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Lepton Flavor Violation in SUSY-SO(10) with Predictive Yukawa Texture

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Abstract

We analyze the scalar lepton mass matrices in a supersymmetric SO(10) grand unified model with soft SUSY breaking terms generated at Planck scale and a Georgi-Jarslkog Yukawa texture at GUT scale induced by higher dimensional operators. This model predicts lepton flavor violation. The predictive features of the Georgi-Jarlskog texture are used to estimate branching ratios for the radiative decays $e_a \to e_b + \gamma$, and we find rates that could provide an experimental test for this kind of model.

1 Introduction

Supersymmetric Grand Unified Theory (SUSY GUT), supported by the unification of the coupling constants, provides an interesting framework for extending the Standard Model. In order to enhance predictivity, several *ansatze* for the Yukawa structure can be discussed in the context of SUSY GUT models inspired by superstrings. It is worthwhile to study whether some of the exact predictions of the Standard Model (SM) are modified in phenomenologically interesting ways by these proposals.

One of the most predictive *ansatze* for the Yukawa structure at GUT is the Georgi-Jarlskog [1] texture, which assumes symmetric quark and lepton mass matrices of different forms for the up and down quark sectors. Since in SO(10) all the fermions of a family are unified in an irreducible representation, the **16**, the symmetric texture is naturally accommodated. The **16** representation can also accommodate a right handed neutrino, which leads to an interesting phenomenology.

The generation of predictive textures with renormalizable couplings for the Yukawa sector in SO(10) unified models implies the introduction of large representations (like $\overline{\mathbf{126}}$) at about the GUT scale. This can destroy the asymptotic freedom of the theory; moreover, without further input, the hierarchies among the different couplings remain unexplained. In order to deal with both these issues, it has been suggested that the usual trilinear Yukawa couplings be replaced by higher dimension operators [2] which involve lower representations of SO(10). Recently, it has been shown that such models can arise in the free fermionic formulation of superstrings [3, 4]. The authors of these papers classify specific SO(10) representations which can emerge as massless chiral multiplets below the Planck scale. Such constraints can be used to generate the Yukawa texture at GUT , and a particular example was provide in ref. [5]. We will use the model proposed in this reference in an illustrative manner to generate the Georgi-Jarskog texture, and as an example for our calculations.

In the Minimal Supersymmetric Standard Model (MSSM), the terms which softly break supersymmetry originate at the Planck scale. Thus, even with flavor diagonal initial conditions at Planck scale, it has been observed [6, 7] that the evolution between

Planck and GUT scales may induce initial conditions in these parameters at GUT scale which are not flavor-neutral, and which eventually generate important contributions to low energy flavor violation. In this work we study lepton flavor-violating (LFV) proceses that arise due to the SO(10) structure of the superpotential above GUT. We illustrate some new important effects that originate with the requirement of the Georgi-Jarlskog texture, and show that these effects could lead to LFV lepton decay rates which are comparable with the experimental limits.

In a recent paper, Barbieri, Hall and Strumia [8] have calculated rates for LFV decays in the context of an SO(10) GUT. There is notable difference between this present work and ref. [8]: the large LFV effects calculated in this work for $\mu \to e\gamma$ processes do *not* depend on the existence of a large top Yukawa coupling - they are in fact completely decoupled from the third generation. Instead, they are directly traceable to the (above-stated) requirement of generating the Georgi-Jarlskog texture in an SO(10) GUT.

2 Slepton Mass Matrix

The minimal supersymmetric extensions of the Standard Model (MSSM) coming from spontaneously broken minimal N=1 Supergravity theories, can be described by the superpotential [9]:

$$W = h^u_{ij} Q_i H_u U_j + h^d_{ij} Q_i H_d D_j + h^e_{ij} L_i H_d R_j + \mu H_d H_u \quad , \tag{1}$$

where group indices have been omitted. In addition one introduces all the allowed soft supersymmetry breaking (SSB) terms involving the scalars and gauginos. These are given by [9]

$$\begin{aligned} -\mathcal{L}_{soft} \;=\; & (\xi^u_{ij} Q_i H_u U_j + \xi^d_{ij} Q_i H_d D_j + \xi^e_{ij} L_i H_d R_j + h.c) + (B\mu H_d H_u + h.c) \\ & + m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + m^2_{\tilde{L}}|\tilde{L}|^2 + m^2_{\tilde{E}}|\tilde{E}|^2 + m^2_{\tilde{\nu}}|\tilde{\nu}|^2 + m^2_{\tilde{Q}}|\tilde{Q}|^2 \\ & + m^2_{\tilde{D}}|\tilde{D}|^2 + m^2_{\tilde{U}}|\tilde{U}|^2 + \tfrac{1}{2}(M_3 \bar{\tilde{g}}\tilde{g} + M_2 \bar{\tilde{W}}^a \tilde{W}^a + M_1 \bar{\tilde{B}}\tilde{B} + h.c) \quad . \end{aligned} \tag{2}$$

with $q = u, d, e$.

From the coupling of the MSSM to the minimal $N = 1$ supergravity the following set of assumptions is plausible at Planck scale:

$$\begin{aligned} M_i(M_{\rm P}) &= m_{1/2}, \quad i = 1, 2, 3 \\ m^2_{\tilde{Q}} &= m^2_{\tilde{u}} = ... = m^2_0 \ . \end{aligned} \tag{3}$$

We make no statement about the ξ_{ij} at Planck scale; boundary conditions on ξ_{ij} at GUT will follow from the physics to be discussed.

It has been shown in refs. [6, 7] that in SUSY-GUT's, the additional couplings to the extra GUT fields between GUT and Planck scales can substantially modify the universality of the SSB parameters at GUT, allowing LFV vertices. In our study we consider an SO(10) SUSY-GUT model in which the Yukawa textures are obtained at GUT from effective higher dimensional operators. In this kind of model, it will be the the different group structure of operators involved in the generation of the Yukawa texture which produces a flavor-dependent evolution of the SSB terms, and which results in LFV vertices.

As previously stated, we choose to illustrate our calculations using the model of ref. [5]. In this model the Georgi-Jarlskog texture is derived from the superpotential:

$$\begin{aligned} W_{Yukawa} = M^{-1}\left(Y_{33}\psi_3\psi_3 H_1 S_1 + h_{33}\psi_3\psi_3 H_2 S_2 + h_{23}\psi_2\psi_3 H_2 S_3\right) \\ + M^{-2} Y_{22}\psi_2\psi_3 A A' H_1 + M^{-3}\left(Y_{12}\psi_1\psi_2 H_1 S_2^3 + h_{12}\psi_1\psi_2 H_2 S_1^3\right) \end{aligned} \tag{4}$$

This superpotential is SO(10) invariant, and respects a set of discrete symmetries (given in [5]). M is an intermediate mass between Planck and GUT that could arise by integrating out fermions like $\overline{\mathbf{16}} + \mathbf{16}$. We will take $M = M_{\rm P} = 2.4 \times 10^{18}$ GeV. The fields S_1, S_2, S_3 are singlets, H_1, H_2 are $\mathbf{10}$'s, and A, A' $\mathbf{45}$'s of SO(10). It is important to note that there are *no renormalizable interactions* in the Yukawa sector.

