

# Investigation on the Dual-Phase Lag Effects in Biological Tissues during Laser Irradiation

Hamed Ziaei Poor <sup>\*1</sup>, Hassan Moosavi <sup>1</sup>, Amir Moradi <sup>2</sup>

Hossein Ghorbani Menghari <sup>1</sup>, Mahboubeh Parastarfeizabadi <sup>3</sup>

<sup>1</sup>Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran

<sup>2</sup>Department of Mechanical Engineering, Bu-Ali Sina University, Hamadan, Iran

<sup>3</sup>Department of Biomedical Engineering, Isfahan University, Isfahan, Iran

\*Corresponding Author: Tel: +98-918 9633493, Fax: +98-311 3912628, h.ziaeipoor@me.iut.ac.ir<sup>1</sup>

**Abstract-** This paper studies the dual-phase lag (DPL) bioheat transfer model in biological tissues during laser irradiation. The energy equation is analytically solved using the separation of variables and Duhamel's integral method. The problem is considered for both highly absorbed and strongly scattered tissues. The analytical results indicate that the DPL bioheat transfer model describe different thermal responses from the thermal wave (hyperbolic) and Pennes (parabolic) bioheat transfer models, depending on the values of the two lagging times. The results show that the DPL bioheat transfer model approaches to Pennes bioheat transfer model when just both thermal lagging times approach zero, whilst different thermal responses are estimated by the DPL model than that of calculated by Pennes bioheat transfer model when  $\tau_q = \tau_r$ . The reliability of the proposed analytical method is verified through comparison with the existing results in the literature and the results show that there is a good agreement between our outcomes and the past studies. Impacts of different time delays and laser parameters such as the laser irradiance and laser exposure time on temperature history of biological tissues are also studied. The effect of the blood perfusion on the transient temperature distribution of tissue is carried out and it is found that the effect of blood perfusion rate is similar to that of the time delay  $\tau_r$ .

**Keywords-** Dual-Phase Lag (DPL) Model; Laser Irradiation; Duhamel's Integral Method; Thermal Relaxation; Biological Tissue

## I. INTRODUCTION

During past decades, laser is widely used in surgical techniques and employed in different clinical areas. Laser treatments such as laser hyperthermia, coagulation, and surgery involved thermal effect. The prediction of temperature in biological tissues during laser treatments is very important for personal safety and thermal damage. In order to obtain deep understanding of thermal behavior involved in laser irradiated tissues numerous studies have been conducted [1-4]. Most of the researchers applied Pennes bioheat transfer model to predict temperature response in biological tissues. A tree steps model for predicting laser-induced damage in tissue was presented by Welch [5]. Many researchers [6-8] employed Welch's approach to study the bioheat transfer in biological tissues. But all of them used Pennes bioheat conduction (Fourier heat conduction) theory, assuming the thermal disturbance propagates at an infinite speed.

Although, the Fourier heat conduction model provides good results in most engineering problems, but in some cases that the assumption of infinite heat propagation speed is inadequate such as those involving extremely short times or high heat flux [9-10], this model gives nonphysical results. For analysis of bioheat transfer in nonhomogeneous biological tissues, hyperbolic (thermal wave) bioheat transfer model has been proposed [11-13]. The bioheat transfer problem of short pulses of laser irradiation on body tissues was numerically studied by Jaunich et al. [14]. Their results showed that the experimentally measured temperature distributions agreed with the predictions made through the hyperbolic heat conduction model. Trujillo et al. [15] proposed an analytical study of thermal-optic model for laser heating of biological tissue using the hyperbolic heat conduction model. They used the Laplace transform method to solve the bioheat transfer equation for irradiated tissues and derived their results for a laser pulse duration of 200  $\mu s$ . Effect of the thermal wave in radiofrequency ablation modeling was analytically studied by Molina et al. [16]. They analytically solved the electrical-thermal coupled problem by including the power source in both equations and compared the parabolic and hyperbolic results. Ahmadikia et al. [17] applied the Laplace transform method to solve the hyperbolic and parabolic heat conduction in biological tissue during laser irradiation. They solved the bioheat transfer model for both highly absorbing and scattering tissues and found that the non-Fourier effect should be considered during laser irradiation for highly absorbing tissues to prevent error in the predicted tissue temperature. Zhou et al. [18] numerically investigated the non-Fourier heat conduction effect on laser-induced thermal damage in biological tissues. They solved the thermal wave and Pennes bioheat transfer model and found that thermal damaged predicted by non-Fourier model is totally different with that of assessed by classical Fourier model. Kim [19] in his PhD thesis studied the thermal response of biological tissue subjected to short-pulsed laser irradiation. He solved a combined transient hyperbolic heat

conduction with radiation to simulate a heat transfer in biological tissues. He used the experimental results to evaluate the hyperbolic heat conduction model and found that some experimental results verify the hyperbolic model by the temperature suddenly dropping rather than gradual temperature change. Zhang et al. [20] analytically studied the non-Fourier heat conduction studying on high short pulse laser. They demonstrated that the non-Fourier effect becomes more remarkable when relaxation time increases. Molina et al. [21] presented an analytical model for pulsed radiofrequency ablation based on hyperbolic heat conduction model.

Although the hyperbolic model (thermal wave model) can describe the delayed response between the heat flux and the temperature gradient, it still assumes an immediate response between the temperature gradient and the energy transport. Because biological tissues consist of complicated and nonhomogeneous structure, there is not only delayed response between heat flux and temperature gradient but also a delayed response exists between the temperature gradient and the energy transport. Tzou [22, 23] firstly proposed the dual phase lag (DPL) model that introduces two time relaxations to account for two types of delayed response among the heat flux, the temperature gradient and the energy transport. The agreement of DPL model with experimental data has been shown in [24]. In recent years, DPL model has attracted many interests of researchers in a wide variety of scientific and engineering problems. Xu et al. [25] applied the DPL model to study the bioheat transfer in skin tissue during surface heating. They compared three bioheat transfer models (Pennes, thermal wave and DPL model) with each other and found that the thermal relaxations have significant effects on transient temperature and thermal damage. Zhou et al. [26] numerically investigated dual phase lag effect on thermal damage of biological tissues during laser irradiation. They studied the effect of thermal relaxations, blood flow and laser parameters on temperature history and thermal damage of biological tissues. Their results indicated that effects of blood flow on thermal response and thermal damage are similar to those of the time delay between the temperature gradient and the energy transport. Afarin et al. [27] presented a numerical simulation of thermal damage to living biological tissue caused by laser irradiation. They solved a generalized DPL bioheat transfer model based on the nonequilibrium heat transfer in living biological tissues. They indicated that the generalized DPL model predicts different temperature and thermal damage than that of predicted by the classical DPL model. Liu and Wang [28] used the DPL bioheat transfer model to analyze thermal response for estimating thermal damage in laser-irradiated biological tissue. They numerically solved the energy equation and explored the effects of blood perfusion and the metabolic heat generation on thermal response and thermal damage of biological tissue. In addition to these studies, some researchers during the recent years have conducted an analytical study on dual phase lag bio-heat transfer in tissues. [29-31]

This paper evaluates the dual phase lag bioheat transfer model in biological tissues during laser irradiation. An analytical method based on the separation of variables and Duhamel's integral method is developed for the solution of the energy equation for two types of scattering and absorbing tissues. This method provides a more accurate and faster solution for temperature response of biological tissues induced by laser irradiation. To the authors' knowledge, such a development has not been pursued in the literature yet. All of works in this field employed numerical techniques for solution of the DPL bioheat transfer model during laser irradiation. Subsequently, the analytical results for different time delays, blood perfusion and laser parameters are derived and compared with existing results obtained from literature.

