In this paper, we consider a cooperative relaying scenario with multiple sources transmitting to one or more destination nodes through several relay terminals. Each relay is equipped with multiple receive and transmit antennas. We assume that the relays can estimate their uplink (relay-destination) channels with enough accuracy and that they have access to the training sequences transmitted by the sources. We present two adaptive training-based algorithms for multiuser relay beamforming. Both algorithms use Kalman filtering to estimate the beamforming matrices iteratively. The first algorithm is centralized where the relay terminals forward their received data to a processing center that computes the beamforming coefficients and feeds them back to the relays. In the second algorithm, each relay terminal can estimate its beamforming matrix locally using its received data and some common information that is broadcasted by the other relays. We present numerical simulations that validate the good performance of the proposed beamforming algorithms in stationary and nonstationary signal environments.

Index terms—Array signal processing, cooperative relay beamforming, Kalman filtering.

1. INTRODUCTION

Cooperative relaying brings a large number of advantages to wireless communication systems [1]. For instance, it increases the range of communication which can be further extended via relay beamforming [2]. Moreover, it provides spatial diversity which can be exploited by applying distributed space-time coding [3]. Cooperative relaying can also be used to provide spatial multiplexing in multiuser communication scenarios where multiple signal sources are targeting one or more destination nodes [4].

Many noncooperative multiuser zero forcing relay beamforming algorithms have appeared in the literature, e.g., [5] and [6]. Multiuser cooperative zero forcing relay beamforming was also proposed in [7]. All these relaying techniques use beamforming to eliminate the interference between different source and destination pairs. Zero forcing relay beamforming requires full knowledge of the channels from the sources to the relays and from the relays to the destination nodes. However, zero forcing beamforming is known to be suboptimal when the signal-to-noise ratio (SNR) of the sources is relatively low as it results in increased noise power [8].

In [9], we have developed a multiuser cooperative relay beamforming algorithm for wireless communication networks. In this algorithm, the beamforming matrices of the relay terminals are jointly designed such that both the noise received at each destination node and the interference caused by the sources not targeting this node are minimized. Each source signal is preserved at its targeted destination node via linear constraints. The resulting optimization problem was formulated in [9] as a convex second-order cone program (SOCP) that could be efficiently solved with polynomial complexity using interior point methods [10]. However, the shortcoming of the algorithm in [9], and also of that in [7], is that it does not have a direct online implementation. Hence, every time one of the source-relay or relay-destination channels changes, the beamforming matrices have to be recomputed. This might not be efficient, especially in nonstationary environments.

In this paper, we develop an iterative beamforming algorithm for multiuser cooperative MIMO-relaying. We assume that the relays can estimate and track their relay-destination channels with enough accuracy. This assumption is well justified in outdoor wireless communication scenarios where the relays and the destination base stations are not mobile. We also assume that the sources transmit training sequences that are known by the relays. These sequences provide an indirect estimate of the source-relay channels. Using the information about the relay-destination channels and the training sequences, the relays can estimate the actual and the desired received signals at each destination node. The beamforming matrices of the relays are jointly designed such that the sum of the squared difference between the desired and actual received signals at all the destination nodes is minimized. We use a state-space modelling approach to solve the resulting optimization problem similar to that used in [11].

We present two adaptive algorithms to solve the training-based relay beamforming problem. Both algorithms use Kalman filtering to estimate the beamforming matrices iteratively [12]. The first beamforming algorithm is centralized; in the sense that the relay terminals forward their received data to a local processing center that also has knowledge of the training sequences of the sources. The processing center then computes the beamforming matrices and feeds them back to the relay terminals. In the second algorithm, each relay terminal computes its beamforming coefficients locally with the help of some common information that is broadcasted by the other relays. We present simulation results showing the efficacy of our approach in terms of the received signal-to-interference-plus-noise ratio (SINR) at the destination nodes. Simulation results also show the ability of the proposed beamformers to track rapid changes in the operating environment.

