Fig.5 for $U = 1$ and 16 as a function of the average received bit energy-to-AWGN noise power spectrum density ratio $E_b/N_0 (= \infty)$.

<table>
<thead>
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<th>Table 1</th>
<th>Simulation condition.</th>
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<tr>
<td>Transmitter</td>
<td>Data modulation</td>
</tr>
<tr>
<td>Number of FFT points</td>
<td>$N_c = 256$</td>
</tr>
<tr>
<td>Guard interval length</td>
<td>$N_g = 32$</td>
</tr>
<tr>
<td>Spreading sequence</td>
<td>Product of Walsh sequence and PN sequence</td>
</tr>
<tr>
<td>Spreading factor</td>
<td>$\text{SF} = 16$</td>
</tr>
<tr>
<td>Code multiplexing order</td>
<td>$U = 1, 16$</td>
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<tr>
<td>Pilot chip sequence</td>
<td>PN sequence</td>
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<td>Channel Fading</td>
<td>Frequency-selective block Rayleigh</td>
</tr>
<tr>
<td>Power delay profile</td>
<td>$L = 16$-path exponential power delay profile</td>
</tr>
<tr>
<td>Decay factor $\alpha$</td>
<td>$0, 3$, $\infty$ (dB)</td>
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</table>
0.25(P · SF · $T_c/N_0)(1 + N_g/N_c)N/(N − 1))$. We have assumed block fading (represented by the maximum Doppler frequency of $f_D \to 0$), where the channel gains stay constant over a frame ($N$ chip-blocks). With pilot-assisted MMSE-CE with decision feedback, the $E_b/N_0$ loss from the ideal CE case for BER = $10^{-4}$ is about 0.8 (0.9) dB when $U = 1$ (16). This $E_b/N_0$ loss includes a pilot insertion loss of 0.28 dB. The use of 2-step MLCE improves the BER performance and the $E_b/N_0$ loss can be reduced to about 0.4 dB for both $U = 1$ and 16.

The simulated BER performance is plotted in Fig. 6 with decay factor $\alpha$ as a parameter for the full code-multiplexing case ($U = SF = 16$). $\alpha \to \infty$ corresponds to the single-path case ($L = 1$). Regardless of decay factor $\alpha$, 2-step MLCE provides a better BER performance than conventional MMSE-CE and reduces the $E_b/N_0$ loss from the ideal CE to about 0.4 dB.

As the fading rate increases, it becomes more likely that different chip-blocks in the same frame will have different BERs since the channel estimation tends to lose the tracking ability against fading variation; the BER per chip-block may degrade as the chip-block index $n$ increases, $n = 1 \sim 15$. The simulated BER is plotted in Fig. 7 as a function of the block index $n$ when the normalized Doppler frequency $f_D(N_c + N_g)T_c = 10^{-4}$ and $10^{-3}$. When $f_D(N_c + N_g)T_c = 10^{-4}$, 2-step MLCE provides almost the constant BER while conventional MMSE-CE decreases the BER as the chip-block index $n$ increases. This is because the effect of averaging the noise enhancement is increased as the chip-block index $n$ increases in conventional MMSE-CE. On the other hand, the proposed 2-step MLCE provides always smaller BER than the conventional MMSE-CE. However, when $f_D(N_c + N_g)T_c = 10^{-3}$, the proposed 2-step MLCE is inferior to conventional MMSE-CE for $n > 9$. This is because the proposed 2-step MLCE assumes the constant channel gain over a frame of $N = 16$ chip-blocks.

So far we have assumed a block fading where the channel gain stays constant over a frame. However, as the terminal moving speed gets faster, this assumption cannot hold. Here, we assume that the channel gains vary over a frame ($N$ chip-blocks), but still stay constant during each chip-block. Figure 8 shows the impact of fading rate on the achievable BER as a function of the normalized Doppler frequency $f_D(N_c + N_g)T_c$ at $E_b/N_0 = 24$ dB for the full code-multiplexing case ($U = SF = 16$). It is seen from Fig. 8 that 2-step MLCE provides a better BER performance than conventional MMSE-CE when $f_D(N_c + N_g)T_c < 7 \times 10^{-4}$.
Fig. 8 Impact of fading rate.

Average BER

- 2-step MLCE
- Pilot-assisted MMSE-CE w/ decision feedback [8]

16QAM
$N_c = 256, N_e = 32$
$S_F = U = 16$
$L = 16, $\alpha = 0$ dB
$E_b/N_0 = 24$ dB

Fig. 8 Impact of fading rate.

(this corresponds to a terminal moving speed of 52.5 km/h for a chip rate $1/T_c$ of 100 Mcps and 5 GHz carrier frequency). However, for a higher fading rate, the proposed 2-step MLCE is inferior to conventional MMSE-CE since it assumes the constant channel gain over a frame ($N$ chip-blocks).

5. Conclusions

In this paper, we proposed the 2-step MLCE for multicode DS-CDMA with MMSE-FDE in a very slow frequency-selective fading channel. It was shown by computer simulation that the proposed 2-step MLCE improves the BER performance compared to the conventional pilot-assisted MMSE-CE with decision feedback. The required $E_b/N_0$ loss for BER = $10^{-4}$ from the ideal CE is only 0.4 dB (about 0.28 dB is due to the pilot insertion) irrespective of code multiplexing order and channel decay factor. However, 2-step MLCE assumes that the channel gains stay constant over a frame and therefore, the achievable BER performance degrades as the fading gets faster. In a fast fading environment (the maximum Doppler frequency normalized by the chip-block length $> 7 \times 10^{-5}$), the proposed 2-step MLCE is inferior to the conventional pilot-assisted MMSE-CE with decision feedback.

References


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