In the 2-2 entry, the operator $H_1 A A'$ generates the $\overline{\mathbf{126}}$ required by the Georgi-Jarlskog texture. The vevs of A and A' are along the $(B - L)$ and the I_{3R} directions, respectively, so only the $\overline{\mathbf{126}}$ contributes to the mass matrices [5].

The couplings in the superpotential are chosen to be of order 1. The vevs $\langle S_1 \rangle$, $\langle S_3 \rangle$, $\langle A \rangle$, $\langle A' \rangle$ are supposed to be of the order of the GUT scale, $\langle S_2 \rangle \approx M$ and we assume for simplicity that all of them are real.

As follows from the standard supergravity analysis [10], soft-breaking terms (the equivalent of the usual trilinear term) of the form

$$-\mathcal{L}^{10}_{soft} = M^{-1}\overline{Y}_{33}\psi_3\psi_3 H_1 S_1 + \ldots \tag{5}$$

are generated from (4). Matching to Eq. (2) at GUT, we have at tree level relations such as

$$(\xi^{d,e}_{33})_{\mathrm{G}} = M^{-1}(\overline{Y}_{33})_{\mathrm{G}}\,\langle S_1\rangle \quad . \tag{6}$$

Then we assume that at GUT all the symmetries break at once to $SU(3)_c\times SU(2)_L\times U(1)_Y$, generating the effective Yukawa texture

$$h^u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \tag{7}$$

$$h^d = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix} \tag{8}$$

$$h^e = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \tag{9}$$

If the vevs satisfy $\langle S_1\rangle \sim \langle S_3\rangle \sim \langle A\rangle \sim \langle A'\rangle \sim \epsilon M$, $\langle S_2\rangle \sim M$, then a hierarchy among the Yukawa entries follows:

$$A = h_{33};\; B = h_{33}\epsilon;\; C = h_{12}\epsilon^3;\; D = \epsilon Y_{33};\; E = \epsilon^2 Y_{33};\; F = \epsilon^3 Y_{33}\ .$$

Even though $\langle S_2\rangle / M \sim 1$, we treat the 33 piece of the lagrangian as nonrenormalizable.

For superpotentials like the one in Eq.(4) the universality of the initial conditions (3) is broken at GUT; the soft masses evolve differently according to their representation $\mathcal{R}$ under SO(10), reaching GUT values

$$m^2_{\mathcal{R}} = m_0^2 + 2C_{\mathcal{R}}\; M^2_{\mathrm{G}}\; \tilde{\alpha}_{\mathrm{G}} \left[t_{\mathrm{P}}(2 - b_{10}\tilde{\alpha}_{\mathrm{G}} t_{\mathrm{P}})/(1 - b_{10}\tilde{\alpha}_{\mathrm{G}} t_{\mathrm{P}})^2\right] \quad . \tag{10}$$

$C_{\mathcal{R}}$ is the Casimir for the representation $\mathcal{R}$ ($\mathcal{R} = \mathbf{16}, \mathbf{10}$), and $b_{10} = 4$. We define $\tilde{\alpha}_{\rm G} = \alpha_G/4\pi$, and the variable $t = 2\log(Q/M_{\rm GUT})$, with $t_{\rm P} = 2\log(M_{\rm P}/M_{\rm GUT})$. $M_{\rm G}$ is the gaugino mass at GUT, related to $m_{1/2}$ by:

$$M_{\rm G} = (1 - b_{10}\tilde{\alpha}_{\rm G} t_{\rm P})\; m_{1/2} \quad .$$

Radiative corrections coming from (4) must also be included in obtaining a slepton mass matrix at GUT; as we shall see, these will substantially alter the flavor-diagonal expression (10) in the case of $\mathcal{R} = \mathbf{16}$.

The charged slepton mass matrix can be written at Fermi scale as:

$$\begin{aligned} -\mathcal{L}_m^{sl} &= \tilde{e^\dagger}_L \left(m_L^2 + \delta m_{16}^2 + m_e m_e^+\right) \tilde{e}_L + \tilde{e^\dagger}_R \left(m_R^2 + \delta m_{16}^2 + m_e m_e^+\right) \tilde{e}_R \\ &\quad + \tilde{e^\dagger}_L \left(\frac{v_d}{\sqrt{2}} (\xi^e + \delta\xi^e) + \mu m_e \tan\beta \right) \tilde{e}_R + h.c. \end{aligned} \tag{11}$$

where all entries are matrices in flavor space. In a standard notation, $\tan\beta = v_u/v_d$. m_e is the charged fermion mass matrix, and δm_{16}^2 and $\delta\xi^e$ are additional contributions generated via radiative corrections between $M_{\rm P}$ and $M_{\rm GUT}$ which are sources of flavor violation; they will be explicitly given in what follows. m_L^2 and m_R^2 are flavor independent and are given to one loop by the expressions:

$$\begin{aligned} m_L^2 &= m_{16}^2 + \left(\tfrac{3}{10}K_1(t) + \tfrac{3}{2}K_2(t)\right) \tilde{\alpha}_{\rm G}\; M_{\rm G}^2 \\ &\qquad - m_Z^2 \left(\tfrac{1}{2} - \sin^2\theta_W\right) \cos 2\beta \end{aligned} \tag{12}$$

$$m_R^2 = m_{16}^2 + \tfrac{6}{5}K_1(t) \cdot \tilde{\alpha}_{\rm G}\; M_{\rm G}^2 - m_Z^2 \sin^2\theta_W \cos 2\beta \tag{13}$$

We have defined $K_i(t) = t(2 + b_i\tilde{\alpha}_{\rm G} t)/(1 + b_i\tilde{\alpha}_{\rm G} t)^2$, with $b_i = (33/5, 1, -3)$ and θ_W =Weinberg angle. m_{16}^2 is given in Eq. (10). The mass matrix for the (left-handed) sneutrinos is given by:

$$(\tilde{\nu}_L^+) \left(m_{16}^2 + \left(\tfrac{3}{10}K_1(t) + \tfrac{3}{2}K_2(t)\right) \tilde{\alpha}_{\rm G} M_{\rm G}^2 + \tfrac{1}{2} m_Z^2 \cos 2\beta + \delta m_{16}^2\right) (\tilde{\nu}_L) \tag{14}$$