2. CENTRALIZED ADAPTIVE RELAY BEAMFORMING

We consider K relay terminals that linearly process the signals received from I statistically-independent narrowband sources and forward each signal to its destination. We assume that the ith relay
terminal is equipped with an $m_k$-element antenna array that is used for receiving from the $I$ sources and another $m_k$-element array transmitting to the destination nodes as shown in Fig. 1. Let $h^{(1)}_l$ denote the $m_k \times 1$ vector containing the channel coefficients from the $l$th source to the $k$th relay. The $m_k \times 1$ received signal vector at the $k$th relay terminal at the $n$th time instant can be written as

$$x_k(n) = \sum_{i=1}^{K} \sqrt{P_i} h^{(1)}_i s_i(n) + n^{(1)}_k(n)$$  \hspace{1cm} (1)$$

where $s_i(n)$ is the unit-power signal transmitted by the $i$th source, $P_i$ is the received power of the $i$th source, $n^{(1)}_k(n)$ is the $m_k \times 1$ vector of white Gaussian noise with zero mean and covariance matrix $\Sigma^{(1)}_k$, and $(\cdot)^{\dagger}$ refers to the $k$th relay terminal. The received signal vector by the $k$th relay terminal is linearly processed by the $m_k \times m_k$ beamforming matrix $W_k(n)$ before transmission to the destination nodes. The function of the beamforming matrices at the $K$ relays is to focus each of the $I$ sources at its targeted destination node while reducing the received noise power and the interference caused by the sources that are not targeting this node.

In this work, we assume that each of the $J$ destination nodes is equipped with one antenna only\(^1,2\). For efficient multiuser beamforming, the number of degrees of freedom available at the relay beamformers has to be greater than or equal to the product of the number of source and destination nodes [7], [9], i.e., $\sum_{k=1}^{K} m_k \geq J$. Let $g^{(1)}_j$ be the $m_k \times 1$ vector containing the complex conjugate of the channel coefficients from the $k$th relay to the $j$th destination node. Therefore, we can write the received signal at the $j$th destination at the $n$th time instant as

$$y_j(n) = \sum_{k=1}^{K} g^{(1)*}_k W_k^H(n) x_k(n) + n^{(2)}_j(n)$$  \hspace{1cm} (2)$$

where $W_k(n)$ is the beamforming matrix employed at the $k$th relay terminal, $n^{(2)}_j(n)$ is the white Gaussian noise with zero mean and variance $\sigma^{(2)}_j$ induced at the $j$th destination node, $(\cdot)^{\dagger}$ refers to the $j$th destination node, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively.

The received signal at the $j$th destination node consists of three different components: the desired signals, i.e., the signals from the sources targeting the $j$th destination node, multiuser interference from the other sources, and noise. Let $q_j$ be the number of sources targeting the $j$th destination. We define the $q_j \times 1$ vector $s_j(n)$ such that it contains the signals transmitted by these sources. We also define the $q_j \times 1$ vector $f_j(n)$ such that it contains the complex conjugate of the desired responses for these sources at the $j$th destination node. Hence, the desired signal that should be received at the $j$th destination is given by

$$y^{(D)}_j(n) = f_j^H(n) s_j(n).$$  \hspace{1cm} (3)$$

Note that if multiple sources are targeting the same destination node, i.e., if $q_j > 1$, we can use time or code division multiplexing in order to be able to separate these sources at the destination node. This can be achieved by spreading each source symbol over $N$ time slots and selecting the components of the desired response vectors $(f_j(n))^\nu_{\nu=1}$, such that they correspond to the chips of $q_j$, multiplexing (spreading) codes of length $N$. At the receiver side, i.e., at the $j$th destination node, the desired sources can be separated by applying multiuser detection techniques on the spread symbols.

Given the training sequences transmitted by the $I$ sources, i.e., the desired received symbols for each source at its targeted destination node, we jointly design the beamforming matrices of the relay terminals such that the sum of the squared difference between the desired and actual received signals at all the destination nodes is minimized. Therefore, we can write the cost function of the training-based relay beamforming problem as

$$C(W_k(n)) = \sum_{j=1}^{J} \left| f_j^H(n) s_j(n) - \sum_{k=1}^{K} g^{(1)*}_k W_k^H(n)x_k(n) \right|^2$$  \hspace{1cm} (4)$$

where $f_j(n)$ are designed offline, $s_j(n)$ are known through training, the relay-destination channels $g^{(1)}_k$ are assumed to be known with enough accuracy, and the relay beamforming matrices $\{W_k(n)\}_{k=1}^{K}$ are the design parameters. We start by vectorizing the design parameters. Let $w_k(n) = \text{vec}(W_k(n))$ where $\text{vec} \{ \cdot \}$ is the vectorization operator that stacks the columns of a matrix on top of one another. Using the matrix identity $\text{vec}(ABC) = (C^T \otimes A) \text{vec}(B)$, we can write the cost function in (4) as

