

Should Banks Own Equity? A Corporate Finance Perspective^{*}

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Abstract:

This paper examines the question whether banks should hold a share of their borrowing firms' equity. A bank's equity participation can help overcome a well documented agency problem: without the equity participation, the bank would use its informational advantage over other sources of finance (e.g. competing banks) to extract profits from the client firm whenever the firm needs additional investment funds. This, in turn, reduces the incentives of the borrowing firm to generate profits. It is shown that even a small, minority equity stake held by the bank significantly reduces the propensity of the bank to extract profits, which then improves the incentives of the firm. A small equity stake suffices, because the private information that the bank has about the firm makes the bank's profit extraction extremely sensitive to the nature of the bank's claim. The benefit of bank equity participation is related to firm characteristics (e.g. size, growth, capital needs, etc.), characteristics of the banking sector (e.g. competition), as well as information issues (e.g. the quality of the prevailing accounting practices). The paper addresses, *from a corporate finance perspective*, the current debate about whether banks should be allowed to own equity stakes, and if yes how large these should be.

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1. Introduction

This paper analyzes situations in which banks should also hold a share of their borrowing firms' equity. The equity participation can help overcome well documented agency problems. The paper takes a corporate finance perspective on the question, i.e. it looks at the loan providing side of the banking business and abstracts from the deposit taking side of the business. Nevertheless, it contributes to an ongoing debate about the merits of having banks simultaneously be lenders and shareholders. In terms of the US banking system, the paper argues for changes to the current regulation which prohibits commercial banks from holding equity.¹ It outlines the conditions under which it is most desirable to have banks hold small, minority equity stakes. In terms of other economies (e.g. most of Europe and Japan), the paper provides a theoretical model with which to analyze the *optimal* equity participation in firms to which the banks have provided substantial loans.²

The basic idea of the paper is as follows: a firm raises funds from a competitive bank, which can later extract profits from the firm whenever the firm needs further financing. This profit extraction is possible, because due to its banking relationship the bank acquires privileged information about the firm that other potential investors do not have. In essence, any *uninformed* alternative provider of finance will be scared off, if the *informed* original bank were to reject its own firm's requests for new funds. Hence, the original bank is de-facto the only affordable source of future funding and is able to extract extra profits. In as much as the extraction of profits by the bank reduces the incentives of the firm (or its managers) to create these profits, the bank relationship is costly.³ The main point of this paper is to show that this cost is significantly smaller when the bank holds a mix of debt and equity as opposed to pure debt. Even a *small minority equity stake* significantly changes the bargaining power of the bank and reduces the propensity of the bank to extract rents, thus leading to greatly improved incentives for the firm. This point, which is discussed further below, is important because in

¹ Of course, there are concerns about allowing banks to own equity positions. Mostly, there is the fear that banks would use equity to inefficiently increase the riskiness of their (deposit insured) portfolios. *This issue is addressed in the context of this paper at the end of the model section.* Basically, the effects pointed out in this paper should be seen as complementary to the effects arising on the deposit taking side of banks' business activities. See James (1995) for a good discussion of US banking regulations, especially the equity ownership restrictions.

² By extension, this paper also speaks to the current debate about the merits of financial systems similar to the US versus systems similar to most of Europe and Japan. Since the financial systems of some transition economies are still being created, this paper outlines an important corporate finance consideration in terms of the allowable/desirable relationship between banks and firms.

³ These issues are similar to those discussed in Rajan (1992) and von Thadden (1995), who do **not** examine the

many countries banks are reluctant (or prohibited by regulators) to hold *controlling* shares of equity.

The privileged information that the bank has about the firm's prospects plays a crucial role. Not only is this privileged information the source of power for the bank, but it also makes the bank's ability to extract rents very sensitive to the nature of the bank's claim. In particular, a bank is particularly strong in any bargaining with the firm over future funding, whenever the bank holds a claim to the *lower* tail of the cashflow distribution of the firm (i.e. debt). This is because the bank can be sure to receive a reasonable payoff even if bargaining were to break down. Shifting the bank's claim towards the upper tail of the cashflow distribution (i.e. holding some equity) significantly improves the firm's bargaining position. This is true even for small equity stakes, because the improvement is more dramatic the larger the amount of privileged information that the bank has about the firm.⁴

In as much as the extraction of profits by the bank happens only when new financing is needed, the predictions of this paper best apply to firms with growth opportunities and the accompanying financing needs. It is precisely these firms that have been shown to rely on equity holdings by their banks. Furthermore, the benefits of a bank's equity participation arise when it's difficult or costly for a firm to access multiple or non-bank sources of funds. This constraint applies mostly to young or small firms,⁵ to firms in economies where public financial markets are not as well developed or information about firms is opaque, and to firms and industries which cannot easily access venture capital. By all accounts, this appears to be the majority of all growing, small and medium size firms, even in the US.

In more general terms, this paper argues that banks should hold equity stakes whenever (i) the firm-bank relationship is close and long-lasting, (ii) the initial bank has information that other potential providers of financing do not have, and (iii) banks have non-negligible bargaining power when negotiating with individual firms. In terms of cross-country comparisons, other authors have shown that in Germany and Japan, firm-bank relationships are closer and longer

role of banks' equity holdings.

⁴ Since the lower tail of the cashflow distribution is *safe*, it is not much affected by information issues. The upper tail, however, is very sensitive to private information as it is highly uncertain.