## II. THE MATHEMATICAL FORMULATION

The classical Fourier heat conduction model in biological tissues was firstly introduced by Pennes [32] as follows:

$$\rho_t c_t \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \rho_b \omega_b c_b (T_a - T) + Q_{met} + Q_{ext} \quad (1)$$

where  $\rho_b$  and  $c_b$  are the density and specific heat of the blood, respectively;  $\rho_t$ ,  $c_t$  and  $k$  are the density, specific heat and the thermal conductivity of skin tissue, respectively; and  $\omega_b$  is the blood perfusion rate;  $T$  and  $T_a$  are skin tissue and blood temperatures, respectively;  $Q_{met}$  and  $Q_{ext}$  are the metabolic heat generation in skin tissue and the heat generated by external heating sources, respectively. Vernott [31] and Cattaneo [34] proposed the modification of classical Fourier's law by considering the concept of finite propagation speed of thermal waves as follows:

$$q(x, t) + \tau_q \frac{\partial q(x, t)}{\partial t} = -k \nabla T(x, t) \quad (2)$$

Here  $\tau_q > 0$  is a material property and is called the relaxation time. Considering Eq. (2) as well as the Pennes equation, a general form of thermal wave model of bioheat transfer in living biological tissues can be written as [35]:

$$\tau_q \rho_t c_t \frac{\partial^2 T}{\partial t^2} + (\rho_t c_t + \tau_q \rho_b \omega_b c_b) \frac{\partial T}{\partial t} + \rho_b \omega_b c_b (T - T_a) = k \frac{\partial^2 T}{\partial x^2} + (Q_{met} + Q_{ext} + \tau_q \frac{\partial Q_{met}}{\partial t} + \tau_q \frac{\partial Q_{ext}}{\partial t}), \quad (3)$$

Where  $\tau_q = \alpha / C^2$ ,  $\alpha$  is the thermal diffusivity and  $C$  is the thermal wave speed in the medium [23,36]. The thermal wave model has only considered fast transient process of heat transfer and has ignored the microstructural interactions. These two effects can be reasonably proposed by the dual phase lag (DPL) between  $q$  and  $\nabla T$ , also, further modification of the classical Fourier's model gives:

$$q(x, t) + \tau_q \frac{\partial q(x, t)}{\partial t} = -k \left( \nabla T(x, t) + \tau_T \frac{\partial \nabla T(x, t)}{\partial t} \right), \quad (4)$$

In which is the relaxation time which is the phase-lag in establishing the temperature gradient across the medium during which conduction occurs through its small-scale structures. This equation reduces to the Fourier model when both lag times approach zero. Based on Eq. (4) for temperature gradient including the characteristic time as well as the thermal wave model of bioheat transfer equation, a general form of the DPL (dual phase lag) model of bioheat transfer in living tissues is expressed by [25]

$$\tau_q \rho_t c_t \frac{\partial^2 T}{\partial t^2} + (\rho_t c_t + \tau_q \rho_b \bar{\omega}_b c_b) \frac{\partial T}{\partial t} + \rho_b \bar{\omega}_b c_b (T - T_a) = k \left( \frac{\partial^2 T}{\partial x^2} + \tau_T \frac{\partial^3 T}{\partial x^2 \partial t} \right) + (\dot{Q}_{met} + \dot{Q}_{ext} + \tau_q \frac{\partial \dot{Q}_{met}}{\partial t} + \tau_q \frac{\partial \dot{Q}_{ext}}{\partial t}) \quad (5)$$

Here, in above equation  $\dot{Q}_{ext}$  is heat generated by irradiated laser ( $q_L$ ). The laser volumetric heat source is expressed by [26, 37].

$$q_L(x, t) = \mu_a \phi(x, t) \quad (6)$$

Here  $\mu_a$  and  $\phi(x, t)$  are the absorption coefficient of material and local light distribution, respectively. Two types of highly absorbing and scattering tissues are considered here. For highly absorbing tissues, the light distribution can be obtained by Beer's law [37]:

$$\phi(x, t) = (1 - R)I(t)e^{-\mu_a x} \quad (7)$$

Where,  $I(t)$ ,  $R$  and  $\mu_a$  are the laser intensity, light reflectance of surface and attenuation coefficient. For scattering tissues the light distribution is expressed by diffusion theory as follows:

$$\phi(x, t) = I_0(t) \left( C_1 e^{\frac{-k_1 x}{\delta}} - C_2 e^{\frac{-k_2 x}{\delta}} \right) \quad (8)$$

Where  $C_1, C_2, K_1, K_2$  are calculated based on the Monte Carlo solutions [38], depending on the diffuse reflectance;  $\delta$  the effective optical penetration depth, which is defined from the diffusion theory as:

$$\delta = \frac{1}{\sqrt{3\mu_a[\mu_a + \mu_s(1-g)]}} \quad (9)$$

Equation (9) is valid when  $0.1 \leq \mu_s / (\mu_a + \mu_s) \leq 0.999$  and  $0.7 \leq g \leq 0.9$

Where  $\mu_s$  and  $g$  are scattering coefficient and anisotropy factor, respectively.