3 Non-universal Soft Scalar Masses At GUT

The entries for the above-GUT correction δm_{16}^2 come from one loop diagrams like the one in Fig 1, and can be generated by evolving the composite operators from Planck to GUT. We illustrate our procedure by considering the 2-3 entry. The interactions required for calculating this one-loop contributions can be obtained from the relevant F-terms

$$
\begin{aligned}
F_{H_2} F^*_{H_2} &= \left(\frac{h_{23}}{M}\right)\left(\frac{h_{33}}{M}\right) \psi_3 \psi_3^* \psi_3 \psi_2^* S_2 S_3^* + h.c \\
F_{\psi_3} F^*_{\psi_3} &= 2\left(\frac{h_{23}}{M}\right)\left(\frac{h_{33}}{M}\right) \psi_3 H_2 H_2^* \psi_2^* S_2 S_3^* + h.c
\end{aligned}
$$

Its effective contribution at GUT can be estimated by integrating the loop between the two scales, so that

$$
(\delta m^2_{16})_{23} = -2 \cdot 5 \, \frac{m^2_{16}}{8\pi^2} \, \log \frac{M_{\mathrm{P}}}{M_{\mathrm{GUT}}} \, (2 h_{23} h_{33}) \frac{\langle S_2 \rangle}{M} \frac{\langle S_3 \rangle}{M} \tag{15}
$$

The factor of 5 counts the fields of the multiplet runing in the loop and is the same for the **16** and for the **10**. There is an additional symmetry factor of 2 when ψ_3 runs in the loop. In this fashion we find for the complete matrix δm^2_{16}

$$
\delta m^2_{16} = -10 \begin{pmatrix} y^2_{12} & 0 & BC \\ 0 & y^2_{12} & 2BA \\ BC & 2AB & 4A^2 \end{pmatrix} \frac{m^2_{16}}{8\pi^2} \, \log \frac{M_{\mathrm{P}}}{M_{\mathrm{GUT}}} \tag{16}
$$

The 0 value for the 1-2 entry is due to the orthogonality of the vevs of A, A'. We have displayed only the lowest power of ϵ in every matrix element, which is equivalent to neglecting the down Yukawas.

We pause for two remarks concerning Eq. (16):

- In models with renormalizable couplings, where the hierarchy is enforced by having different Higgs fields to generate the Yukawa entries, there will be no off-diagonal elements induced in δm^2 at one-loop level. This contrasts with our result, as evident in Eq. (16).

- The third generation receives a large contribution (δm^2_{33}) from the top Yukawa A. (In our model, $A \approx 3$.) This leads to a significant (indeed, nonperturbative) splitting of $m^2_{\tilde{\tau}}$ from $m^2_{\tilde{\mu}} \simeq m^2_{\tilde{e}}$. (This possibility was noted in ref. [8]). For

$\langle S_2 \rangle / M \sim 1$, there are other possible contributions to (δm^2_{33}) [11]. In the present model, flavor violations in the $\mu - e$ sector will not involve the $\tilde{\tau}$ or its mass. Flavor violations in $\tau - \mu$ or $\tau - e$ processes will involve loops with $\tilde{\tau}$, and this mass splitting will be simply parameterized when these processes are discussed.

Both the coupling constants in the superpotential (4) and the related soft-breaking parameters (5) receive radiative corrections in evolving from Planck to GUT. These are obtained by integrating the loops in Fig 2 between the two scales (we display only the 22 entry). Displayed as differential equations, the results are

$$\frac{dY_{ab}}{dt} = 2G_{ab}\tilde{\alpha}_{10}Y_{ab} \tag{17}$$

$$\frac{d\overline{Y}_{ab}}{dt} = -2G_{ab}\tilde{\alpha}_{10}(2M_{10}Y_{ab} - \overline{Y}_{ab}) \ , \tag{18}$$

where $\tilde{\alpha}_{10}$ is the (running) SO(10) coupling constant (divided by 4π). and M_{10} is the (running) SO(10) gaugino mass. Their behavior with scale t is governed by the usual SO(10) R-G equations, appropriate to the representation content of the model of ref. [5].

The factors G_{ab} depends on the group structure of the Yukawa operator, and one can show that they are

$$G_{ab} = -\frac{1}{2}\sum_r C_r \tag{19}$$

where C_r is the Casimir operator for the representation of every field present in the operator. An integration of Eq.(18) using Eq. (3) and the matching conditions (6) gives for the effective trilinear parameter at GUT

$$(\xi^e_{ab})_G = m_0 A_0 (h^e_{ab})_G + 8\tilde{\alpha}_{\rm G}\ G_{ab}\ m_{1/2}(h^e_{ab})_G \log\left(M_{\rm P}/M_{\rm GUT}\right) \tag{20}$$

where the subscript G denotes evaluation at GUT. We have used as a boundary condition $(\overline{Y}_{ab})_{M_{\rm P}} = m_0 A_0 (Y_{ab})_{M_{\rm P}}$.

In our example the factors G_{ab} are $-63/8$ for all $\{ab\}$ except for $\{ab\} = \{22\}$, in which case it is $-127/8$. Matching our results at GUT to the MSSM lagrangian, the expressions for the trilinear soft terms for the sleptons becomes:

$$(\xi^e_{ab})_G = m_0 A_0 \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}$$

$$+ 8\tilde{\alpha}_{\rm G}\ m_{1/2}\ \log\left(\frac{M_{\rm P}}{M_{\rm GUT}}\right) \begin{pmatrix} 0 & (-63/8)F & 0 \\ (-63/8)F & (-127/8)(-3E) & 0 \\ 0 & 0 & D \end{pmatrix} \tag{21}$$

In what follows, it is convenient to work in a superfield basis in which the lepton Yukawa matrix is diagonal at GUT. In this basis, only δm^2_{16} and $\delta\xi^e$ mix the generations. Also in this basis, no new off-diagonal entries are generated during the evolution from GUT to Fermi scale, so that only the terms δm^2_{16} and $\delta\xi^e$, containing GUT-scale physics, contribute to lepton flavor mixing. We also note here the different nature of the mixing between different generations: δm^2_{16} is responsible for the mixing of the third generation with the first two, and this mixing is permitted through the use of non-renormalizable operators above GUT. In models where the G-J texture arises from renormalizable operators (including Higgses in **10**'s and $\overline{\mathbf{126}}$'s), the Higgs stucture chosen to enforce the desired Yukawa texture at GUT prevents the mixing of the third generation with the others in the slepton matrix at GUT. Enhancement in the mixing between the first two generations (in $\delta\xi^e$) comes from sizeable SO(10) group theoretic factors.