⁵ Petersen and Rajan (1994) evaluate a large sample of US small and medium firms, which are responsible for almost 40% of US GNP, but borrow between 75% and 95% of outside funds from a single **bank**. Further empirical support for *both* the assumptions *and* the predictions of the paper is discussed in detail after the model is developed.

lasting than in the US. Furthermore, in Germany and Japan, financial information about firms is not as readily available to agents outside the firm-bank relationship as it is in the US. Hence, this paper provides a rationale for the association of relationship banking and equity shares in Germany and Japan on the one hand, and arm's length financing and no equity shares in the US on the other.⁶ Additionally, banks are likely to have significant bargaining power versus their borrowers if the banking sector is not very competitive (as is the case for instance in the UK). Again, in that case small equity stakes owned by the bank would alleviate the hold-up problem.

Finally, the model can be applied by banks which want to establish the *optimal* level of equity participation over the life-cycle of their borrowing firms. In particular, equity participation is more important when (i) the firm is likely to have future financing needs which exceed its internal cashflow, (ii) the bank has significant amounts of private information, and (iii) the bank has a strong bargaining position versus the firm. As these variables change over the life of a firm, so does the optimal equity stake implied by the model. Consequently, the paper also allows for a further interpretation of existing and future empirical studies of firm-bank relationships. Since in most countries banks do hold some equity, this paper provides a way to incorporate the effects of equity participation on such variables as investment, profitability, and efficiency.

The paper proceeds as follows: the next sub-section provides a short account of some of the most related papers. Section 2 develops the model. Section 3 analyzes the model and derives the main results. Section 4 evaluates both the assumptions and the predictions of the model in light of the available empirical research. Section 5 concludes. All proofs are relegated to the appendix.

1.1. Related Literature

Many papers outline the conditions under which debt and/or equity are optimal contracts between investors and a firm (or its managers). They include⁷ papers by Innes (1990), and

⁶ Interestingly, the flip side of the pro-equity arguments in this paper has implications for those countries where equity holdings by banks are commonplace. The recent *increased* access of firms in Europe to non-bank financing, and the *increased* use of transparent accounting rules, *reduces* the potential regulatory costs of limiting future equity participation by banks (say in countries like Germany, where it is currently unrestricted).

⁷ This section can only mention a small fraction of the relevant papers. A complete list can be found in surveys as e.g. Harris and Raviv (1992).

Aghion and Bolton (1992) for debt, Myers (2000), Fluck (1997) and Burkart, Gromb, and Panunzi (1997) on equity, and Dewatripont and Tirole (1994) and Mahrt-Smith (2000) on a mix of debt and equity. There are also many papers that analyze the costs and benefits of having financing provided by banks rather than other agents or markets. They include papers by Diamond (1984), Mayer (1988), Sharpe (1990), Rajan (1992) and von Thadden (1995). However, there are relatively few theoretical papers that explicitly analyze the terms of the contracts that should optimally be signed between a firm and a bank. While Diamond (1984) shows that a standard debt contract is optimal in his model, the focus of his paper is on the feasibility and efficiency of delegated monitoring, and his static model cannot address the dynamic issues outlined in this paper.⁸

The recent papers which do address the contractual form between banks and firms examine issues somewhat different from the current paper. Berlin, John, and Saunders (1996) examine the quality-signaling effect of having an informed bank choose particular equity positions in its borrowers. This signaling leads to efficient re-negotiations during financial distress by the firm with previously uninformed third parties (such as suppliers or employees), and it is thus somewhat unrelated to the issues of hold-up in this paper. Berlin, et al. do point out, however, that “the [...] relationship between the debt-equity structure of the bank's claim and its power to influence firm behavior is an important topic for research.”

Boyd, Chang, and Smith (1998) examine the effect that equity participation has on project choice. In their model, the bank will, by virtue of holding equity, share in some of the non-contractible perquisites which the firm can withhold from other investors, and the firm's project choice is then influenced by the bank's strategy. Their model is set in a costly state verification framework, and banks will be either lenders or owners, but not both at the same time (the central feature of the current paper). The issues addressed in Boyd et al. are more related to deposit insurance and general equilibrium considerations.

Many economists have worried about the effect that equity ownership would have on the riskiness of a bank's (often government insured) portfolio. While most authors do not look explicitly at the corporate finance side of the story, John, John, and Saunders (1994) and Santos (1999) do examine the effect of equity ownership by the bank on the risk choice of the firm. In these papers, the firm will choose less risky projects when the bank holds some of the

⁸ Other related papers in this area are Diamond (1991, 1993) and von Thadden (1995).

firm's equity (the firm's own, now reduced equity share will make it less prone to 'risk shifting'). In fact, contrary to the concerns of deposit insurers, a bank's overall portfolio risk may be lower when it holds some equity claims, as its borrowers now follow safer strategies. The focus of these papers is more on the regulatory aspect of the banking sector and bank portfolio risk.

In papers most related to this one, Sharpe (1990), Rajan (1992), and von Thadden (1995) consider situations where banks have private information about their borrowers and may potentially use this information to extract profits in the future. However, Sharpe and Rajan take the form of the contract between the firm and the bank as given (they assume debt). Hence, they are silent on the issues discussed in the current paper. While von Thadden does consider the optimal type of the firm-bank contract, he uses a different model set in a complete contracting world, which leads to very different results from the current setup. Equity participation is not an issue in von Thadden's paper, and the hold up problem posed in his paper can be 'contracted away'.

Finally, the prudential regulation of a bank's ability to hold equity (from a non-corporate finance perspective) is discussed in Dewatripont and Tirole (1994), Bhattacharya and Thakor (1993), Hellwig (1991) and Roe (1990). A historical perspective on changes in the US system (which is moving closer to possibly allowing *some* equity ownership by banks) is given in Berger, Kashyap and Scalise (1995).

The relationship between both the assumptions and the conclusions of this paper and the empirical literature is discussed in section 4 below.