### III. THE ANALYTICAL SOLUTION OF THE PROBLEM

#### A. The dual-phase lag (DPL) bioheat transfer model for highly absorbing tissues

Here, the energy equation is analytically solved in a finite domain of a biological tissue during laser irradiation. The adiabatic conditions are considered for boundaries. The initial surface temperature is  $T_a = 37^\circ\text{C}$  here. So that, the initial and boundary conditions can be expressed by:

$$T(x, 0) = T_a \quad (10) \quad \frac{\partial T}{\partial t} \Big|_{t=0} = \frac{\partial^2 T}{\partial t^2} \Big|_{t=0} = 0 \quad (11)$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0 \quad (12) \quad \frac{\partial T}{\partial x} \Big|_{x=l} = 0 \quad (13)$$

Using Eqs. (5-7), the DPL bioheat transfer equation for highly absorbing tissues during laser irradiation is expressed by:

$$\begin{aligned} \tau_q \cdot \rho_t C_t \frac{\partial^2 T}{\partial t^2} + (\rho_t C_t + \tau_q \rho_b \varpi_b C_b) \frac{\partial T}{\partial t} + \rho_b \varpi_b C_b (T - T_a) \\ = k \left( \frac{\partial^2 T}{\partial x^2} + \tau_T \frac{\partial^3 T}{\partial x^2 \partial t} \right) + q_{met} + \left( I(t) + \tau_q \frac{\partial I(t)}{\partial t} \right) \times (1 - R) \mu_a e^{-\mu_a x} \end{aligned} \tag{14}$$

For simplicity, the dimensionless variables are defined by:

$$\begin{aligned} W_b = \rho_b \varpi_b \quad \xi = p x \quad , \quad \theta = \frac{T - T_a}{I_0} \sqrt{k W_b C_b} \quad , \quad p = \sqrt{\frac{W_b C_b}{k}} \quad , \quad L = p l \\ \eta = \frac{W_b C_b}{\rho_t C_t} t \quad , \quad \Lambda_q = \frac{W_b C_b}{\rho_t C_t} \tau \quad , \quad \Lambda_i = \frac{W_b C_b}{\rho_t C_t} \tau_i \quad , \quad \Lambda_T = \frac{W_b C_b}{\rho_t C_t} \tau_T \end{aligned} \tag{15}$$

Eq. (14) can be rewritten in terms of dimensionless variables and then gives:

$$\begin{aligned} \Lambda_q \frac{\partial^2 \theta}{\partial \eta^2} + (1 + \Lambda_q) \frac{\partial \theta}{\partial \eta} + \theta = \frac{\partial^2 \theta}{\partial \xi^2} + \Lambda_T \frac{\partial^3 \theta}{\partial \xi^2 \partial \eta} + \frac{q_{met}}{p I_0} \\ + \underbrace{\left( I \left( \frac{\rho_t C_t}{W_b C_b} \eta \right) + \Lambda_q \frac{\partial I \left( \frac{\rho_t C_t}{W_b C_b} \eta \right)}{\partial \eta} \right)}_{F(\eta)} \frac{(1 - R) \mu_a e^{-\left(\frac{\mu_a}{p}\right) \xi}}{p I_0} \end{aligned} \tag{16}$$

While the dimensionless initial and boundary conditions are expressed by:

$$\theta(\xi, 0) = 0 \quad , \quad \left. \frac{\partial \theta}{\partial \eta} \right|_{\eta=0} = \left. \frac{\partial^2 \theta}{\partial \eta^2} \right|_{\eta=0} = 0 \tag{17}$$

$$\left. \frac{\partial \theta}{\partial \xi} \right|_{\xi=0} = 0 \quad , \quad \left. \frac{\partial \theta}{\partial \xi} \right|_{\xi=L} = 0 \tag{18}$$

This problem consists of nonhomogeneous differential equation and nonhomogeneous heat source. Transient heat conduction problems with time-dependent boundary conditions or heat source cannot be solved by employing the separation of variables. The available methods for the solution of such problems are the Laplace transform method, Green’s function and Durhamel’s superposition integral. Here we use the latter one to solve Eq. (16) regarding the time-dependent laser heat source. According to this method, continuous time-dependent disturbance is reduced to the sum of the time stepwise disturbances. So that, the general solution of  $\theta(\xi, \eta)$  with reference to the time-dependent laser heat source is obtained with sum of stepwise solution of  $\theta(\xi, \eta)$  in each step. With decreasing the time step  $d\tau$ , the total effect at time  $\eta$  is achieved by integrating the effect of time dependent laser heat in the time intervals of  $d\tau$  and summing it with the beginning effect of  $F(0)$  as follows [39]:

$$\theta(\xi, \eta) = F(0) \theta(\xi, \eta) + \int_0^\eta \theta(\xi, \eta - \tau) \frac{dF(\tau)}{d\tau} d\tau, \tag{19}$$

where  $\theta(\xi, \eta)$  is the solution of Eq. (16) with initial and boundary conditions (17) and (18) considering  $F(\eta) = 1$ . Subsequently for solution of Eq. (16) with initial and boundary conditions (17) and (18) the problem is formulated in terms of a steady and transient parts as the following:

$$\theta(\xi, \eta) = \theta_1(\xi, \eta) + \theta_2(\xi) \tag{20}$$

Substituting Eq. (20) into Eq. (16) yields

$$\Lambda_q \frac{\partial^2 \theta_1}{\partial \eta^2} + (1 + \Lambda_q) \frac{\partial \theta_1}{\partial \eta} + \theta_1 = \frac{\partial^2 \theta_1}{\partial \xi^2} + \Lambda_T \frac{\partial^3 \theta_1}{\partial \xi^2 \partial \eta} \tag{21}$$

$$\frac{\partial^2 \theta_2}{\partial \xi^2} - \theta_2 = -\frac{q_{met}}{p I_0} + \frac{(1 - R) \mu_a e^{-\left(\frac{\mu_a}{p}\right) \xi}}{p I_0} \tag{22}$$

The boundary conditions for the steady-state part (Eq. (22)) are:

$$\left. \frac{\partial \theta_2}{\partial \xi} \right|_{\xi=0} = 0, \quad \left. \frac{\partial \theta_2}{\partial \xi} \right|_{\xi=L} = 0 \tag{23}$$

The solution for Eq. (22) regarding the boundary conditions (23) is obtained by:

$$\mathcal{G}_2(\xi) = \frac{(1-R)\mu_a e^{-\frac{\mu_t \xi}{p}}}{p \left(1 - \frac{\mu_t^2}{p^2}\right)} + \frac{q_{mer}}{p I_0} + \frac{(1-R)\mu_a \frac{\mu_t}{p} \left( \text{Cosh}(\xi) e^{-\frac{\mu_t L}{p}} - \text{Cosh}(\xi - L) \right)}{p \left(1 - \frac{\mu_t^2}{p^2}\right) \text{Sinh}(L)} \quad (24)$$

Now the initial and boundary conditions for the transient part (Eq. (21)) are given by:

$$\mathcal{G}_1(\xi, 0) = -\mathcal{G}_2(\xi), \quad \frac{\partial \mathcal{G}_1}{\partial \eta} \Big|_{\eta=0} = \frac{\partial^2 \mathcal{G}_1}{\partial \eta^2} \Big|_{\eta=0} = 0 \quad (25)$$

$$\frac{\partial \mathcal{G}_1}{\partial \xi} \Big|_{\xi=0} = 0, \quad \frac{\partial \mathcal{G}_1}{\partial \xi} \Big|_{\xi=L} = 0 \quad (26)$$

