On the Degrees of Freedom of Parallel Relay Networks

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Abstract—We study the degrees of freedom (DOF) of a single-antenna $M$-user time-varying parallel relay network, where the communications between $M$ pairs of unconnected sources and destinations are provided by a large number of half-duplex decode-and-forward (DF) relays. Unlike the conventional relaying strategy which demands all the relays to simultaneously assist the sources, we divide the relays into two clusters and permit them to take turns forwarding the source messages. With appropriate interference alignment design, it is proved that the $M$-user time-varying relay network has $M$ DOF, provided that the number of relays is infinitely large.

I. INTRODUCTION

In the last a few years, the concept of interference alignment (e.g. [1] [2] [3]) has rapidly become one of the most popular tools in research on interference management because it promises significant gains in wireless networking, but requires only relatively simple techniques compared to conventional interference management techniques. The basic idea of interference alignment is to construct signals such that the undesired interference signals at each receiver cast overlapped space that is distinct from the desired signal space. Successful transmission can be realized simply with distributed linear beamforming at the transmitters and distributed linear zero-forcing at the receivers.

An important application of interference alignment is in a single-antenna $M_s$-source $M_d$-destination network where each source intends to send an independent message to each destination (i.e. an $M_s \times M_d$ channel). Through interference alignment, reference [2] proved that in a time-varying channel environment, the sum capacity can be approximated as

$$C(\rho) = \frac{M_s M_d}{M_s + M_d - 1} \log_2 \rho + o(\log_2 \rho),$$

where $\rho$ represents the signal-to-noise ratio (SNR). This result implies that the $M_s \times M_d$ channel has $\frac{M_s M_d}{M_s + M_d - 1}$ degrees of freedom (DOF), which is dramatically better than what was previously known. Furthermore, if $M_s \gg M_d$ or vice versa, the DOF in (1) is approximately $\min\{M_s, M_d\}$. Conventionally it was believed that the DOF $\min\{M_s, M_d\}$ is achievable only when joint processing is permitted in the sources and/or the destinations [3]. However, the interference alignment technique provides evidence that pure distributed processing is actually able to obtain the same performance.

The above interference alignment scheme can be directly applied to an $M$-user parallel relay network, where the communication of $M$ unconnected source-destination pairs is provided by multiple intermediate relay terminals [2], as displayed in Fig. 1. For this multi-hop system, the transmission is normally divided into two phases. In the first phase, the sources broadcast their transmit signals to all the relays, and then the relays forward these signals to the destinations in the second phase. Using decode-and-forward (DF) relays, the transmission in each phase can be viewed as that in an $X$ channel so that $M$ DOF can be obtained through interference alignment as long as the number of relays is sufficiently large. Due to the practical half-duplex limitation, relays cannot transmit and receive simultaneously in the same frequency band. Thus the two phases span orthogonal channels (normally different time channels) and the achievable DOF of the system is $\frac{M}{2}$. This confirms the result presented in [4] where amplify-and-forward (AF) relays are used. However, one may ask whether $\frac{M}{2}$ is the highest DOF that can be achieved.

For single-source multiple-relay systems, it is well known that requiring the source and all relays to transmit orthogonally (e.g. in [5]) is spectrally inefficient. A large number of advanced relaying transmission protocols have been proposed to improve the DOF (or multiplexing gain) performance of the orthogonal relaying strategy. For the considered multi-user relay network, can the DOF $\frac{M}{2}$, presented in [2] [4] also be increased by properly designed spectrally efficient transmission schemes?

The answer to this question is positive. In this paper, we apply the concept of successive relaying (e.g. presented in [6] [7] for two-relay DF networks) in the system. We divide the relays into two equal-size clusters and require them to take

$^1$This paper considers the system where the sources and the destinations are unconnected. In addition, to achieve a higher DOF than $\frac{M}{2}$, two clusters of relays take turns forwarding the source messages to the destinations. It is worth noting that when the sources and the destinations are also fully-connected and/or all the relays transmit simultaneously (as assumed in most previous work on parallel relay networks), the $\frac{M}{2}$ DOF shown in [2] [4] is actually the optimal result [3].
turns forwarding the source messages to the destinations. The sources and one relay cluster may transmit concurrently so that the channel is efficiently used. Consequently, the interference alignment scheme is different from that proposed in [2]. For example, the beamforming designed in the forwarding relay cluster shall aim to align interference at both the listening relay cluster and the destinations rather than simply the destinations. In this way, we show that as long as the number of relays is sufficiently large, the considered $M$-user relay channel has $M$ DOF, which doubles the result provided in [2] and [4].