2. The Model

The model is designed to be compatible with two separate ideas in the previous literature as reference points. First, it is similar in spirit to the model of Rajan (1992), in order to re-examine the issues raised in that paper when equity is introduced as a possible contract. It is shown that the costs of relationship banking are significantly reduced when banks are allowed to hold small equity stakes.

A second design feature of the model is as follows: In the **absence** of private information for the bank, debt **would** be an optimal contract.⁹ This benchmark allows for the equity stake, which arises as optimal in the model, to be interpreted as resulting directly from the private information possessed by the bank. It is interesting to note that, in contrast to the previous literature, *outside equity actually has a positive effect on managerial incentives* when an adverse selection problem is present.

The model has three dates, $t \in \{0,1,2\}$. Everyone is risk-neutral.

At $t = 0$, an initial investment (i) must be made. The entrepreneur-manager (EM) raises funds from one of many competitive banks.¹⁰ This bank becomes the main or inside bank (IB). Also at this date, EM chooses to exert effort (e) or shirk (s). The non-contractible effort choice is privately costly to EM and entails a monetary equivalent utility loss of k .

The “effort” choice is to be understood as a metaphor for *any* moral hazard problem that affects the long-term profitability of the firm. It is really a euphemism for ‘firm-specific investments’ by the manager. Thus, if (say) corporate restructuring and large-scale layoffs, or the implementation of a moderate-growth business strategy are less desirable to the manager than other alternatives, then this moral hazard set-up would capture the resulting effects of managerial self-interest

At $t = 1$, a second project becomes available. The nature of this project is unknown at $t = 0$, and no contracts can be written at $t = 0$, which are contingent on the nature of the project

⁹ In this respect, the paper extends the literature on debt as an optimal contract to overcome agency problems. This paper uses a model in which debt helps overcome a moral hazard problem as e.g. in Innes (1990). One could equally well choose a model in which the initial debt issue is optimal for other reasons (e.g. Myers and Majluf (1984)).

¹⁰ The benefits of borrowing from a bank have been discussed elsewhere (see also section 4). They include monitoring, lowering the costs of financial distress, and the ability to engage in beneficial long-term relationship specific contracts. In particular, the role of banks in terms of monitoring their borrowers leads naturally to the information issues which drive the results. Also, as is shown below, many of the empirical findings about bank relationships are consistent with the assumptions and predictions of this paper. The assumption of having only one bank is made for convenience and realism (most small and medium firms have only a small number of banks - usually one). The model could be extended to the case where the firm has multiple banks, but it borrows a large proportion of its funds from only one of the banks. A reason for having one main bank could be that having multiple lenders entails free-rider problems with regard to monitoring which are too costly (except for the largest firms). Thus, endogenizing the costs and benefits of monitoring (where part of the cost is fixed) would naturally lead to the optimality of having only one bank (or at least one dominant, monitoring bank). An extension to the venture capital industry (where equity participation is the norm) is also certainly possible, but bank finance is the dominant source of finance almost everywhere.

which turns up (see Grossman & Hart (1986) for the seminal discussion of this specification). If EM chose to exert effort at $t = 0$, then a high quality project (h) turns up. If EM shirked, then only with probability p $\hat{I}(0,1)$ is a high quality project available, and with probability $(1-p)$ a low quality project (l) is available. A high quality project could be interpreted as being available if a new, highly differentiated product was successfully developed. A low quality project would then correspond to a less profitable new product. The projects require an additional investment of I .¹¹ This investment can be raised from IB or from the competitive outside financial market (OM). For simplicity, it is assumed that EM will first bargain over new finance with IB. If this bargaining fails, EM can raise funds from OM.

Also at this time, both EM and IB receive a signal about the profitability of the second period project. The *project specific* signal can be **good** or **bad** (an interpretation of the signal could be that it is about the costs and results of small pilot investments). It is good with probability Q . In other words, due to its lending relationship with the firm, IB acquires an informational advantage over other, potential lenders. The signal is assumed to be about the profitability of the project itself, and not about EM's effort choice. This allows for an interesting extension of the results in the previous literature. If the signal were about whether or not EM chose to exert effort, then all results would carry through.

At $t = 2$, the firm realizes its payoffs. Cashflows are uncertain and depend on which project was available at $t = 1$ and on the signal that was received. Both projects $j \in \{h, l\}$ pay either an amount $C + X$ (success) or an amount C (failure).¹² If there is no private information, then the probability of success is P_j . If IB and EM have a private signal about profitability, then the probability of success goes to $P_j - Q_j P_j$ (following a bad signal) or to $P_j + Q_j(1 - P_j)$ (following a good signal), where $Q_j \hat{I}(0,1)$.¹³

¹¹ Making I project specific does not add any insights (the results hold even if I is zero for the low quality project, but some extensions become meaningless). Also, it is implicitly assumed that I cannot be raised at $t = 0$ (i.e. *unconditional, irrevocable* lines of credit - which are rare anyway in the real world - are ruled out). This could easily be endogenized by adding a negative NPV project at $t = 1$, which EM would nevertheless take if money were available - say because EM derives private benefits from investing. Jensen (1986) seems to be convinced that this is a very real problem.

¹² Normalizing C to zero would actually make the algebra more messy (see assumption **A6**).

¹³ This specification ensures that, for all $Q \hat{I}(0,1)$ the probabilities remain between 0 and 1. On the downside, it has the inconsequential but unfortunate side-effect that the ex-ante probability of success is not P_j but $Q Q_j + (1 - Q_j) P_j$. No comparative statics results are affected by this issue.

Q_j is the natural measure of the informativeness of the signal. $Q_j = 0$ means that the signal is not informative (a good/bad signal does not change the probability of high cashflows), while $Q_j = 1$ means that the signal is perfectly informative (the good signal increases the probability of high cashflows to 1, a bad signal reduces it to 0).