By considering boundary conditions (26), we expand the function  $\mathcal{G}_1(\xi, \eta)$  into the following Fourier series:

$$\mathcal{G}_1(\xi, \eta) = \sum_{i=0}^{\infty} T_i(\eta) \text{Cos} \frac{i\pi}{L} \xi \quad (27)$$

Substitution of Eq. (27) into Eq. (21) gives:

$$\Lambda_q T_i''(\eta) + (1 + \Lambda_q + \Lambda_T \lambda_i^2) T_i'(\eta) + (1 + \lambda_i^2) T_i(\eta) = 0, \quad \lambda_i = \frac{i\pi}{L} \quad (28)$$

The general solution of the differential equation (28) is given by

$$T_i(\eta) = a_i f_1(\eta, \lambda_i) + b_i f_2(\eta, \lambda_i) \quad (29)$$

By setting

$$\Delta = (1 + \Lambda_q + \Lambda_T \lambda_i^2)^2 - 4\Lambda_q(1 + \lambda_i^2) \quad (30)$$

The functions  $f_1$  and  $f_2$  are expressed in the following fashion:

if  $\Delta > 0$

$$f_1(\eta, \lambda_i) = e^{\frac{-(1+\Lambda_q+\Lambda_T \lambda_i^2)-\sqrt{\Delta}}{2\Lambda_q}\eta}, \quad f_2(\eta, \lambda_i) = e^{\frac{-(1+\Lambda_q+\Lambda_T \lambda_i^2)+\sqrt{\Delta}}{2\Lambda_q}\eta} \quad (31)$$

if  $\Delta < 0$

$$f_1(\eta, \lambda_i) = e^{\frac{-(1+\Lambda_q+\Lambda_T \lambda_i^2)}{2\Lambda_q}\eta} \text{Cos} \left( \frac{\sqrt{-\Delta}}{2\Lambda_q} \eta \right), \quad f_2(\eta, \lambda_i) = e^{\frac{-(1+\Lambda_q+\Lambda_T \lambda_i^2)}{2\Lambda_q}\eta} \text{Sin} \left( \frac{\sqrt{-\Delta}}{2\Lambda_q} \eta \right) \quad (32)$$

Employing the initial conditions (25) one has:

$$\sum_{i=1}^{\infty} (a_i f_1(0, \lambda_i) + b_i f_2(0, \lambda_i)) \cos(\lambda_i \xi) = -\mathcal{G}_2(\xi) \quad (33)$$

$$\sum_{i=1}^{\infty} \left( a_i \frac{\partial f_1}{\partial \eta}(0, \lambda_i) + b_i \frac{\partial f_2}{\partial \eta}(0, \lambda_i) \right) \cos(\lambda_i \xi) = 0 \quad (34)$$

Utilizing the orthogonality of the set  $\cos \lambda_i \xi$  we have

$$a_i f_1(0, \lambda_i) + b_i f_2(0, \lambda_i) = \frac{2}{L} \int_0^L -\mathcal{G}_2(\xi) \cos(\lambda_i \xi) d\xi \quad (35)$$

$$a_i \frac{\partial f_1}{\partial \eta}(0, \lambda_i) + b_i \frac{\partial f_2}{\partial \eta}(0, \lambda_i) = 0 \quad (36)$$

$$a_0 f_1(0, 0) + b_0 f_2(0, 0) = \frac{1}{L} \int_0^L -\mathcal{G}_2(\xi) d\xi \quad (37)$$

$$a_0 \frac{\partial f_1}{\partial \eta}(0, 0) + b_0 \frac{\partial f_2}{\partial \eta}(0, 0) = 0 \quad (38)$$

Solving the algebraic equation system (35)–(36) and (37)–(38) one arrives at:

$$a_i = \frac{2}{L} \int_0^L -g_2(\xi) \cos(\lambda_i \xi) d\xi \left( \frac{\frac{\partial f_2(0, \lambda_i)}{\partial \eta}}{f_1(0, \lambda_i) \frac{\partial f_2(0, \lambda_i)}{\partial \eta} - f_2(0, \lambda_i) \frac{\partial f_1(0, \lambda_i)}{\partial \eta}} \right), \tag{39}$$

$$b_i = -\frac{2}{L} \int_0^L -g_2(\xi) \cos(\lambda_i \xi) d\xi \left( \frac{\frac{\partial f_1(0, \lambda_i)}{\partial \eta}}{f_1(0, \lambda_i) \frac{\partial f_2(0, \lambda_i)}{\partial \eta} - f_2(0, \lambda_i) \frac{\partial f_1(0, \lambda_i)}{\partial \eta}} \right), \tag{40}$$

$$a_0 = \frac{1}{L} \int_0^L -g_2(\xi) d\xi \left( \frac{\frac{\partial f_2(0, 0)}{\partial \eta}}{f_1(0, 0) \frac{\partial f_2(0, 0)}{\partial \eta} - f_2(0, 0) \frac{\partial f_1(0, 0)}{\partial \eta}} \right), \tag{41}$$

$$b_0 = -\frac{1}{L} \int_0^L -g_2(\xi) d\xi \left( \frac{\frac{\partial f_1(0, 0)}{\partial \eta}}{f_1(0, 0) \frac{\partial f_2(0, 0)}{\partial \eta} - f_2(0, 0) \frac{\partial f_1(0, 0)}{\partial \eta}} \right). \tag{42}$$

The solution of Eq. (16) subjected to the initial and boundary conditions (17) and (18) considering  $F(\eta) = 1$  is given by:

$$g(\xi, \eta) = \frac{(1-R) \mu_a e^{-\frac{\mu_a \xi}{p}}}{p \left( 1 - \frac{\mu_r^2}{p^2} \right)} + \frac{q_{met}}{p I_0} + \frac{(1-R) \mu_a \frac{\mu_l}{p} \left( \text{Cosh}(\xi) e^{-\frac{\mu_l L}{p}} - \text{Cosh}(\xi - L) \right)}{p \left( 1 - \frac{\mu_r^2}{p^2} \right) \text{Sinh}(L)} + \tag{43}$$

$$(a_0 f_1(\eta, 0) + b_0 f_2(\eta, 0)) + \sum_{i=1}^{\infty} (a_i f_1(\eta, \lambda_i) + b_i f_2(\eta, \lambda_i)) \cos(\lambda_i \xi)$$

Now the solution of Eq. (16) with time-dependent laser heat source  $H$  is a unit step function, after utilizing Eq. (19) is obtained by

$$\theta(\xi, \eta) = \int_0^{\eta} g(\xi, \eta - \tau) F(\tau) d\tau \tag{44}$$

There is an alternative Duhamel's theorem which is expressed by [38]

$$\begin{aligned} \theta(\xi, \eta) &= F(0) g(\xi, \eta) + \int_0^{\eta} g(\xi, \eta - \tau) \frac{dF(\tau)}{d\tau} d\tau \\ &= F(\eta) \theta(\xi, 0) - \int_0^{\eta} \frac{\partial g(\xi, \eta - \tau)}{\partial \tau} F(\tau) d\tau \end{aligned} \tag{45}$$