We denote with a Greek index the lepton mass eigenstates, and rotate the superfields:

$$\begin{aligned} \hat{e}_{R_\alpha} &= U^R_{\alpha j}\ \hat{e}_{R_j} \\ \hat{e}_{L_\alpha} &= U^L_{\alpha j}\ \hat{e}_{L_j} \end{aligned}$$

such that

$$h^e_{Diag} = U^L h^e U^{R\dagger} \tag{22}$$

Since h^e is symmetric, up to a phase we can identify U^L and U^R with a matix U_e such that $U_e h^{e\dagger} h^e U_e^\dagger = (h^e_{diag})^2$. In the Georgi-Jarskog texture this matrix is of the form:

$$U_e = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{23}$$

where

$$\tan\theta = -\frac{2F}{3E} \tag{24}$$

Then the non-diagonal terms in the slepton matrix are contained in:

$$\Delta^{LL} \ = \ \Delta^{RR} = U_e \ \delta m_{16}^2 \ U_e^\dagger \tag{25}$$

$$\Delta^{LR} \ = \ U_e \ \frac{v_d}{\sqrt{2}} \ (\delta\xi^e)_{\rm G} \ U_e^\dagger \tag{26}$$

This makes explicit the property that all flavor mixing in the slepton sector is a reflection of physics above GUT. Using expressions (16), (23) we find for the mixing of the third generation:

$$\Delta_{\tau e} \ \equiv \ \Delta_{13}^{LL} = \Delta_{13}^{RR} = -(BC\cos\theta - BA\sin\theta) \ 10 \ \frac{m_{16}^2}{8\pi^2} \ \log\left(\frac{M_{\rm P}}{M_{\rm GUT}}\right) \tag{27}$$

$$\Delta_{\tau\mu} \ \equiv \ \Delta_{23}^{LL} = \Delta_{23}^{RR} = -(BC\sin\theta + 2BA\cos\theta) 10 \frac{m_{16}^2}{8\pi^2} \log\left(\frac{M_{\rm P}}{M_{\rm GUT}}\right) \tag{28}$$

From the trilinear terms we find (using (21) and (26)) for the mixing of the first two generations:

$$\Delta_{\mu e} \equiv \Delta_{12}^{LR} = F \frac{v_d}{\sqrt{2}} \cos 2\theta (-63/8 + 127/8) \ 8\tilde{\alpha}_{\rm G} \ m_{1/2} \ \log\left(\frac{M_{\rm P}}{M_{\rm GUT}}\right) \tag{29}$$

We can see that in our model:

$$F \cdot v_d \approx \sqrt{m_e m_\mu}, \quad E \approx m_\mu \quad .$$

Then (29) is of the order of

$$\sqrt{m_e m_\mu} \cdot m_{1/2} \cdot Clebsch$$

This result can be compared with the effective insertion obtained by Barbieri *al* [8]: in their model (which does not contain the G-J texture)

$$\Delta_{12}^{LR} \approx m_\tau V_{\tau\mu} V_{\tau e} m_{1/2}$$

Parametric agreement with our result is obtained for

$$V_{\tau\mu}V_{\tau e} \approx \sqrt{\frac{m_\mu}{m_\tau}}\sqrt{\frac{m_e}{m_\tau}}$$

which is the approximate situation for the model in [8]. This explains why using a different approach our results are numerically comparable with theirs.

4 Radiative Decay $e_a \to e_b\ \gamma$

The amplitude for the decay can be written as a magnetic transition

$$T(e_a \to e_b\gamma) = \epsilon^\lambda \bar{e}_b(k-q)\ [iq^\nu \sigma_{\lambda\nu}(A + B\gamma_5)]\ e_a(k) \tag{30}$$

In the limit of vanishing mass for the outgoing leptons the left handed and right handed decay amplitudes do not interfere therefore Eq. (30) can be written as:

$$T(e_a \to e_b\gamma) = \epsilon^\lambda \bar{e}_b(k-q)\ \{2k\epsilon[A_R\left(\frac{1+\gamma_5}{2}\right) + A_L\left(\frac{1-\gamma_5}{2}\right)]\ \}\ e_a(k) \tag{31}$$

Thus the decay rate is given by:

$$\Gamma(e_a \to e_b\gamma) = \frac{{m_{e_a}}^3}{16\pi}(|A_R|^2 + |A_L|^2) \tag{32}$$

Since the mass insertions depend on the generations, the diagrams (a, b, c) of Fig 3 contribute to $\tau \to e\gamma$ and $\tau \to \mu\gamma$, while the diagram of Fig 4 contributes to $\mu \to e\gamma$. For the τ decays we obtain

$$A_L = A_L^{3b}; \qquad A_R = A_R^{3a} + A_R^{3c}\ \ , \tag{33}$$

where

$$A_R^{3a} = i\frac{g^2 e}{32\pi^2 c_W^2}\frac{\Delta_{\tau\mu}m_\tau}{(m^2_{\tilde{\tau}_L} - m^2_{\tilde{\mu}_L})}\frac{F_{3a}}{m^2_{\tilde{\chi}^0_j}} \tag{34}$$

$$A_L^{3b} = i\frac{g^2 e}{32\pi^2 c_W^2}\frac{\Delta_{\tau\mu}m_\tau}{(m^2_{\tilde{\tau}_R} - m^2_{\tilde{\mu}_R})}\frac{F_{3b}}{m^2_{\tilde{\chi}^0_j}} \tag{35}$$

$$A_R^{3c} = -i\frac{g^2 e}{16\pi^2}\frac{\Delta_{\tau\mu}m_\tau}{(m^2_{\tilde{\nu}_\tau} - m^2_{\tilde{\nu}_\mu})}\frac{F_{3c}}{m^2_{\tilde{\chi}^+_j}}\ \ , \tag{36}$$

while for $\mu \to e\gamma$

$$A_R^{4a} = A_L^{4b} = +i\frac{g^2 e}{32\pi^2 c_W^2}\frac{\Delta_{\mu e}^{LR}}{m_{\tilde{\mu}_L}^2 - m_{\tilde{e}_R}^2}\frac{F_4}{m_{\tilde{\chi}_j^0}} \quad . \tag{37}$$