The following definition simplifies the presentation: the way that Q_j enters all results is as the following measure of the relative informativeness of the signal:

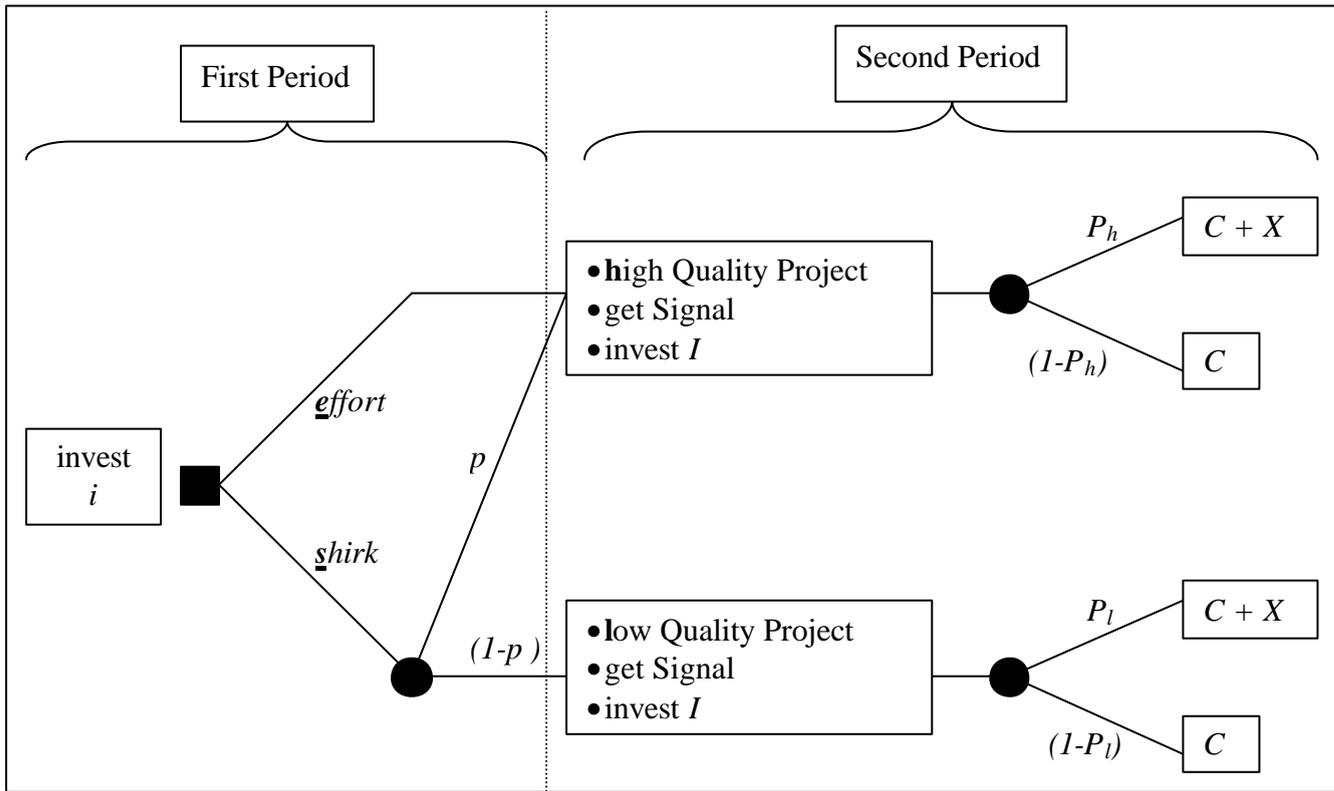
$$\mathbf{b}_j \equiv \frac{Q_j}{(1-Q_j)\Pi_j}. \quad (1)$$

\mathbf{b}_j measures the *increase* in the probability of the high cashflow when a good signal is observed, *relative* to the probability of the high cashflow when a bad signal is observed. $\mathbf{b}_j = 0$ means that the signal contains no information, while as \mathbf{b}_j approaches infinity the signal becomes perfectly informative.

Denote as $P_j \equiv \Theta Q_j + (1-Q_j)\Pi_j$ the ex ante *expected* probability that the project $j \in \{h, l\}$ will be successful (Θ is the probability that a good signal is received).

All variables associated with the second period projects are capitalized, while variables associated with the first period choices are in lower case.

The model is summarized in the figure below.



A few assumptions are needed for tractability and to rule out degenerate cases.

A1: It is efficient for EM to choose to exert effort and not shirk. Formally, this implies $(1-p)(P_h - P_l)X > k$

A2: Bargaining between EM and IB is done in a Nash bargaining game, where IB has bargaining power α relative to EM (i.e. IB will capture a fraction α of the surplus). Alternative bargaining games would be equally feasible.

A3: The firm can raise the initial funds (i) by issuing debt (which is senior to new securities) as well as equity.

Assumption **A3** raises two issues: first, it assumes that firms and banks are limited to a subset of all possible contracts. This is not a severe problem, because the main message of the paper is to show that allowing banks access to equity stakes is beneficial relative to limiting them to only debt contracts. These benefits can easily be examined and analyzed in the current model. To develop a framework where debt and equity are *uniquely* optimal, which allows all parties

to optimize over all general contracts (and potentially over messages sent to each other) would, in this dynamic context, necessitate significantly more complicated assumptions. These would invariably detract from the main message of the paper. The fact that the most commonly observed contracts are indeed debt and equity suggests that focusing on these alternatives is a useful exercise in its own right. Finally, the issue that the form of the contract (debt or equity) held by the bank may affect the incentives of the bank to become privately informed in the first place is briefly addressed in the extensions below.