II. SYSTEM MODEL

We consider a single-antenna relay network with $M$ ($M > 1$) source-destination pairs and $2K$ ($K > M$) half-duplex DF relays. Each source intends to communicate with one dedicated destination but no physical connection exists between any source and any destination. The sources and the relays are fully connected, i.e. the channel fading coefficient between any source-relay pair is drawn according to a continuous positive minimum value and a finite maximum value [2]. The same assumption holds also for the inter-relay and relay-destination channels. A narrow-band transmission is considered and the transmissions of all nodes use the same frequency band. A time-varying fading environment is assumed such that the channel coefficient between any two nodes changes independently across consecutive time slots. In addition, we assume that the CSI is known at all the nodes in the network.

Reference [2] proposed a transmission scheme and its associated interference alignment design for such a network. The transmission is divided into two orthogonal phases. In the first phase, the sources transmit their messages to all the relays and it can be considered as an $M \times 2K$ X channel. After the relays decode the received signals, they retransmit these messages to the destinations in the second phase. This phase can be viewed as a $2K \times M$ X channel. One possible way to conduct the communication is that during the first $(2K - 1)(n + 1)^T + Mn^T)$ time slots (i.e. the first phase), each source sends $n^T$ independent codeword streams to each relay, where $\Gamma = (M - 1)(2K - 1)$ and $n$ is a positive integer. In the next $(M - 1)(n + 1)^T + 2Kn^T)$ time slots (i.e. the second phase), each relay decodes the $n^T$ streams it received from each source and retransmits them to the source’s dedicated destination. A total of $2MKn^T$ streams are delivered using a total of $(M + 2K - 2)(n + 1)^T + (2K + M)n^T)$ time slots. Defining the DOF as that in [2], such a scheme achieves the DOF $d$ as

$$d = \frac{2MKn^T}{(M + 2K - 2)(n + 1)^T + (2K + M)n^T} \approx \frac{MN}{M + 2K - 1},$$

(2)

where the approximation holds when $n$ approaches infinity. Clearly, $d$ approaches $\frac{M}{2}$ when $K \rightarrow \infty$.

However, we argue that the above transmission scheme is spectrally inefficient because the sources and the relays transmit orthogonally. In fact, we can apply an advanced transmission strategy to more efficiently use the multiple relays and obtain a higher DOF than that shown in (2). We first summarize the main result in the following theorem.

Theorem 1: When the number of relays $2K \rightarrow \infty$, the sum capacity $C(\rho)$ of the considered single-antenna $M$-user time-varying relay network can be approximated by

$$C(\rho) = M \log \rho + o(\log \rho).$$

(3)

The proof of the converse of Theorem 1 is trivial. Clearly, the DOF is upper bounded by a multiple-input multiple-output (MIMO) relay channel where the communication between an $M$-antenna source and its intended $M$-antenna destination is through a $2K$-antenna full-duplex relay. Following the cut-set bound analysis [8], when $2K \geq M$, it is easy to see that the DOF of the MIMO relay channel is $M$.

In the following, we will show that this DOF upper bound is actually asymptotically achievable. The transmission scheme that achieves the optimal DOF is built on a successive relaying strategy with associated interference alignment design.

III. RELAYING PROTOCOL DESCRIPTION

We denote the source cluster and destination cluster as $S$ and $D$, respectively. The relays are divided into two equal-size clusters $R_1$ and $\bar{R}_2$, each containing $K$ relays. We assume that each source intends to deliver $L$ frames of messages to its desired destination. Each frame has $\bar{N}$ independent codeword streams. The whole transmission is completed in $(L+1)$ “super slots”, each of which contains $T$ individual time slots.

The transmission follows a successive relaying strategy. Each source continuously broadcasts one frame in each “super slot” during the first $L$ “super slots”. From the second “super slot”, the two relay clusters are activated successively to forward the source frames to the destinations. The detailed transmission process is described as follows:

“Super slot” 1: Each source sends its 1st frame to $R_1$.

“Super slot” 2: Each source sends its 2nd frame to $R_2$. $R_1$ forwards the 1st frames of the sources to $D$. $\bar{R}_2$ is interfered by the transmission of $R_1$.