We have defined:

$$\begin{aligned}
F_{3a} &= |c_W Z_{2j} + s_W Z_{1j}|^2 \left[g\left(m_{\tilde{\tau}_L}/m_{\tilde{\chi}_j^0}\right) - g\left(m_{\tilde{\mu}_L}/m_{\tilde{\chi}_j^0}\right)\right] & (38)\\
F_{3b} &= |2 s_W Z_{1j}|^2 \left[g\left(m_{\tilde{\tau}_R}/m_{\tilde{\chi}_j^0}\right) - g\left(m_{\tilde{\mu}_R}/m_{\tilde{\chi}_j^0}\right)\right] & (39)\\
F_{3c} &= |V_{1j}|^2 \left[f\left(m_{\tilde{\nu}_\tau}/m_{\tilde{\chi}_j^+}\right) - f\left(m_{\tilde{\nu}_\mu}/m_{\tilde{\chi}_j^+}\right)\right] & (40)\\
F_4 &= (c_W Z_{2j}^* + s_W Z_{1j}^*)(2 s_W Z_{1j}^*) \left[h\left(m_{\tilde{\mu}_L}/m_{\tilde{\chi}_j^0}\right) - h\left(m_{\tilde{e}_R}/m_{\tilde{\chi}_j^0}\right)\right] & (41)
\end{aligned}$$

where Z_{ij} and V_{ij} are the neutralino and chargino mixing matrix respectively defined as in Haber and Kane [9], and

$$\begin{aligned}
f(a) &= \frac{2 + 3a^2 - 6a^4 + a^6 + 6a^2\log(a^2)}{12(a^2-1)^4} & (42)\\
g(a) &= \frac{5 - 9a^4 + 4a^6 + 6a^2[2\log(a^2) - a^2\log(a^2)]}{(a^2-1)^4} & (43)\\
h(a) &= \frac{1 - a^4 + 2a^2\log(a^2)}{2(a^2-1)^3} & (44)
\end{aligned}$$

Eq. (32) gives for the ratio

$$\frac{\Gamma(e_a \to e_b\gamma)}{\Gamma(e_a \to e_b\nu_a\bar{\nu}_b)} = \frac{12\pi^2}{m_{e_a}^2 G_F^2}(|A_R|^2 + |A_L|^2) \tag{45}$$

Since the percentage of the total decay for the reactions in the denominator are 99% for $\mu \to e\gamma$, 18.01% for $\tau \to e\gamma$, 17.65% for $\tau \to \mu\gamma$ [12], we write for the relevant branching ratios:

$$\mathrm{BR}(\mu \to e\gamma) = \frac{3\alpha}{\pi}\left(\frac{\Delta_{\mu e}}{m_{\tilde{\mu}_L}^2 - m_{\tilde{e}_R}^2}\right)^2 \frac{M_Z^4}{m_\mu^2 m_{\tilde{\chi}_j^0}^2}\ |F_4|^2 \tag{46}$$

$$\mathrm{BR}(\tau \to \mu\gamma) = \frac{3\alpha}{2(.18)\pi}\ M_Z^4\ (\Delta_{\tau\mu})^2 \quad \cdot$$

$$\left(\left|\frac{F_{3a}}{(m_{\tilde{\tau}_L}^2 - m_{\tilde{\mu}_L}^2)m_{\tilde{\chi}_j^0}^2} - 2c_W^2\frac{F_{3c}}{(m_{\tilde{\nu}_\tau}^2 - m_{\tilde{\nu}_\mu}^2)m_{\tilde{\chi}_j^+}^2}\right|^2 + \left|\frac{F_{3b}}{(m_{\tilde{\tau}_R}^2 - m_{\tilde{\mu}_R}^2)m_{\tilde{\chi}_j^0}^2}\right|^2\right) \tag{47}$$

Since in our model the slepton (mass)^2 of the two first generations differ only in the square of the lepton masses, we can use Eqs. (27) and (28) to obtain

$$\mathrm{BR}(\tau \to e\gamma) = \left(\frac{\Delta_{\tau e}}{\Delta_{\tau\mu}}\right)^2 \cdot \mathrm{BR}(\tau \to \mu\gamma) \tag{48}$$

5 Inputs

In order to estimate the branching ratios of expressions (46),(47),(48) we require values for the GUT coupling α_{G}, the common scalar mass at Planck scale m_0^2, the gaugino mass at Planck scale $m_{1/2}$, the (effective) Higgs bilinear coupling at GUT μ, the ratio of vevs $\tan\beta$, the flavor-symmetric soft-breaking parameter A_0, and the Yukawa matrices at GUT.

The value of μ up to its sign can be expressed in terms of the other parameters by means of the symmetry breaking relation:

$$\mu^2 = -\frac{m_Z^2}{2} - \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{1 - \tan^2\beta} \tag{49}$$

In this expression we neglect the one loop corrections to the the effective potential, although these could be significant for some values of the parameters $m_{1/2}, m_0$, and $\tan\beta$ [13]. Its inclusion unnecessarily complicates our calculations, since in our case the branching ratios are *not* proportional to μ or to A (unlike the situation in ref. [8]). In our case μ is primarily used in determining the mass matrix for the neutralinos.

To obtain α_{G} and M_{GUT} we integrate the R-G equations assuming for integration purposes that all the supersymmetric masses, as well as the heavier Higgs, are degenerate at m_t. We consider the light Higgs to have a mass of the order of M_Z. On integrating the MSSM to two loops in the coupling constants [14] from GUT to m_t and the SM from there to M_Z, we find

$$\alpha_{\mathrm{G}} = .041; \quad M_{\mathrm{GUT}} = 2 \times 10^{16}$$

obtained for the three coupling constants at M_Z

$$\alpha_s(M_Z) = .120; \quad \alpha_1(M_Z) = .0169; \quad \alpha_2(M_Z) = .0332$$

The value of A_0 is used in the integration of the R-G equations to obtain μ. Varying it from -3 to 3 produces a change of less than the 5% in μ; in the range of approximation we are using we can fix $A_0 = 1$ for the rest of the calculation. As we said before the branching ratios are not proportional to this parameter.

There are in the literature several studies of the predictivity of the Georgi-Jarlskog texture [16, 17]. In these, the Yukawa matrices are scaled from GUT to Fermi scale, giving reasonably good agreement with the experimental data. We concentrate on low values of $\tan\beta$ (< 10) so we can assume that $h_t >> h_\tau, h_b$ and use the one loop semianalytical analysis of ref. [16] to determine our imputs. The predictivity of this model is still impressive although the value obtained for V_{cb} is at the upper limit of the most recent experimental values [18]. We follow here the one loop analysis as in [16].

We use as inputs

$$m_\tau = 1.784 \text{ GeV};\ m_e = .511 \text{ MeV};\ \ m_\mu = 105.658 \text{ MeV};$$

$$m_u/m_d = .55;\ m_b = 4.23 \text{ GeV};\ m_c = 1.26 \text{ GeV}$$

Working to three-loop corrections in QCD [19] and one loop in QED we find $h_t(m_t) = 1.11$ for $A = 3.08$ (where A is defined in Eq. (7)). For $1.6 \leq \tan\beta \leq 10$, we obtain $.0446 \leq V_{cb} \leq .048$. These lie at the upper limit of the experimental data. The physical mass for the top lies in the range 174 GeV $\leq m_t^{phy} \leq$ 204 GeV, consistent with experimental bounds.