We can also use

$$\frac{\partial g(\xi, \eta - \tau)}{\partial \tau} = -\frac{\partial g(\xi, \eta - \tau)}{\partial \eta} \tag{46}$$

Thus the alternative form of Duhamel's theorem is expressed by

$$\theta(\xi, \eta) = F(\eta) \theta(\xi, 0) + \int_0^{\eta} \frac{\partial g(\xi, \eta - \tau)}{\partial \eta} F(\tau) d\tau \tag{47}$$

For the piecewise continuous function  $F(\eta)$ , Eq. (47) appears

$$\begin{aligned} \theta(\xi, \eta) &= \theta(\xi, 0) + \int_0^{\eta} \frac{\partial g(\xi, \eta - \tau)}{\partial \eta} d\tau + \Lambda_q \frac{\partial g(\xi, \eta)}{\partial \eta}, & \eta \leq \Lambda_i \\ \theta(\xi, \eta) &= \theta(\xi, 0) + \int_0^{\Lambda_i} \frac{\partial g(\xi, \eta - \tau)}{\partial \eta} d\tau + \Lambda_q \left( \frac{\partial g(\xi, \eta)}{\partial \eta} - \frac{\partial g(\xi, \eta - \Lambda_i)}{\partial \eta} \right), & \eta > \Lambda_i \end{aligned} \tag{48}$$

The above closed form analytical solution reduces to the thermal wave solution when  $\Lambda_T = 0$ .

*B. The dual-phase lag (DPL) bioheat transfer model for scattering tissues*

Using Eqs. (5-6) and Eq. (8), the DPL bioheat transfer equation for scattering tissues is expressed by:

$$\Lambda_q \frac{\partial^2 \theta}{\partial \eta^2} + (1 + \Lambda_q) \frac{\partial \theta}{\partial \eta} + \theta = \frac{\partial^2 \theta}{\partial \xi^2} + \Lambda_T \frac{\partial^3 \theta}{\partial^2 \xi \partial \eta} + \frac{q_{met}}{p I_0} + \underbrace{\left( I \left( \frac{\rho_t C_t}{W_b C_b} \eta \right) + \Lambda_q \frac{\partial I \left( \frac{\rho_t C_t}{W_b C_b} \eta \right)}{\partial \eta} \right)}_{F(\eta)} \mu_a \left( C_1 e^{\frac{-k_1 \xi}{p \delta}} - C_2 e^{\frac{-k_2 \xi}{p \delta}} \right) \tag{49}$$

The solution of Eq. (49) with reference to boundary conditions (17) and (18) is similar to that of the highly absorbing tissue (section 3.1). But, the steady part of Eq. (5) is expressed by

$$\mathcal{G}_2(\xi) = \frac{C_1 \mu_a e^{\frac{-k_1 \xi}{p \delta}}}{p \left( 1 - \frac{k_1^2}{p^2 \delta^2} \right)} + \frac{q_{met}}{p I_0} + \frac{C_1 \mu_a \frac{k_1}{p \delta} \left( \text{Cosh}(\xi) e^{\frac{-k_1 L}{p \delta}} - \text{Cosh}(\xi - L) \right)}{p \left( 1 - \frac{k_1^2}{p^2 \delta^2} \right) \text{Sinh}(L)} - \frac{C_2 \mu_a e^{\frac{-k_2 \xi}{p \delta}}}{p \left( 1 - \frac{k_2^2}{p^2 \delta^2} \right)} - \frac{C_2 \mu_a \frac{k_2}{p \delta} \left( \text{Cosh}(\xi) e^{\frac{-k_2 L}{p \delta}} - \text{Cosh}(\xi - L) \right)}{p \left( 1 - \frac{k_2^2}{p^2 \delta^2} \right) \text{Sinh}(L)} \tag{50}$$

And the series coefficient in Eqs. (39-42) is calculated based on Eq. (50).

*C. The Pennes bioheat transfer model for highly absorbing and scattering tissues*

Using Eqs. (1) and (6-7), the Pennes bioheat transfer equation for highly absorbing tissues during laser irradiation is expressed by

$$\rho_t C_t \frac{\partial T}{\partial t} + \rho_b \omega_b C_b (T - T_a) = k \frac{\partial^2 T}{\partial x^2} + q_{met} + \left( I(t) + \tau_q \frac{\partial I(t)}{\partial t} \right) \times (1 - R) \mu_a e^{-\mu_a x} \tag{51}$$

The initial and boundary conditions in this solution are the same as that of section 3.1. Similar to the section 3.1, the solution of above equation is given by:

$$\mathcal{G}(\xi, \eta) = \frac{(1 - R) \mu_a e^{\frac{-\mu_a \xi}{p}}}{p \left( 1 - \frac{\mu_a^2}{p^2} \right)} + \frac{q_{met}}{p I_0} + (C_0 f(\eta, 0)) + \sum_{i=1}^{\infty} (C_i f(\eta, \lambda_i)) \cos(\lambda_i \xi), \quad \lambda_i = \frac{i \pi}{L}$$

$$C_i = \frac{2}{L} \int_0^L -\mathcal{G}_2(\xi) \cos(\lambda_i \xi) d\xi, \tag{52}$$

$$C_0 = \frac{1}{L} \int_0^L -\mathcal{G}_2(\xi) d\xi, \quad f(\eta, \lambda_i) = e^{-(1 + \lambda_i^2) \eta}$$

Then the solution for Eq. (51) for a piece wise function  $F(\eta)$  is obtained similar to Eq. (48). For the scattering tissues, the similar fashion is carried out to solve the energy equation. In this case, the first term of the right side of Eq. (52) is substituted by Eq. (50) and the series coefficients in Eq. (52) are calculated based on Eq. (50).

IV. RESULTS AND DISCUSSION

In this article, temperature distributions for three bioheat transfer models (Pennes, thermal wave and DPL) are obtained for both highly absorbed and strongly scattered tissues. Also, the adiabatic conditions are considered for both sides of the target tissue [26, 28]. Then the obtained analytical results for Pennes, thermal wave and DPL bioheat transfer models are compared with each other. Firstly, the reliability of the present results for highly absorbed tissues are evidenced through comparison with the numerical results reported by Tung et al. [40]. These comparison is conducted for a tissue of human cornea which the properties of this tissue given in Table 1. In this case, the predicted tissue temperature profiles at two different depths are provided in Fig.1.