“Super slot” 3: Each source sends its 3rd frame to $R_1$, $\bar{R}_2$ forwards the 2nd frames of the sources to $D$. $R_1$ is interfered by the transmission of $\bar{R}_2$.

“Super slot” 4: Each source sends its 4th frame to $\bar{R}_2$, $R_1$ forwards the 3rd frames of the sources to $D$. $\bar{R}_2$ is interfered by the transmission of $R_1$.

The process continues until the $L$th “super slot”. “Super Slot” $L + 1$: $R_\ell$ ($r = \mod{L + 1, 2} + 1$) forwards the $L$th frames of the sources to $D$.

The transmission during the $l$th ($1 \leq l \leq L + 1$) “super slot” can be plotted as Fig. 2. (If $l = 1$, the forwarding relay cluster does not transmit, and if $l = L + 1$, $S$ does not transmit. During the next “super slot”, the listening relay cluster becomes the forwarding relay cluster and vice versa.) If successful communication is feasible, the proposed scheme obtains the DOF

$$d = \frac{L}{L + 1} \frac{MN}{T}.

(4)$$
The channel matrix between $S_{M+j}$ (2 ≤ $j$ ≤ $K$) to $D_{i+K}$ (1 ≤ $i$ ≤ $M$) as an $n^T \times 1$ vector $r_{i+K}^j$ and denote the sub-frame from $S_{M+j}$ to $D_{i+K}$ (1 ≤ $i$ ≤ $M$) as an $(n+1)^T \times 1$ vector $r_{i+K}$. Their beamforming matrices are denoted as $U_{M+j}^{k+K}$ (with size $T \times n^T$) and $U_{M+1}^{k+K}$ (with size $T \times (n+1)^T$), respectively.

At each $D_i$ (1 ≤ $i$ ≤ $K$; i.e., each relay in the listening relay cluster), the $T \times 1$ received signal vector $Y_i$ is expressed as

$$Y_i = \sum_{j=1}^{M} H_j^i V_j^i s_j^i + \sum_{j=1}^{M} H_j^i \left( \sum_{k=1, k \neq i}^{K} V_j^k s_j^k \right) + \sum_{j=1}^{K} H_{M+j}^i \left( \sum_{k=1}^{M} U_{M+j}^{k+K} a_{M+j}^{k+K} \right) + Z_i,$$  

(5)

where $Z_i$ is the noise vector. The first part in the right hand side (RHS) of (5) is the $Mn^T$ desired signal streams (transmitted from the sources) for each listening relay. The second part is the interference signals transmitted by the sources. And the third part is the interference signals transmitted by the forwarding relay cluster.

On the other hand, the destinations are not interfered by the sources’ transmissions since there is no physical connection between. The received signal at $D_i$ (1+K ≤ $i$ ≤ $M+K$) is

$$Y_i = \sum_{j=1}^{K} H_{M+j}^i U_{M+j}^i r_{M+j}^i + \sum_{j=1}^{K} H_{M+j}^i \left( \sum_{k=1}^{M} U_{M+j}^{k+K} a_{M+j}^{k+K} \right) + Z_i,$$  

(6)

where the first part in the RHS is the $Kn^T$ desired streams and the second part is the interference signals.

B. Beamforming Design

To obtain the optimal DOF, we design the beamforming matrices at each source to align the interference at each destination. Roughly speaking, during the considered “super slot”, we create a $(K(n+1)^T + Mn^T)$-dimensional receive signal space at each receiver. At each $D_i$ (1 ≤ $i$ ≤ $M$, i.e. each of the first $M$ listening relays), we align the interference into a $(n+1)^T$-dimensional space and leave the $Mn^T$ desired streams in the remaining interference-free $Mn^T$-dimensional space. At each $D_i$ (M+1 ≤ $i$ ≤ $K$, i.e. each of the remaining $K$-listening relays), the interference signals are aligned into a $(K-1)(n+1)^T$-dimensional space and the desired signals are in the remaining $((n+1)^T + Mn^T)$-dimensional space. Finally, at each $D_i$ (1+K ≤ $i$ ≤ $M+K$, i.e. each of the $M$ destinations), we align the interference into an $(M−1)(n+1)^T$-dimensional space and leave the $Kn^T$ desired streams in an interference-free $(K−M+1)(n+1)^T + Mn^T)$-dimensional space. The alignment is illustrated in Fig. 3.