$$C(w_k(n)) = \sum_{j=1}^{J} \left| f_j^H(n) s_j(n) - \sum_{k=1}^{K} a^{(j)*}_k(n) w_k(n) \right|^2$$  \hspace{1cm} (5)$$

where the $m_k^2 \times 1$ vector $a^{(j)}_k(n) = g^{(1)}_k \otimes x_k^*(n)$ and $(\cdot)^*$ denotes the complex conjugate.

In order to minimize the cost function in (5), we will use a state-space modelling approach similar to that used in [11]. We define the stacked beamforming vector $\mathbf{w}(n) = [w_1^T(n), \ldots, w_K^T(n)]^T$. Therefore, a state-space model describing the relay beamforming design problem is given by the following process equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mathbf{n}_w(n)$$  \hspace{1cm} (6)$$

and the associated measurement equation is

$$z(n) = \mathbf{A}(n) \mathbf{w}(n) + \mathbf{n}_m(n)$$  \hspace{1cm} (7)$$

In (7), the $J \times 1$ measurement vector $z(n)$ is given by

$$z(n) = [s_1^H(n)f_1^*(n), \ldots, s_J^H(n)f_J^*(n)]^T,$$  \hspace{1cm} (8)$$

\(^1\)Physically, each relay terminal can have only one antenna array operating in some appropriate duplex mode.

\(^2\)The extension to the case in which the destination nodes are equipped with multiple antennas will be considered in our future work.
and the $J \times (\sum_k m_k^2)$ measurement matrix

$$A(n) = \begin{bmatrix} A_1(n) & \ldots & A_K(n) \end{bmatrix}$$

(9)

where the $J \times m_k^2$ matrix $A_k(n)$ is given by

$$A_k(n) = \begin{bmatrix} a_k^{(1)}(n), \ldots, a_k^{(J)}(n) \end{bmatrix}^T.$$  

(10)

In the above state-space model given by (6) and (7), the $(\sum_k m_k^2) \times 1$ vector $w(n)$ is the state vector, $n_w(n)$ is the process noise that allows tracking of the beamforming vector in nonstationary environments and is assumed to be white Gaussian with zero mean and covariance $Q = \sigma_w^2 I$, and $n_m(n)$ is the measurement noise assumed to be white Gaussian with zero mean and covariance $R = \sigma_m^2 I$ and independent of the process noise. Note that the measurement matrix in (9) is composed of $K$ submatrices $\{A_k(n)\}_{k=1}^K$ where $A_k(n)$ contains information about the received data vectors and the relay-destination channels of the $k$th relay at the $n$th time instant.

Based on the above state-space model, a state estimator, e.g., the Kalman filter, can be used to estimate and track the beamforming vector $w(n)$. The estimator will yield a vector that minimizes the mean square values of the components of the measurement noise, i.e., the mean square difference between the desired and actual received signal at each destination node. Hence, the cost function in (5) will be minimized. The process noise variance $\sigma_m^2$ should be chosen to reflect the degree of nonstationarity of the environment. Also, $\sigma_w^2$ should be chosen small enough such that the cost function is sufficiently minimized, e.g., $\sigma_w^2 = 10^{-8}$ in our simulations.

The recursion for the estimated weight vector starts with an initial random weight vector estimate $\hat{w}(0)$ with the associated covariance $P(0|0)$, and updates the weight vector estimate through

$$\hat{w}(n) = \hat{w}(n-1) + G(n) \left( z(n) - A(n)\hat{w}(n-1) \right)$$

(11)

where $\hat{w}(n)$ is the estimate of the stacked beamforming vector at the $n$th time instant and the filter gain $G(n)$ is given by

$$G(n) = P(n|n-1)A^H(n)S^{-1}(n).$$

(12)

The innovation covariance matrix and the covariance matrix of the predicted weight vector are given respectively by

$$S(n) = A(n)P(n|n-1)A^H(n) + R$$

(13)

$$P(n|n-1) = P(n-1|n-1) + Q,$$

(14)

and the updated state covariance matrix is given by

$$P(n|n) = P(n|n-1) - G(n)S(n)G^H(n).$$

(15)