TABLE 1 Thermal and optical properties of the human cornea [40]

| Parameters                         | Value   |
|------------------------------------|---|
| Density                            | 1060 (Kg/m <sup>3</sup> )                     |
| Conductivity                       | 0.556 (W/ m. K)                               |
| Diffusivity                        | 1.3695 x 10 <sup>-7</sup> (m <sup>2</sup> /s) |
| Fresnel Reflectance R              | 0.024   |
| Absorption coefficient             | 2000 (1/m)                                    |
| Thermal relaxation time            | 10 (s)  |
| Initial temperature T <sup>o</sup> | 35 (o C)                                      |

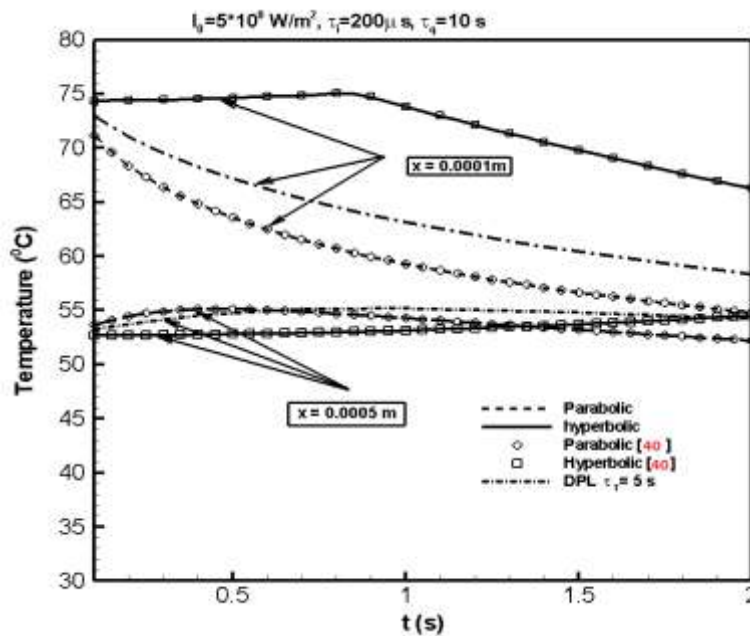


Fig. 1 Temperature profile, T, in two different locations, x, when a cornea is exposed to a pulse laser with

$$I_0 = 5 \times 10^8 \frac{W}{m^2} \text{ and } \tau_i = 200 \mu s$$

The analytical thermal wave and Pennes bioheat transfer results are compared with the numerical results [40] and an excellent agreement is observed. It should be noticed that the obtained DPL temperatures are lower than that of calculated by the hyperbolic (thermal wave) model. It is indicated that the DPL bioheat transfer model considers a time delay between the temperature gradient and the energy transport and decays the wave-fronts effect of hyperbolic model. Here, the CO<sub>2</sub> laser with 10600 nm wavelength, 20 Kw/m<sup>2</sup> radiant intensity and time duration  $\tau_i = 30 s$  is considered for predicting temperature response in skin tissue as a highly absorbed tissue [41]. This type of laser is widely used in biomedical applications such as dermatology, dentistry, and neurosurgery.

In present study, the blood and skin tissue properties are similar those employed by Xu et al. [25] listed in Table 2. The blood perfusion rate and blood temperature are  $W_b = 0.5 \text{ kg} / \text{m}^3 \text{ s}$  and  $T_a = 37^\circ \text{C}$  respectively. The temperature distribution



through skin depth for three bioheat transfer models are depicted in Fig. 2 when CO<sub>2</sub> laser with exposure time  $\tau_i = 30$  s accidents to skin surface.

TABLE 2 THERMOPHYSICAL PROPERTIES OF SKIN TISSUE AND BLOOD

| Parameters                   | Value                       | References |
|------------------------------|-----------------------------|------------|
| Skin Density                 | 1190 (Kg/m <sup>3</sup> )   | [25]       |
| Dkin Specific Heat           | 3600 (J/ Kg. K)             | [25]       |
| Thermal Conductivity of Skin | 0.235 (W/ m. K)             | [25]       |
| Metabolic Heat Generation    | 368.1(W/m <sup>3</sup> )    | [25]       |
| Blood Density                | 1060 (Kg/m <sup>3</sup> )   | [25,42]    |
| Blood Specific Heat          | 3770 (J/ Kg. K)             | [25,42]    |
| Blood Perfusion Rate         | 0.5 (Kg/ m <sup>3</sup> .s) | [42]       |

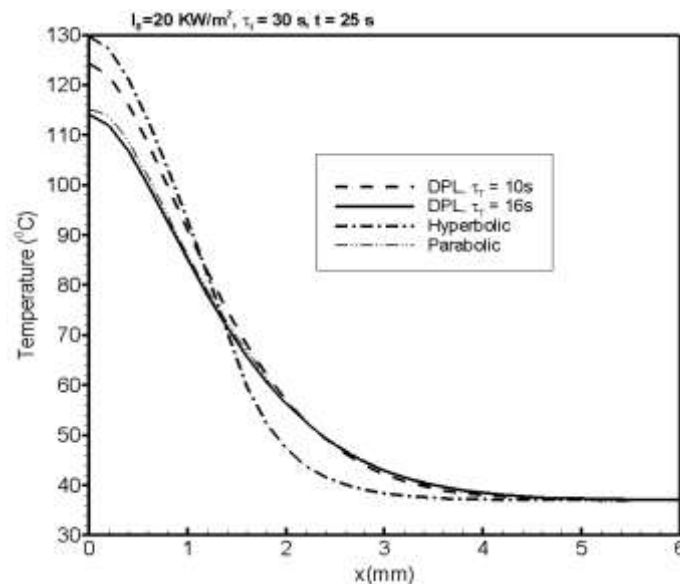


Fig. 2 Temperature distribution along skin tissue depth for three bioheat transfer model when a it is exposed to pulse laser with  $I_0 = 20 \text{ kW} / \text{m}^2$  and  $\tau_i = 30 \text{ s}$

This laser is located in the far infrared interval. Given the high absorption coefficient in this interval, light is absorbed by the skin surface and pervades into a small depth. The heat transfer process in laser-tissue interaction includes two parts; first the heat is absorbed by surface of the biological tissue and then it penetrates into the tissue depth by heat conduction. In three models, heat generation is concentrated at the surface of skin tissue. The same amount of energy is produced continuously in all models. In case of hyperbolic (thermal wave) model more energy is concentrated in skin surface due to the thermal relaxation time  $\tau_q$  that leads to a finite thermal wave velocity. Therefore the temperature rise for the thermal wave model is greatest. The effect of thermal wave model is diminished as the lagging time  $\tau_T$  increases. Unlike the thermal wave model, no wave-like behavior is observed in the DPL model as expected, but a non-Fourier diffusion-like behavior is considered due to the second thermal relaxation  $\tau_T$  whose influence weakens the thermal wave behavior. So that, the predicted temperature by the DPL model is lower than that of the hyperbolic model for points closer to the surface of tissue and difference between the thermal wave and DPL results becomes greater as the thermal relaxation  $\tau_T$  gets larger. But discrepancy amongst results gets negligible as heat penetrates into the skin depth. In this paper, the laser with  $I_0 = 30 \text{ W} / \text{cm}^2$  radiant intensity and time exposure  $\tau_i = 5 \text{ s}$  is considered for studying strongly scattered tissues. The thermo-physical, optical and thermal properties of the biological tissue and the blood perfusion rate, the blood temperature and the metabolic heat generation are similar those used by Zhou et al. [26]. The diffusion reflectance  $R = 0.05$  is used for calculation of laser disturbance in the scattered tissue. The thickness of biological tissue is  $l = 5 \text{ cm}$  and the initial temperature is  $T_0 = 37^\circ \text{C}$  [26].