Secondly, **A3** restricts debt to being senior to new issues. This restriction can be endogenized by extending the model. In particular, using debt which is junior to new issues to finance the firm at $t = 0$ would enable EM to raise new funds for additional projects by issuing new *senior* debt at $t = 1$. If the new projects are risky enough, then this would transfer some of the expected payoffs from IB to EM. In other words, there are strong incentives for EM to expropriate existing debt holders by starting new, risky projects which may even have significantly *negative* expected value. If the model is extended to include the possibility of new, risky, *negative* NPV projects at $t = 1$, then issuing debt at $t = 0$ which is junior to debt issued at $t = 1$ is no longer a viable option. While even senior debt issued at $t = 0$ is subject to this problem, the danger is much less severe, because EM can no longer issue *more senior* debt at $t = 1$.¹⁴ Again, given that in reality almost all bank debt is senior to new issues (or alternatively, secured against specific assets, making it de facto senior), it is likely that an analysis under the current restrictions adds significantly to our understanding of actual lending relationships.

A4: $C < I + i$ (to rule out degenerate cases where funds can always be raised riskfree)

A5: $C + P_t X > I + i$ (to rule out the case where the second period project cannot be financed at all. It also avoids the separate issue of debt overhang (Myers 1977))

A6: $C > \text{Max}[i, I]$ (purely for analytical tractability)

3. Analysis

¹⁴ Equity, with its unlimited upside, is not subject to this expropriation via increased firm risk.

The following lemma is important for the analysis and captures the adverse selection problem (lemons problem) arising from the fact that IB has some informational advantage relative to OM.

Lemma 1: *If OM is asked to provide funds at $t = 1$, then OM will infer that EM and IB received a bad signal about the profitability of the firm.*

Proof: See appendix.

This lemma formalizes the source of the bargaining power for IB. Since OM will infer that only bad firms will ask for funding (good firms continue to obtain funding from IB), the terms of finance will be bad. Hence, even if a firm is in good shape (the signal was good), the firm is 'locked in' the relationship with IB. Going to OM when the firm is in good shape would lead to positive rents for OM, which can instead be captured by EM and IB through bargaining with each other.

The analysis will proceed as follows: first, the case of pure debt financing is examined. This establishes a useful benchmark. In particular, the model is set up so that debt would be *optimal*, **if** there were no adverse selection problem. Second, EM is allowed to issue equity to IB at $t = 0$ to mitigate the adverse selection problem. Finally, we examine under what conditions issuing equity increases firm value and what amount of equity is optimal.

3.1. Debt only

Assume EM chose to efficiently exert effort at $t = 0$ (we will show below under what conditions this will happen).

To determine the amount of profits which are extracted by IB when it bargains with EM over continuation funding, we need to establish the outside option for EM - namely getting funds from the uninformed OM. Given Lemma 1, OM, if asked, would offer financing at $t = 1$ at

terms which allow it to break even if it faces a bad firm. This leads to a face value of second period debt (D_h) demanded by OM given by¹⁵

$$I = (1 - Q_h) \Pi_h D_h + [1 - (1 - Q_h) \Pi_h] [C - d], \quad (2)$$

where d is the amount of debt held by IB.¹⁶ Of course, if EM were to approach OM following a *good* signal, then these terms of financing would leave some rents with OM. EM and IB will bargain over these rents rather than letting them accrue to OM.

For IB to break even overall (taking into account the expected profits it will extract at $t = 1$), IB will demand an initial face value of debt (d) given by

$$i = d + \Theta a R_h. \quad (3)$$

R_h are the potential profits over which IB and EM bargain at $t = 1$ if they received a good signal. They are equal to the potential rents which *would* accrue to OM, if EM *were* to have to go to OM for funds following a *good* signal. Since EM and IB will bargain over these rents, a is the share of these rents going to IB. Q is the probability of receiving a good signal (which is the time when the rents are available). R_h is given by

$$R_h = -I + [\Pi_h + Q_h (1 - \Pi_h)] D_h + [1 - \{\Pi_h + Q_h (1 - \Pi_h)\}] [C - d] = Q_h (D_h - [C - d]). \quad (4)$$

Equations 2, 3, and 4 can be solved for d :

$$d = i - (I + i - C) \frac{a \Theta b_h}{a \Theta b_h + 1} \quad (5)$$

Equation 5 shows that if there is no chance for IB to hold-up EM at $t = 1$ (i.e. the bank's bargaining power a is zero, the informativeness b_h of the signal is zero, or the probability of receiving a good signal Q is zero), then the original debt contract, which is senior, simply specifies a face value equal to the borrowed amount.

To determine if EM will indeed choose to exert effort at $t = 0$, we compute the (monetary equivalent) utility - U_e - for EM following high effort. Given that IB and OM just break even

¹⁵ Restricting second period financing to pure debt is without loss of generality in the current setting. In fact, it is straight forward to show that debt is an *optimal* contract for any financing raised from OM for the second period: given that there is adverse selection about profitability, OM should hold the claim which receives most of its payback from cashflows about which there is no asymmetric information (C), and the least amount of payback from cashflows about which OM is at an informational disadvantage (X). In other words, OM should hold debt, which receives *all* of the riskfree cashflows, plus a small amount of risky payback.

¹⁶ **A4** ensures that the debt issued to OM will always be risky, and hence OM will obtain its full face value D_h only following success and will receive less ($C - d$) following failure of the project.

in expectations, we get

$$U_e = (C + P_h X - I - i) - k \quad (6)$$

We can use the same arguments as above to derive EM's utility in the (hypothetical) case where EM shirks at $t = 0$, which we call U_s .