The above iterative algorithm in (11)–(15) estimates the beamforming matrices of the $K$ relays jointly. The estimation process is performed at a local processing center which has access to all the received data vectors $\{x_k(n)\}_{k=1}^K$ of the relays, i.e., the relays forward $\sum_k m_k^2 N_k$ complex values to the processing center every time instant. The processing center then employs a Kalman filter that estimates the beamforming matrices of the relays with a computational complexity of $O(JM^2)$ per iteration. Note that this computational complexity is less than the $O\left( J^2 (M+1)(M^2 + J)^2 \right)$ complexity of the algorithm in [9]. The beamforming matrix of each relay is obtained by feedback from the processing center, i.e., a total number of $\sum_k m_k^2$ coefficients are fed back from the processing center.

### 3. DECENTRALIZED ADAPTIVE RELAY BEAMFORMING

The centralized beamforming algorithm presented in the previous section requires the existence of a local processing center that performs a considerable amount of data exchange with the relays. This might not be feasible in many practical communication systems. In this section, we will develop a decentralized adaptive beamforming algorithm that allows each relay terminal to compute its beamforming matrix locally with reduced communication with the other relays.

We assume that the $k$th relay terminal has access only to its received data vector $x_k(n)$ and the estimates of its uplink channels $\{d_j^{(k)}\}_{j=1}^J$. Each relay is required to estimate its beamforming coefficients locally such that the cost function in (5) is minimized. Thus, the process equation of the beamforming vector of the $k$th relay is given by

$$w_{k}(n+1) = w_{k}(n) + \tilde{n}_{w_{k}}(n).$$

(16)

where $w_k(n)$ is the state vector, $\tilde{n}_{w_{k}}(n)$ is the process noise associated with the beamforming vector of the $k$th relay. It is also assumed to be white Gaussian with zero mean and covariance $Q_k = \sigma_{w_k}^2 I$.

Next, we will construct the measurement equation for the $k$th relay by modifying equation (7). The $k$th relay has access to all the training sequences transmitted by the sources and to the desired response vectors, and hence, it can form the measurement vector in (8). However, it can only construct the submatrix $A_k(n)$ of the measurement matrix $A(n)$ in (9), as it does not have any information about the received data and uplink channels of the other $K-1$ relay terminals. In what follows, we will propose a scheme that condenses this information and allows each relay terminal to estimate its beamforming matrix locally with minimum information exchange with the other relays. For the $k$th relay terminal, we can write the measurement equation in (7) as

$$z(n) = A_k(n)w_k(n) + \sum_{l \neq k} A_l(n)w_l(n) + n_m(n).$$

(17)

We can also write the optimal beamforming vector of the $l$th terminal at the $n$th time instant as

$$w_l(n) = w_l(n-1) + \tilde{n}_{w_l}(n-1)$$

(18)

$$= w_l(n-1) + \tilde{w}(n-1) + \tilde{n}_{w_{l}}(n-1)$$

(19)

where (18) was obtained using the state equation for the beamforming coefficients of the $l$th relay, and we have decomposed the optimal
vector \( w_i(n-1) \) into the sum of its estimate \( \hat{w}_i(n-1) \) and the error \( \tilde{w}_i(n-1) \) associated with this estimate. Substituting with the above expansion for the \( K-1 \) beamforming vectors \( \{w_i(n)\}_{i \neq k} \) into (17), we can write the measurement equation associated with the beamforming vector of the \( k \)th relay as

\[
\hat{z}_k(n) = A_k(n)w_k(n) + \tilde{n}_m(n) \tag{20}
\]

where the \( J \times 1 \) modified measurement vector \( \hat{z}_k(n) \) is given by

\[
\hat{z}_k(n) = z(n) - \sum_{l \neq k} A_l(n)\tilde{w}_l(n-1) \tag{21}
\]

and the modified measurement noise \( \tilde{n}_m(n) \) is given by

\[
\tilde{n}_m(n) = \sum_{l \neq k} A_l(n)\tilde{w}_l(n-1) + n_m(n). \tag{22}
\]

whose covariance matrix can be approximated as

\[
R_k(n) = \sum_{l \neq k} A_l(n) \left( P_l(n-1|n-1) + Q_l \right) A_l^H(n) + \sigma_m^2 I \tag{23}
\]

where \( P_l(n|n) \) is the covariance matrix of the estimated beamforming vector of the \( l \)th terminal at time \( n \). Note that in (23), we have made the approximation that the errors in the estimated beamforming vectors of different relay terminals are uncorrelated.