Fig. 3 displays the temperature history at the surface of tissue for three bioheat transfer models when the laser light is scattered by the tissue. It can be clearly seen that the surface temperature is the same for all models during laser heating, while there are considerable discrepancies amongst temperatures estimated by different bioheat transfer models when the laser is switched off. In case of the thermal wave model, the obtained temperature decreases with higher rate than that of other models.

But the DPL model decreases with lower rate because the thermal relaxation  $\tau_T$  tend to damp the effect of thermal wave model. Further, the effect of diffusion-like of the DPL model gets stronger when the thermal relaxation  $\tau_T$  increases. So that the predicted temperature for the DPL model with the larger  $\tau_T$  is lower than that case obtained by the lower one. When the lagging time  $\tau_T$  is larger than  $\tau_q$ , an over-diffusion phenomenon occurs, leading to a much lower temperature rise (see Fig. 3).

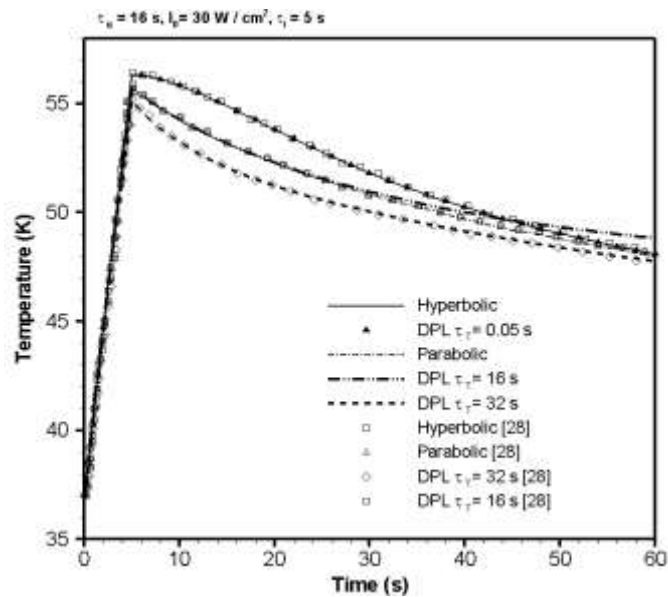


Fig. 3 Temperature distribution for three bioheat transfer model when a scattering biological tissue is exposed to pulse laser with  $I_0 = 30 \text{ W} / \text{cm}^2$  and  $\tau_i = 5 \text{ s}$

This can be seen from Fig. 3 that the DPL model approaches the hyperbolic model when  $\tau_T$  approaches zero. Moreover, unlike the pure material, the DPL model predicts different results than that of anticipated by the parabolic model when  $\tau_T = \tau_q$  (see Fig. 3). This difference results from the blood perfusion rate in the energy equation of biological materials. For the case of the strongly scattered tissue, a comparison between the present results with the numerical result calculated by Liu and Wang [28] is carried out to validate the accuracy of the present results. In this case an excellent agreement is observed among the bioheat transfer model. But as stated above, the proposed analytical method estimates different results for the parabolic and the DPL models when  $\tau_q = \tau_T$ , while Liu and Wang [28] predicted the same results. Zhou et al. [43] in another research considered the axisymmetric effect of the DPL model in biological tissues and concluded that the DPL bioheat transfer model would predict different results for  $\tau_q = \tau_T$  than that of the parabolic model. The effect of thermal relaxation  $\tau_q$  on temperature history of the scattered tissue is shown in Fig. 4.

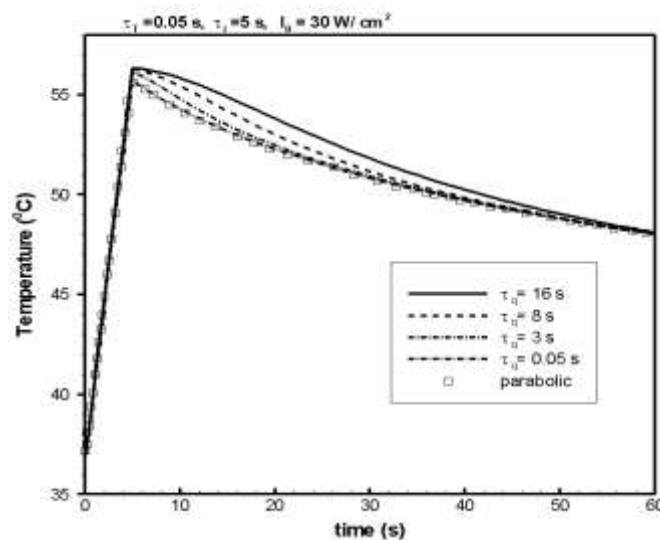


Fig. 4 Temperature distribution at irradiated surface of a scattering tissue obtained by DPL model for different values of  $\tau_q$

It can be seen the longer thermal relaxation  $\tau_q$ , the more significant the thermal wave effect. It is interesting to note that when both thermal lagging are as small as possible, the transient temperature estimated by the DPL model approaches the parabolic model (see Fig. 4). Further, comparing the results in Figs 3 and 4 depicts that the thermal relaxation  $\tau_q$  has more impact on the tissue temperature in early time while the thermal relaxation  $\tau_r$  has more impact on the tissue temperature in the later time. Convective cooling of the blood flow in vascular tissues has a significant role in laser-tissue interaction in biomedical treatments. A high efficiency treatment procedure cannot occur without considering the effects of the blood flow. Therefore here, the effect of the blood flow on the DPL bioheat transfer in the scattered tissue caused by laser irradiation is studied. Fig. 5 provides how the blood perfusion rate affects the transient temperature by the DPL model.