Lemma 2: *EM will choose to exert effort at $t = 0$ iff $U_e \geq U_s$, which is equivalent to*

$$(P_h - P_l)X - (I + i - C) \frac{a\Theta(\mathbf{b}_h - \mathbf{b}_l)}{a\Theta\mathbf{b}_h + 1} \geq \frac{k}{(1-p)} \quad (7)$$

Proof: See Appendix.

The lemma describes the incentives for EM to exert effort, which are affected by the moral hazard problem in the first period and the adverse selection (hold-up) problem in the second. The first term on the LHS of equation 7 is the standard moral hazard term, which would be present if effort elicitation were the only problem. In the presence of adverse selection at $t = 1$, the second term on the LHS becomes an issue. While the main results of the paper follow in later sections, a first result (unrelated to bank equity ownership) follows immediately:

Proposition 1: *The fact that IB has private information about EM and non-zero bargaining power decreases efficiency if $\mathbf{b}_h > \mathbf{b}_l$, and it increases efficiency if $\mathbf{b}_h < \mathbf{b}_l$.*

Proof: See Appendix.

Proposition 1 extends the arguments in Rajan (1992). In particular, in Rajan's specification, the bank's private information and the associated ex post extraction of rents *always* had a negative effect on EM's incentives. This is because, implicitly, Rajan's model assumed that IB had more of an informational advantage over OM following high effort by EM.¹⁷ This corresponds to the case where $\mathbf{b}_h > \mathbf{b}_l$. It can be interpreted as IB having information about the profitability of future investment projects, and EM's efforts produce these future projects. It is in this case that equity participation by banks will turn out to be especially important (as we

¹⁷ Rajan's model was somewhat different (in particular, the signal was *about* effort), as he wanted to address several other issues. The interpretation given here about Rajan's model applies, when some of the extra features

will see below).

However, there may be cases where IB has a true *relative* informational advantage over OM only if the manager shirks and does not generate large, profitable future projects. This information structure may be interpreted as IB having private information only about assets in place and current products/processes. In that case, the signal that IB receives satisfies $b_h < b_l$. Consequently, the bank's power to extract rents actually increases incentives and efficiency (and, as it turns out, equity participation by the bank is *never* desirable). In other words, the fact that IB becomes especially strong versus EM following low effort makes EM work especially hard.¹⁸

This fact that the bank's inside information can actually increase efficiency can be stated more formally as follows: The nature of the projects is non-contractible by assumption. Nevertheless, since the availability of a particular project contains information about EM's choice of effort, we would *like* to write contracts that depend explicitly on the projects in order to allocate rents (away from EM in the case of shirking in order to discourage it). Now, if IB's informational advantage over OM depends on the available project, then it will lead to rent extraction which depends on project quality, leading (non-contractually, i.e. via bargaining and not via ex ante specified sharing rules) to a payoff structure for EM which *does* depend on which project becomes available.

Overall, however, it seems likely that the bank has information about future investment opportunities, or equivalently that high quality projects require more investments. This will lead to a **costly** hold-up problem and will eventually lead to the optimality of equity.

3.2. Debt and Equity

To establish the benefit of having IB hold an equity stake, we will now evaluate if firm value can be increased by allowing equity claims. Recall that the model has been set up in such a way as to make debt ownership by IB an optimal contract (while equity is not!) in the *absence*

in Rajan's model are stripped away. Rajan did not, as this paper does, focus on allowing the more general equity contracts. Interestingly, Rajan does mention in the end the potential interest of such an analysis.

¹⁸ Note that this simple classification into two cases, $b_h > b_l$ and $b_h < b_l$, assumes that investment needs (I) are the same for both projects. If the investment needs are mostly for the high quality project, then this is equivalent

of an adverse selection problem, i.e. without private information for IB.¹⁹ Hence, the comparative statics in the following sections can be interpreted as dealing with the benefit of equity participation in terms of solving the adverse selection problem.

The analysis is similar to the case where IB holds only debt. All variables which are affected by IB's equity stake will have a 'hat'. Assume EM chose to efficiently exert effort at $t = 0$ (we will show below under what conditions this will happen). Given that IB and OM again just break even, we still get that EM's utility from exerting effort is

$$\widehat{U}_e = U_e = (C + P_h X - I - i) - k \quad (8)$$

We can use the same arguments as above to derive EM's utility in the (hypothetical) case where EM shirks at $t = 0$, which we call \widehat{U}_s .

We can then derive the following lemma (\mathbf{g} is the equity stake held by IB)

Lemma 3: *EM will choose to exert effort at $t = 0$ iff $\widehat{U}_e \geq \widehat{U}_s$, which is equivalent to*

$$(1 - \mathbf{g})(P_h - P_l)X - (I + i - C - \mathbf{g}P_h X) \left[\frac{\mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a}\Theta\mathbf{b}_h + (1 - \mathbf{g})} \right] > \frac{k}{(1 - p)} \quad \forall \mathbf{g} \leq \mathbf{g}^* \quad (9)$$

and

$$(1 - \mathbf{g})(P_h - P_l)X > \frac{k}{(1 - p)} \quad \forall \mathbf{g} > \mathbf{g}^* \quad (10)$$

where

$$\mathbf{g}^* \equiv \frac{(I + i - C)}{P_h X} \in (0, 1) \quad (11)$$

Proof: See Appendix.

The lemma again describes the incentives for EM to exert effort, which are affected by the moral hazard problem in the first period and the adverse selection problem in the second. For

to always having $\mathbf{b}_h > \mathbf{b}_l$, which is when bank hold-up is costly and equity participation will turn out to be valuable.