Based on the state equation given in (16) and the modified measurement equation in (20), the \( k \)th terminal employs a Kalman filter to estimate its beamforming coefficients iteratively. The computational complexity associated with one iteration is of \( O(JmK^2) \).

Note that at each time instant, each relay computes (and broadcasts) \( J^2 + J \) parameters using its received data vector, its uplink channels estimate, and its previous state estimate and covariance. These parameters are broadcasted to the other relays to be used in the next time instant. Specifically, at the \( n \)th time instant, the \( l \)th relay terminal computes and broadcasts both the \( J \times 1 \) vector \( A_l(n)\tilde{w}_l(n-1) \) that is used for measurement correction by the other relays in (21), and the \( J \times J \) matrix \( A_l(n) \left( P_l(n-1|n-1) + Q_l \right) A_l^H(n) \) required for computing the measurement noise covariance matrix in (23).

\[
\text{SINR}_k = \frac{P_l \left| \sum_k g_k^{(1)} h_k^{(1)H} W_k^H h_k^{(2)} \right|^2}{P_m \left| \sum_k g_k^{(1)} h_k^{(2)H} W_k^H h_k^{(2)} \right|^2 + \sum_k \sigma_k^2 \left| W_k g_k^{(1)} \right|^2} \tag{22}
\]
the two proposed algorithms does not severely degrade over a large range of the parameter $\sigma^2_m$, i.e., $\sigma^2_m \in [10^{-10}, 10^{-4}]$. For higher values of $\sigma^2_m$, the cost function is not sufficiently minimized, whereas, smaller values of $\sigma^2_m$ lead to convergence problems as the filter considers the measurement vector a perfect measurement.

Next, we investigate the performance of the proposed algorithms for different values of the received SNR at the relay terminals. The received SNRs of all the sources are kept equal and are varied between $-10$ and $6$ dB. Fig. 4 shows the average received SINR of the first source at its targeted destination (after convergence of the Kalman filter) versus different values of the received SNR at the relay terminals. We can see that the proposed centralized beamforming algorithm has a good performance for all values of the received SNR. On the other hand, at high SNR, above $5$ dB, the performance of our decentralized beamforming algorithm degrades. This can be attributed to the assumption we have made along with (23), i.e., the errors in the estimated beamforming vectors of different terminals are uncorrelated. In fact, as the received SNR increases, the relay beamforming matrices focus more on suppressing the interference received at the destination nodes than on reducing the received noise power. Since the interference suppression is accomplished by the relay terminals cooperatively, hence the errors in the beamforming vectors of different relays are correlated.

Finally, we consider a nonstationary signal environment. The simulation setup is similar to the one we have considered in the previous simulation in terms of the configuration of the sources, relays, and destination nodes. The source-relay channels and relay-destination channels are fixed during the first $500$ time instants. After that, completely independent realizations are generated for all these channels. The parameters of our Kalman filter-based algorithms are selected as $\sigma^2_d = 10^{-8}$ and $\sigma^2_m = 10^{-8}$. Fig. 5 displays the average received SINR of the first source at its targeted destination versus iteration number. We can clearly see the capability of both of our beamformers to readapt to the new signal environment and rapidly converge back to yield high SINR.

5. CONCLUSION

We have presented two adaptive beamforming algorithms for multiuser cooperative MIMO-relaying wireless systems. We assume that the relays can estimate their uplink channels with enough accuracy and that the sources are transmitting training sequences to the relay terminals. We have jointly designed the beamforming matrices of the relays such that the sum of the squared difference between the desired and actual received signals at all the destination nodes is minimized. We have presented two adaptive beamforming algorithms for multiuser relay beamforming based on Kalman filtering. The first algorithm is centralized where a local processing center computes the beamforming coefficients of all the relays. However, this algorithm requires a significant amount of communication between the processing center and the relays. In the second algorithm, each relay terminal broadcasts some information to the other relays which helps them to estimate their beamforming coefficients locally. We have presented numerical simulations that illustrate the capability of our algorithms to perform their beamforming tasks efficiently and track rapid changes in the operating environment.

6. REFERENCES