For the G-J texture (at GUT) we find:

$$A = 3.08;\quad B = .094(\sin\beta)^{-1/2};\quad C = 1.67\ 10^{-4}(\sin\beta)^{-1};$$

$$D = 6.84\ 10^{-3}(\cos\beta)^{-1};\quad E = 1.35\ 10^{-4}(\cos\beta)^{-1};$$

$$F = 2.82\ 10^{-5}(\cos\beta)^{-1}\quad .$$

We observe that the large value of obtained for A makes the δm^2 radiative correction (16) for the third generation highly non-perturbative (although the corrections to the Yukawa coupling remain perturbative for this value of A [17]). We deal with this problem in the following manner: we consider as degenerate the first two generations of sleptons, and describe the reduction of the third generation slepton masses with a parameter x as follows:

$$\begin{aligned} m^2_{\tilde{\tau}_{L(R)}} &= m^2_{\tilde{\mu}_{L(R)}} - (1-x)m^2_{16} + m^2_\tau \\ m^2_{\tilde{\nu}_\tau} &= m^2_{\tilde{\nu}_\mu} - (1-x)m^2_{16} \end{aligned} \tag{50}$$

This is equivalent to taking $m^2_{\tilde{\tau}_{L(R)},\ \tilde{\nu}_\tau}|_{\rm GUT} = x\ m^2_{16}, \quad m^2_{\tilde{\mu}_{L(R)},\ \tilde{\nu}_\mu}|_{\rm GUT} = m^2_{16}$.

6 Results and Conclusions

In Figures 5 and 6 we show the branching ratios obtained for representative values of the input parameters. We use as one parameter $m_{\tilde{\mu}_L}$ at the scale of m_t (related to the soft mass for the **16** at GUT by eq. (12)). We consider for $M_{\rm G}$ (the gaugino mass at GUT) a range from 50 GeV (the approximate lower limit from direct chargino search) to a maximum value that we consider to be the one that drives to zero the mass of the scalars in the **10** at GUT, using Eqs. (10) and (12).

To illustrate our calculations, we have shown results for $\tan\beta = 3$. The change with $\tan\beta$ is not very significant: the branching ratios decrease slowly when we increase $\tan\beta$ to its limit value of 10.

A change of sign in μ is shown in Fig 5 to have little effect on the rates for $\mu \to e\gamma$. The same is true for $\tau \to \mu\gamma$. The difference of the masses of the third generation of scalars at GUT, represented by the parameter x, only indirectly (and negligibly) affects the rate for $\mu \to e\gamma$, by modifying the value of μ. The effects on $\tau \to \mu\gamma$ are more direct, as described in the text, and they are displayed in Fig 6 for $m_{\tilde{\mu}_L} = 300$ GeV. (In the other curves we keep $x = .5$.)

The results for BR($\tau \to e\gamma$) can be obtained by scaling the ones of $\tau \to \mu\gamma$ by a factor of $4.8\ 10^{-3}$ for $\tan\beta = 3$, (see Eq. (48)). The rates we find are obviously much lower than the experimental limits.

In Fig 7 we show the areas of the plane $(m_{\tilde{\mu}_L}, M_{\rm G})$ restricted by our calculations. Already, certain regions of the presently allowed MSSM parameter space are ruled out by the the present experimental limits. We observe that if the current limits were decreased by a factor of 10, a large portion of the slepton mass range of phenomenological interest would be excluded. Conversely, either or both of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ could be observed for superpartner masses in the few hundred GeV range at the price of a factor of 10 improvement in experiment.

To conclude: we have analyzed some phenomenological consequences of embedding the MSSM in SO(10) GUT models in which Yukawa structures are generated by effective composite operators at and above GUT. The use of the Georgi-Jarlskog texture permits the use the low energy data to minimize the number of free parameters, but also imposes important constraints on the GUT physics. In our analysis we have shown how a model which incorporates the G-J texture predicts flavor-violating phenomena that can be tested in the current or in the next generation of experiments on lepton decays. In the case of $\mu \to e\gamma$, the large enhancement of the rate originates with the group structure associated with the presence of an effective $\overline{\mathbf{126}}$ in the higgs sector; we then expect our result to be valid in any SO(10) model which incorporates the Georgi-Jarlskog texture.

Figure Captions

Figure 1: One loop contribution to the 3-2 entry of δm^2_{16}.

Figure 2.1: One loop contributions to the effective trilinear (ξ) term at GUT: 2.1a, 2.1b, 2.1c are gauge, D-term, and gaugino contributions, respectively.
Figure 2.2: One loop contributions to the effective Yukawa.

Figure 3: Diagrams contributing to $\tau \to \mu\gamma$. Diagrams similar to 3a, 3b with the photon line attached to $\tilde{\tau}$ are not displayed.

Figure 4: Diagrams contributing to $\mu \to e\gamma$. Diagrams similar to 4a, 4b with the photon line attached to $\tilde{\mu}$ are not displayed.

Figure 5: BR($\mu \to e\gamma$) for a range of values of $m_{\tilde{\mu}_L}$ (labeling the curves) and gaugino mass at GUT (M_{G}). All curves are for $\tan\beta = 3$ and $x = 1$ (see Eq. (50). Solid line:

$\mu > 0$; dashed line $\mu < 0$.

Figure 6: BR($\tau \rightarrow \mu\gamma$) for a range of values of $m_{\tilde{\mu}_L}$ (labeling the curves) and gaugino mass at GUT ($M_{\rm G}$). All curves are for $\tan\beta = 3,\ \mu > 0$. The value of x (the $m^2_{\tilde{\tau}}$ suppression factor at GUT) has been chosen to be 0.5 for the solid lines. The dashed curves show the changes with x, for $m_{\tilde{\mu}_L} = 300$ GeV.

Figure 7: ($M_{\rm G}, m_{\tilde{\mu}_L}$) parameter space excluded by present and (possible) future data (all curves are for $\tan\beta = 3, \mu > 0, A_0 = 1$). Area above line (a) is excluded by the R-G analysis (see discussion in text). Area between lines (a) and (b) is excluded by present upper limit on BR($\mu \rightarrow e\gamma$). Area between line (c) (drawn for $x = 0.5$) and axes is excluded by present upper limit on $\tau \rightarrow \mu\gamma$. Line (d) is the $x = 1$ equivalent to (c). Lines (e) and (f) show the range of parameters excluded if current limits were decreased by a factor of 10.