¹⁹ The formal proof of this is virtually identical to the proof in the many papers (e.g. Innes (1990)) which show debt as an optimal contract – and hence it is omitted. The intuition is that debt would lead to a maximal payment to IB following shirking relative to the payment following effort. Hence, it leaves EM with relatively more rents following effort. Equity, on the other hand, allows IB to participate in the firm's upside, hence making the upside

$\mathbf{g} < \mathbf{g}^*$ (equation 9), the term $(I - \mathbf{g})(P_h - P_l)X$ is again the same term we would get in a standard moral hazard set up - if IB were to hold an equity stake \mathbf{g} . Any positive equity stake would be inefficient for the moral hazard term. The second term again contains the adverse selection effect. It is again obvious that this component is zero when IB has no private information or bargaining power. The adverse selection component is clearly affected by the amount of equity \mathbf{g} . It collapses to the term in equation 7 (the debt only case) for $\mathbf{g} = 0$. The comparative statics are discussed below (equity has a positive incentive effect via the adverse selection component).

Any increase in equity participation beyond \mathbf{g}^* will always lead to a reduction of efficiency (see equation 10), as only the moral hazard component remains. For the intuition, note that higher equity stakes for IB mean that IB receives relatively fewer returns in the form of debt (d), and consequently there are more 'safe' debt-like returns which can (potentially) be promised to OM at $t = 1$. At some point, \mathbf{g} will be high enough to allow EM to issue risk-free debt to OM at $t = 1$. I.e. there exists²⁰ a d^* (corresponding to \mathbf{g}^*) such that

$$I = C - d^* . \quad (12)$$

When the second period borrowings are risk free, then the adverse selection problem disappears.²¹ As stated above, when there is no adverse selection, debt is an optimal contract and equity is not.

Using Lemmas 2 and 3, we can derive the main result of the paper.

Proposition 2: *Equity participation by the bank can increase efficiency. The benefit of equity participation is given by \mathbf{D} , where*

$$\Delta \equiv -\mathbf{g}(P_h - P_l)X + \mathbf{g}\mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l) \left[\frac{[\mathbf{a}\Theta\mathbf{b}_h + 1]P_h X - (I + i - C)}{[\mathbf{a}\Theta\mathbf{b}_h + 1][\mathbf{a}\Theta\mathbf{b}_h + (1 - \mathbf{g})]} \right] \quad \forall \mathbf{g} \leq \mathbf{g}^* \quad (13)$$

and

less attractive for EM, thus leading to less incentives.

²⁰ Existence follows from assumption **A4**.

²¹ The fact that the adverse selection problem *disappears* is an artifact of the two-state nature of the model. If the cashflow distribution were continuous, then the adverse selection problem would simply *decrease in importance* relative to the first period moral hazard problem when IB holds more and more equity. This would lead to a point where the positive effect of equity participation (it reduces adverse selection effects) would eventually be overcome by the negative effect that equity has on the moral hazard component.

$$\Delta \equiv -g(P_h - P_l)X + a\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{(I+i-C)}{a\Theta\mathbf{b}_h + 1} \quad \forall g > g^*. \quad (14)$$

Proof: See Appendix.

The LHS of equations 13 and 14 is made up of two parts. The first part is the penalty that outside equity imposes on EM's incentive to exert effort. This comes from the moral hazard problem in the first period. It is identical to traditional results which show that outside debt is an optimal contract and equity is suboptimal. The second part is the benefit of equity participation by IB in terms of easing the adverse selection problem and the associated hold-up. It is always positive as long as $\mathbf{b}_h > \mathbf{b}_l$ (see again the discussion following proposition 1) since $P_h X - (I + i - C) > 0$ by assumption **A5**.

The following result is now relatively obvious.

Proposition 3: *Debt-and-equity is never an improvement over debt-only if*

- (i) *IB has no bargaining power ($\mathbf{a} = 0$)*
- (ii) *IB has no private information ($\mathbf{Q} = 0$) or equivalently the signal that IB receives is not informative ($\mathbf{b}_h = \mathbf{b}_l = 0$).*
- (iii) *$\mathbf{b}_h \leq \mathbf{b}_l$ (see the discussion following Proposition 1).*

Proof: See Appendix.

As expected, equity helps only if there is a genuine adverse selection problem.

The next step is to evaluate the *optimal* level of equity participation which maximizes firm value and efficiency. It turns out that the result has some interesting implications for the optimal regulation of bank equity participation as well as for empirical research on firm-bank

relationships.

Proposition 4: *The optimal equity stake that IB should hold to maximize firm value is*

(i) zero if

$$\frac{P_l}{P_h} \leq \frac{\mathbf{a}\Theta\mathbf{b}_l + 1}{\mathbf{a}\Theta\mathbf{b}_h + 1} \quad (15)$$

(ii) \mathbf{g}^* if

$$\frac{P_l}{P_h} > \frac{\mathbf{a}\Theta\mathbf{b}_l + 1}{\mathbf{a}\Theta\mathbf{b}_h + 1}. \quad (16)$$

(iii) Also if, in addition to the condition in 16, we have that

$$(P_h - P_l)X > \mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{(\mathbf{a}\Theta\mathbf{b}_h + 1)P_h X - (I + i - C)}{(\mathbf{a}\Theta\mathbf{b}_h + 1)^2} \quad (17)$$

then positive equity stakes **below** \mathbf{g}^* could lead to a reduction of efficiency even in situations where \mathbf{g}^* itself would lead to an increase in efficiency. Hence, even if **allowed** to hold equity, IB may optimally choose to hold **no** equity if holding the entire amount \mathbf{g}^* were not possible.