More specifically, at any $D_i$ (1 ≤ $i$ ≤ $M$), for any $k_l \in \{1, \ldots, M\}$ ($k_l \neq i$), we design the beamforming vectors to

Fig. 2. The transmission at one “super slot”. The dashed line depicts the interference from the forwarding relay cluster to the listening relay cluster.

It can be seen from Fig. 2 that at any “super slot”, the proposed scheme may be viewed as a network containing two X channels, one of which is interfered by the transmission of the other. In what follows, we will describe the associated interference alignment scheme for this network that guarantees the achievability of the optimal system DOF in Theorem 1.

IV. INTERFERENCE ALIGNMENT DESIGN

Since the transmission at any “super slot” can be plotted as Fig. 2, in this section we only concentrate on the system in Fig. 2 (i.e. one single “super slot”) and present the related interference alignment design.

A. Input-output Relations

To facilitate presentation, for the considered “super slot”, we denote the $M$ sources as $S_1, S_2, \ldots, S_M$. The $K$ relays within the listening relay cluster are denoted as $D_1, D_2, \ldots, D_K$. The source cluster and the listening relay cluster can form an $M \times K$ X channel. We assume that each source divides its transmit frame into $K$ equal-size sub-frames, each containing $N$ independent codeword streams, and transmits each sub-frame to each relay within the listening relay cluster.

In addition, we denote the $K$ relays within the forwarding relay cluster as $S_{M+1}, S_{M+2}, \ldots, S_{M+K}$ and denote the $M$ destinations as $D_{1+K}, D_{2+K}, \ldots, D_{M+K}$. These nodes can form a $K \times M$ X channel. During the considered “super slot”, each relay transmits $N$ independent codeword streams to each destination. Clearly, according to the transmission process we described in the above section, the $N$ streams transmitted from any relay to $D_{k+K}$ is the sub-frame the relay decoded from $S_k$ during the last “super frame”.

We assume that the whole transmission takes $T = K(n+1)^T + Mn^T$ time slots, where $\Gamma = (K−1)(2M + K−1)$. The channel matrix between $S_j$ and $D_i$ is denoted as a $T \times T$ diagonal matrix $H_j^i$. In addition, we require that the number of independent signal streams in one sub-frame is $N = n^T$. Then we denote the transmit sub-frame from $S_j$ (1 ≤ $j$ ≤ $M$) to $D_i$ (1 ≤ $i$ ≤ $K$) when $j \neq i$, and 1 ≤ $i$ ≤ $M$ when $j = 1$ as an $n^T \times 1$ vector $s_j^i$. The $T \times n^T$ matrix $V_j^i$ is the beamforming vector at $S_j$ for $s_j^i$. The transmit sub-frame from $S_1$ to $D_i$ ($M+1 \leq i \leq K$) is denoted as an $(n+1)^T \times 1$ vector $s_i^1$, the first $n^T$ elements of which represent the $n^T$ signal streams and the remaining elements are all zeros. Its associated $T \times (n+1)^T$ beamforming matrix is denoted as $V_1^i$. (Transmitting zeros at the transmitters does not change the system performance. We will see later that this setup facilitates interference alignment design.) Similarly, we denote the transmit sub-frame from $S_{M+j}$ (2 ≤ $j$ ≤ $K$) to $D_{i+K}$ (1 ≤ $i$ ≤ $M$) as an $n^T \times 1$ vector $r_{i+K}^j$ and denote the sub-frame from $S_{M+1}$ to $D_{i+K}$ (1 ≤ $i$ ≤ $M$) as an $(n+1)^T \times 1$ vector $r_{i+K}$. Their beamforming matrices are denoted as $U_{M+j}^{k+K}$ (with size $T \times n^T$) and $U_{M+1}^{k+K}$ (with size $T \times (n+1)^T$), respectively.
Fig. 3. Interference alignment at \( D_i \); (a) \( 1 \leq i \leq M \), (b) \( M + 1 \leq i \leq K \), and (c) \( 1 + K \leq i \leq M + K \). The solid and dashed blocks represent the desired and interference signals, respectively. The lower index \((e.g., j)\) and upper index \((e.g., k)\) in each block represent the sub-frame transmitted from \( S_j \) to \( D_k \).

satisfy the following conditions

\[
\text{span}\left( H^i_j V^{k_1}_j \right) \subset \text{span}\left( H^i_{M+1} U^{k_1+K}_{M+1} \right), \quad \forall 1 \leq j \leq M, (7)
\]

\[
\text{span}\left( H^i_{M+1} U^{k_1+K}_{M+1} \right) \subset \text{span}\left( H^i_{M+1} U^{k_1+K}_{M+1} \right), \forall 2 \leq j \leq K. (8)
\]