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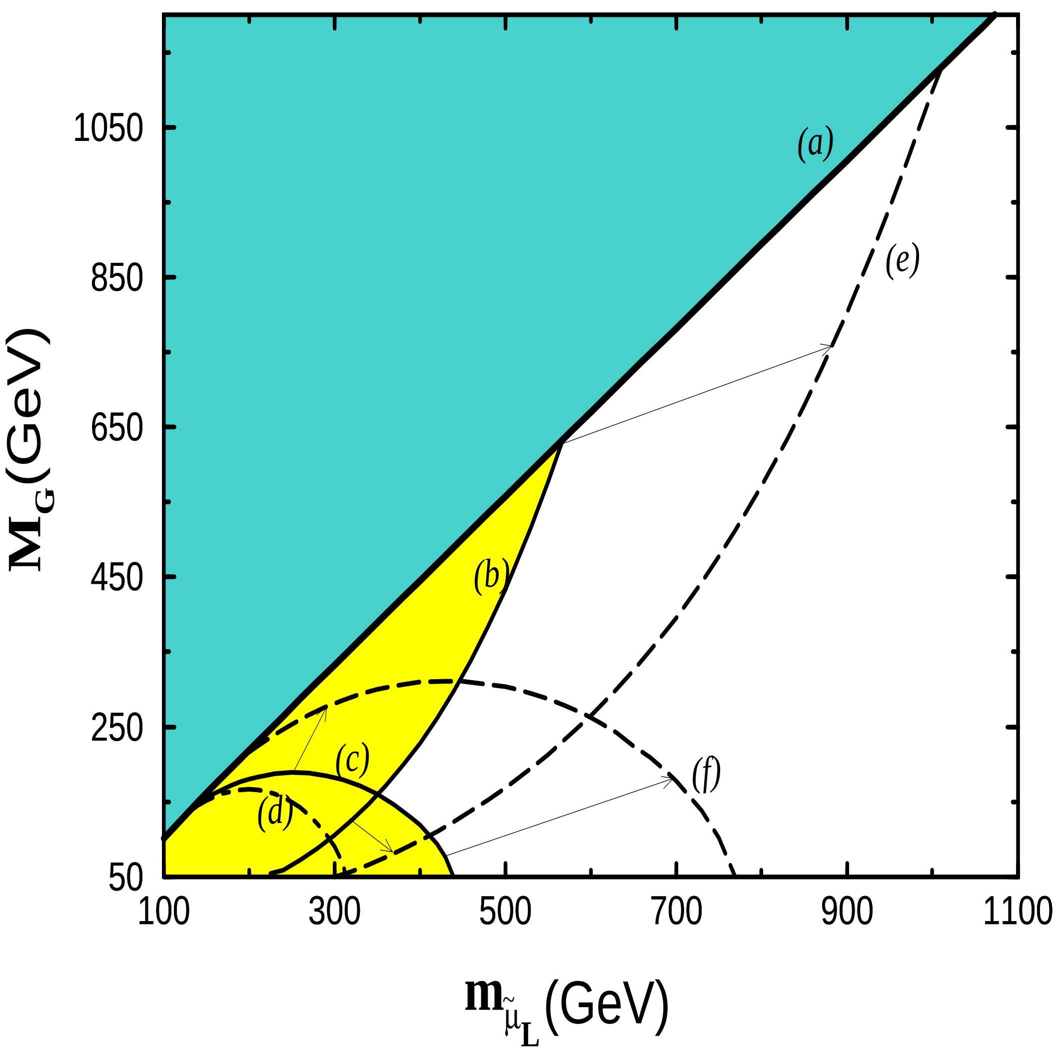


(a)
(b)
(c)
(d)
(e)
(f)
1050
850
650
450
250
50
100
300
500
700
900
1100
M_G(GeV)
$m_{\tilde{\mu}_L}$(GeV)

Fig.7

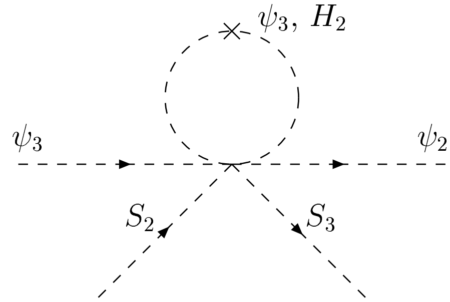
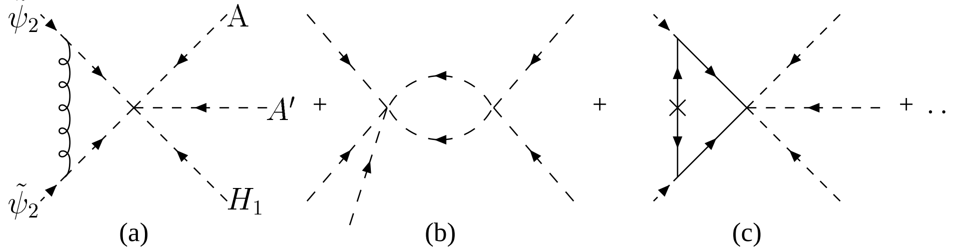
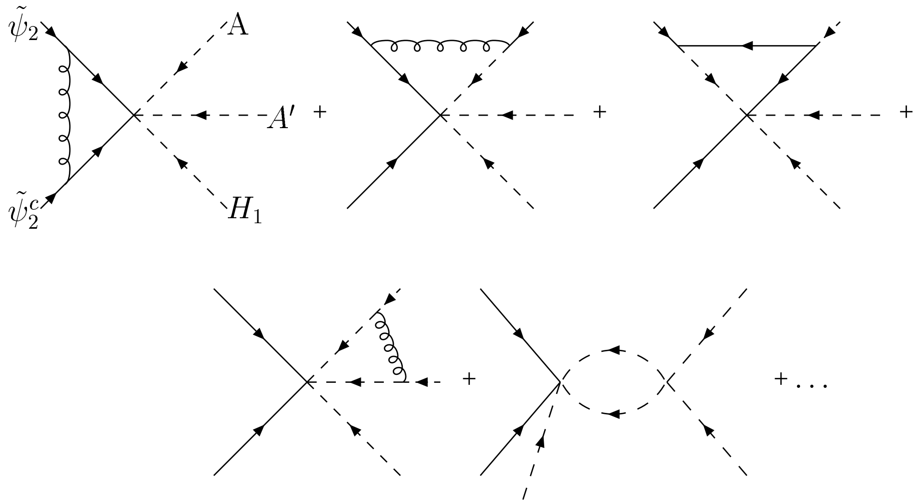