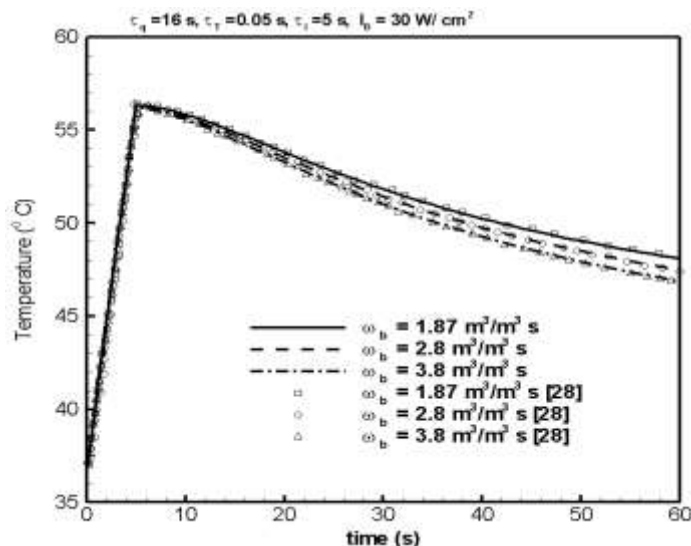


Fig. 5 Time history of temperature at irradiated surface of a scattering tissue obtained by DPL model for different values of blood perfusion rate

It is shown that the higher blood perfusion rate anticipates the lower temperature, because further rate of the blood perfusion can carry away the large amounts of heat, so that as the perfusion rate enhances, the predicted tissue temperature decays as well. It is interesting to note that the effect of blood perfusion rate on tissue temperature is similar to that of the thermal relaxation  $\tau_r$ . Further, the present DPL results for different blood perfusion rate are compared with those reported by Liu and Wang [28] to show reliability of the analytical method. It can be figured out from Fig. 5 that the present results well match with the existing numerical results [28]. The effect of laser intensity on tissue temperature profile anticipated by the DPL bioheat transfer model is shown in Fig. 6 for three different laser intensities.

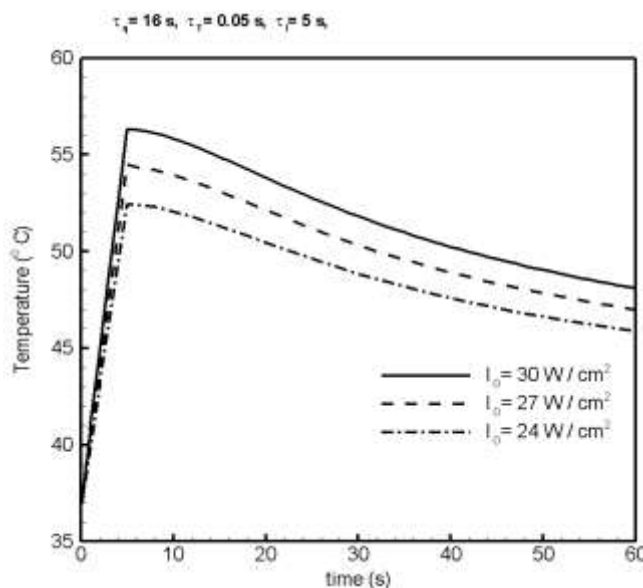


Fig. 6 Temperature distribution at irradiated surface of a scattering tissue obtained by DPL model for different values of laser irradiance

As expected, the predicted tissue temperature increases when the laser irradiant intensity amplifies. By increasing the laser intensity, more heat concentrates on the tissue surface, then the tissue temperature enhances and it needs more time to cool down. Fig. 7 investigates the effects of laser exposure time on the surface temperature of the scattered tissue calculated by the DPL model. In this case three exposure times are considered for the solution. It is demonstrated that the effect of exposure time is similar that of the laser irradiance. It is shown that the temperature rise for the larger exposure time is greatest.

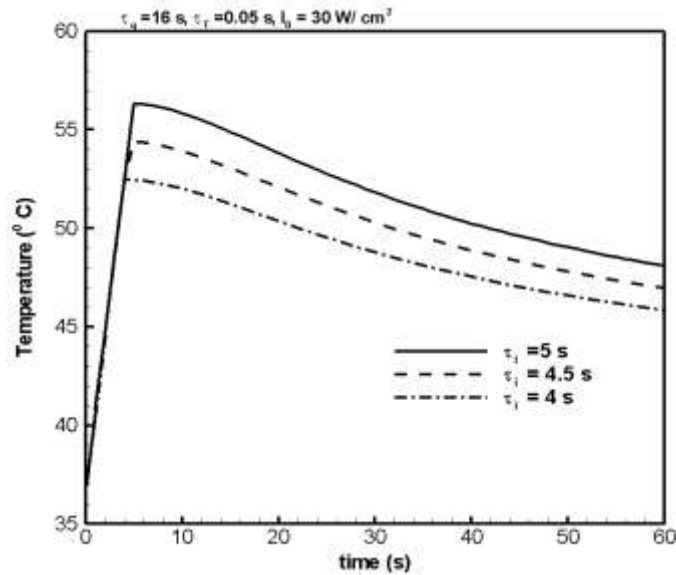


Fig. 7 Temperature distribution at irradiated surface of a scattering tissue obtained by DPL model for different values of laser exposure time

## V. CONCLUSIONS

Thermal analysis of biological tissues during laser irradiation is very important in biomedical applications. In this study, an analytical solution was explored to obtain thermal responses of biological tissues during laser irradiation for both highly absorbed and strongly scattered tissue. The governed energy equation was solved by applying the separation of variables and Duhamel's integral method. The reliability of the present results were validated by comparing the analytical results with those obtained by Tung et al. [40] and Liu and Wang [28] for highly absorbed and strongly scattered tissues, respectively. The results showed that the thermal wave model predicts higher temperature for both absorbed and scattered tissues than that of predicted by the DPL and Pennes bioheat transfer models, because the thermal wave model considers the finite thermal wave speed. While, the DPL bioheat transfer model tends to smooth the sharp wave-like by promoting conduction, leading to a non-Fourier diffusion-like behavior.

It was shown that during laser-tissue interaction, the time delay  $\tau_q$  has considerable impact on the temperature in early times while the time delay  $\tau_r$  has considerable effect on the temperature at the later times. In general, for deeper skin depths, the obtained results from three bioheat transfer models coincide with each other. It is found that, unlike the pure materials the DPL bioheat transfer model does not approach the parabolic model when  $\tau_r = \tau_q$  due to effect of the blood perfusion rate in energy equation. While the DPL bioheat transfer model gets closer to the parabolic model when just both times delay are very small. The impact of the blood perfusion rate is also investigated on tissue temperature and is found that its effect on the temperature is similar to that of the time delay  $\tau_r$ . Furthermore, the results indicated that the effects of laser parameters (laser irradiance and laser exposure time) on tissue temperature are similar to each other. Because by increasing the laser irradiance and laser exposure time, the tissue temperature enhances as well.

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