Then, for each value of \( k_1 \), the signals to be transmitted to \( D_{k_1} \) (i.e. \( s^{k_1}_{1}, \ldots, s^{k_1}_{M} \)) and the signals to be transmitted to \( D_{k_1+K} \) (i.e. \( r^{k_1+K}_{M+1}, \ldots, r^{k_1+K}_{M+K} \)) are aligned into an \((n+1)^{th}\)-dimensional sub-space. We also require

\[
\text{span}\left( H^i_{M+1} U^{i+K}_{M+1} \right) \subset \text{span}\left( H^i_{M+1} U^{i+K}_{M+1} \right), \forall 2 \leq j \leq K, (9)
\]

so that the signals to be transmitted to \( D_{k_1+K} \) (i.e. \( r^{k_1+K}_{M+1}, \ldots, r^{k_1+K}_{M+K} \)) are aligned in an \((n+1)^{th}\)-dimensional sub-space. In addition, for any \( k_2 \in \{M+1, M+2, \ldots, K\} \), if the following conditions are satisfied

\[
\text{span}\left( H^i_j V^{k_2}_j \right) \subset \text{span}\left( H^i_1 V^{k_2}_1 \right), \quad \forall 2 \leq j \leq M, (10)
\]

the signals to be transmitted to each \( D_k \) (i.e. \( s^{k_2}_{1}, \ldots, s^{k_2}_{M} \)) are aligned to an \((n+1)^{th}\)-dimensional sub-space. If all the sub-spaces are independent, the interference signals at \( D_i \) are aligned into a \( K(n+1)^{th}\)-dimensional interference space, as displayed in Fig. 3 (a).

Similarly, at any \( D_i \) \((M + 1 \leq i \leq K)\), we design the beamforming matrices such that for any value of \( k \in \{1, \ldots, K\} \) and \( k \neq i \) the following conditions are satisfied

\[
\text{span}\left( H^i_j V^{k}_j \right) \subset \text{span}\left( H^i_{M+1} U^{k}_{M+1} \right), \forall 1 \leq j \leq M, (11)
\]

\[
\text{span}\left( H^i_{M+1} U^{k}_{M+1} \right) \subset \text{span}\left( H^i_{M+1} U^{k}_{M+1} \right), \forall 2 \leq j \leq K, (12)
\]

\[
\text{span}\left( H^i_j V^{k}_j \right) \subset \text{span}\left( H^i_1 V^{k}_1 \right), \forall 2 \leq j \leq M. (13)
\]

All the interference signals can be aligned into a \((K-1)(n+1)^{th}\)-dimensional interference space as displayed in Fig. 3 (b).

Finally, at each \( D_i \) \((1 + K \leq i \leq M + K)\), for any \( k \in \{1, \ldots, M\} \) and \( k + K \neq i \), we require

\[
\text{span}\left( H^i_{M+1} U^{i+K}_{M+1} \right) \subset \text{span}\left( H^i_{M+1} U^{i+K}_{M+1} \right), \forall 2 \leq j \leq K. (14)
\]

The interference signals to be transmitted to \( D_{k+K} \) (i.e. \( r_{M+1}^{K+1} \), \ldots, \( r_{M+K}^{K+1} \)) are aligned into a \((n+1)^{th}\)-dimensional sub-space (see Fig. 3 (c)).

In order to construct the beamforming vectors to satisfy the relations (7)-(14), for any \( k \in \{1, 2, \ldots, M\} \), we choose

\[
V^k_1 = V^k_2 = \cdots = V^k_M = V^{k+K}_{M+2} = V^{k+K}_{M+3} = \cdots = V^{k+K}_{M+K}. (15)
\]

In addition, for any \( M+1 \leq k \leq K \), we define \( U^{k+K}_{M+1} = V^k_1 \) and choose

\[
V^k_2 = V^k_3 = \cdots = V^k_M. (16)
\]