Fig.1



2.1



2.2

Fig.2

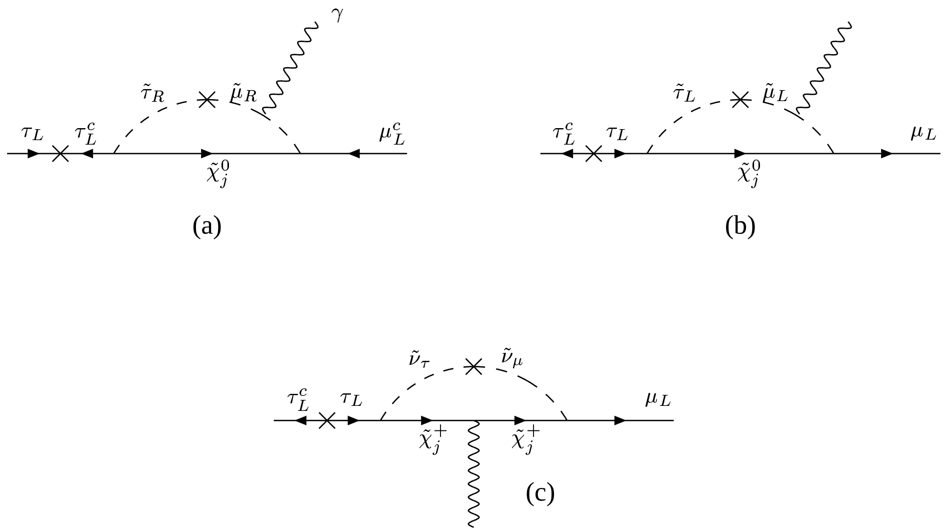

γ
$\tilde{\tau}_R$
$\tilde{\mu}_R$
τ_L
τ_L^c
μ_L^c
$\tilde{\chi}_j^0$
(a)
$\tilde{\tau}_L$
$\tilde{\mu}_L$
τ_L^c
τ_L
μ_L
$\tilde{\chi}_j^0$
(b)
$\tilde{\nu}_\tau$
$\tilde{\nu}_\mu$
τ_L^c
τ_L
μ_L
$\tilde{\chi}_j^+$
$\tilde{\chi}_j^+$
(c)

Fig.3

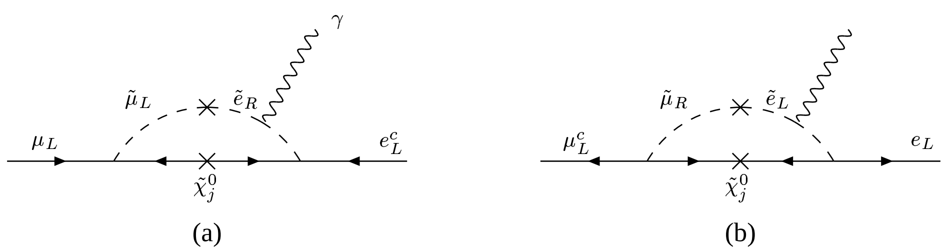
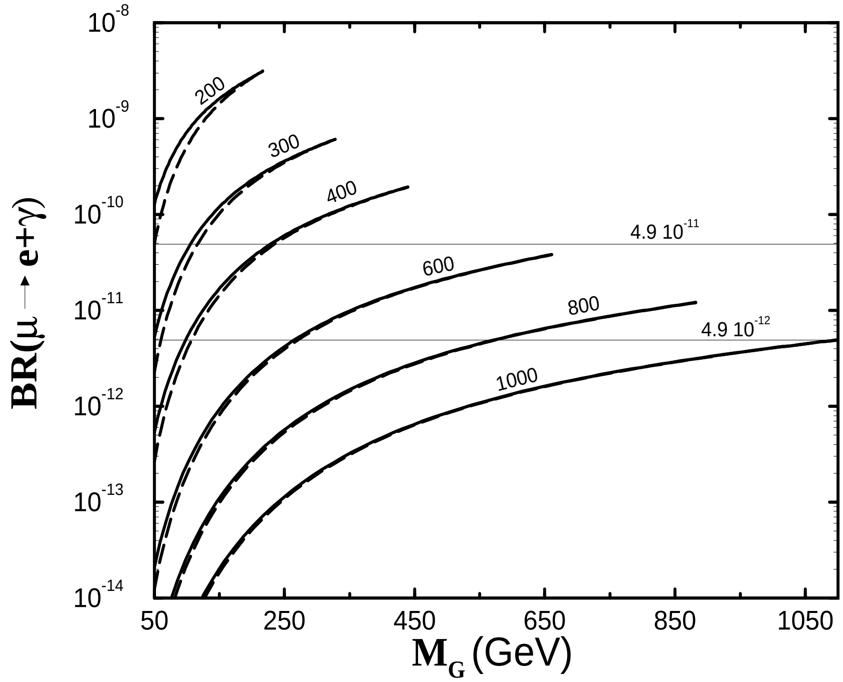
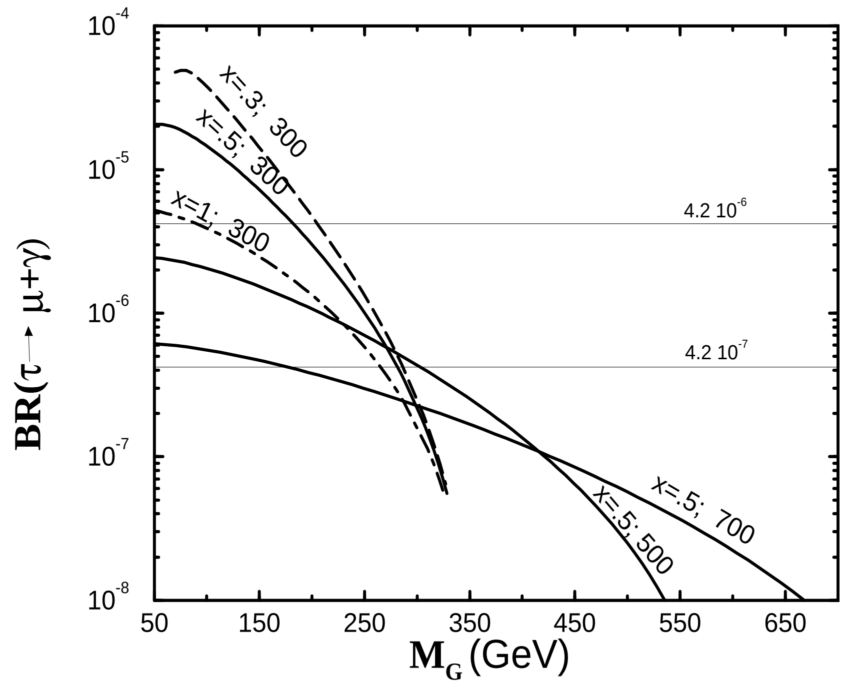

γ
$\tilde{\mu}_L$
$\tilde{e}_R$
μ_L
e_L^c
$\tilde{\chi}_j^0$
(a)
$\tilde{\mu}_R$
$\tilde{e}_L$
μ_L^c
e_L
$\tilde{\chi}_j^0$
(b)

Fig.4



BR(μ → e+γ)
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
200
300
400
600
800
1000
4.9 10^{-11}
4.9 10^{-12}
50
250
450
650
850
1050
M_G (GeV)

Fig. 5



BR(τ → μ+γ)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
x=.3; 300
x=.5; 300
x=1; 300
4.2 10^{-6}
4.2 10^{-7}
x=.5; 700
x=.5; 500
50
150
250
350
450
550
650
M_G (GeV)

Fig.6