The relations from (7) to (14) can be rewritten as

\[
\begin{align*}
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right), \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{\vdots} & \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{\vdots} & \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\forall 1 \leq k \leq K, 1 \leq j \leq M, (17)
\end{align*}
\]

\[
\begin{align*}
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right), \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{\vdots} & \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\text{\vdots} & \\
\text{span}\left( H^i_{M+1} \right)^{-1} H^i_j V^k_j & \subset \text{span}\left( U^{k}_{M+1} \right) \\
\forall 1 \leq k \leq M, 2 \leq j \leq K. (18)
\end{align*}
\]
Therefore, for any $1 \leq k \leq M$, the total number of relations regarding $V^k$ and $U^{k+K}$ is

$$
\Gamma = M(K-1) + (M + K - 1)(K-1)
$$

$$
= (K-1)(2M + K - 1).
$$

(19)

Following the Lemma 2 in [2], we can construct, with probability 1, full rank matrices $V^k_2$ and $U^{k+K}$ of sizes $T \times n^\Gamma$ and $T \times (n+1)^\Gamma$, respectively. For $M + 1 \leq k \leq K$, it is not difficult to prove that constructing full rank matrices $V^k_2$ and $U^{k+K}$ of sizes $T \times n^\Gamma$ and $T \times (n+1)^\Gamma$ respectively is also feasible with probability 1. The beamforming design at the transmitters is complete.

Further, following the interference alignment scheme construction for general X channels in [2], it can be proved that at each receiver, all the desired signal streams are linearly independent of each other and independent of the interference. At any $D_i$ ($1 \leq i \leq M$), since the interference-free space has $Mn^\Gamma$ dimensions, the $Mn^\Gamma$ desired signal streams can be decoded using simple linear zero-forcing equalizers. At any $D_i$ ($M + 1 \leq i \leq M + K$), the number of dimensions of the interference free signal space is larger than the number of desired independent signal streams. The desired signals can also be decoded.

V. SYSTEM DOF PERFORMANCE

We can conclude that during each "super slot", each source delivers $N = n^\Gamma$ signal streams to each relay within the listening relay cluster (i.e. each source frame contains $N = KN = Kn^\Gamma$ signal streams). And each relay within the forwarding relay cluster forwards $N = n^\Gamma$ signal streams it decoded from each source during the previous "super slot" to the source’s desired destination. Considering the whole transmission conducted in the $(L+1)$ "super slots", a total of $L$ frames are transmitted from each source to its desired destination. Since each "super slot" contains $T = K(n+1)^\Gamma + Mn^\Gamma$ time slots, the achievable DOF (4) can be expressed as

$$
d = \frac{L}{L + 1} \frac{MKn^\Gamma}{(K(n+1)^\Gamma + Mn^\Gamma)} \approx \frac{L}{L + 1} \frac{MK}{K + 1}.
$$

(20)

where the approximation holds when $n \to \infty$. Clearly, when the value of the frame length $L$ is sufficiently large and the number of relays at each relay cluster $K \to \infty$, the proposed scheme asymptotically achieves the optimal DOF $M$ shown in Theorem 1.

VI. DISCUSSIONS

In this paper, we follow the signal packing approach presented in [2]. Each interference sub-space has $(n+1)^\Gamma$ dimensions and each transmit sub-frame contains $n^\Gamma$ independent signal streams. When $n$ is chosen as a finite value, the spectral efficiency of the proposed scheme can be improved by the beamforming design proposed in [9]. However, the objective of this paper is to find the optimal DOF of the $M$-user parallel relay network so that we only consider the case when $n \to \infty$. Our scheme is optimal in this sense.

We assume that the CSI is globally known. With this information, we can align the interference signals at each receiver to the maximal extent. However, it might be difficult for the sources to obtain the channel knowledge between the two relay clusters and between the relays and the destinations. In fact, if we assume that each of the four clusters displayed in Fig. 2 knows only the channel knowledge directly related to it, the optimal DOF $M$ is still asymptotically achievable. In this case, the beamforming matrices at the forwarding relay cluster are designed in such a way that at each relay within the listening relay cluster, the aligned interference created by the forwarding relays is independent from the aligned interference created by the sources, and is also independent from the desired signals. By this means, when $K \to \infty$ and $L \to \infty$, the achievable DOF also approaches $M$. Due to the limited space, we omit the detailed description of such a scheme, which can be found in [10].

VII. CONCLUSION

We have studied a single-antenna parallel relay network where the communications between $M$ unconnected source-destination pairs are provided by $2K$ half-duplex DF relays. We have proved that for a time-varying channel environment, the considered $M$-user multi-hop system has $M$ DOF as long as the number of relays approaches infinity. Such an optimal DOF can be asymptotically achieved through a successive relaying strategy with the associated interference alignment scheme.

REFERENCES