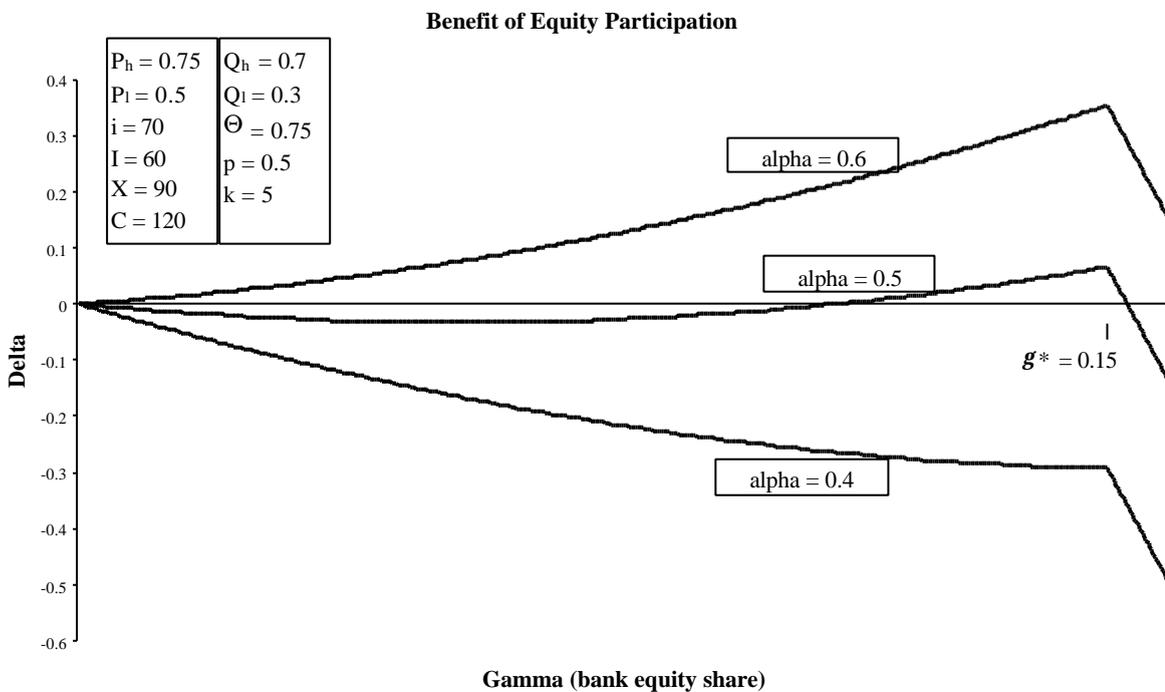
Proof: See Appendix.

The condition for positive equity participation to be optimal (equation 16) states that equity participation is good if the moral hazard problem is not too severe (P_h is not *much* greater than P_l), but the adverse selection problem and the bank's bargaining power are relatively severe (\mathbf{b}_h is larger than \mathbf{b}_l , and \mathbf{a} and \mathbf{Q} are large).

Parts (i) and (ii) of proposition 4 establish that IB should either hold no equity or it should hold the 'maximum' stake \mathbf{g}^* (which is likely to be still a minority stake if the costs of equity participation are non-negligible). This should be taken into account when the observed equity participation of banks is included in empirical investigations of firm-bank relationships. Obviously, the proposition would also inform banks and firms when they decide on how

much equity the bank should hold, especially as this optimal stake is likely to change over the life of the firm. Part (iii) states that, while equity participation at the optimum (γ^*) may be optimal, in some cases a smaller stake would actually lead to a reduction in efficiency.

The following figure illustrates where the results in proposition 4 come from: the 'benefit of equity' function D is convex. As expected, the function is globally higher when the bank has more bargaining power over the firm. Again, the kink is a result of the simple two-state model (see footnote 21). In a more general model it would be a 'smooth' maximum. However, the initial convexity would be preserved.²²



It is interesting to note that the convexity is not the result of a very specific modeling assumption. In fact, it is a direct result of the *standard* moral hazard assumption - the monotone likelihood ratio property (MLRP - in this case simply that $P_h > P_l$). To see this, note that equity participation by the original lender (IB) makes more of the lower tail of the cashflow distribution available to a (potential) second period outside lender (OM). This lower

²² Even though there are clearly some (non-modeled) effects which might attenuate the convexity in the relationship, there is some evidence that it is indeed convex in the data (see section 4 below).

tail is more important for the low quality project ($P_l < P_h$ implies that for the low quality project relatively more of the expected returns lie in the lower tail of the cashflow distribution). Hence, once IB holds some equity, it will have solved *relatively* more of the 'low quality project adverse selection'. In other words, while adverse selection is reduced both for the high quality project and for the low quality one, on a relative basis the adverse selection problem following high effort has increased.²³ But this is precisely what makes it so difficult to provide EM with incentives (as *relatively* more of the rents following high effort are now extracted by IB). Hence, equity participation to solve the hold up problem becomes *even more important* once IB already holds a small equity stake. The result is a natural convexity in the 'benefit of equity' function (D), which means that if a bit of equity for IB is optimal, then even more equity is even more beneficial (up to the maximum, at which point the importance of the adverse selection problem relative to the moral hazard problem diminishes and equity participation becomes counterproductive).

Optimal Regulation

Proposition 4 may have implications for the optimal regulation of banks in the US (and in other countries where equity participation by banks is limited by the regulator).²⁴ One can imagine two ways to relax the constraint on equity participation: (i) banks would be allowed to hold *small* equity stakes in any firm, or (ii) banks would be allowed to hold moderate (minority) equity stakes *only* in medium sized, younger firms in industries with growth opportunities where most financing is provided by one or a few banks, and firms for which banks with significant bargaining power are likely to have private information. Given the convexity of the benefit of equity participation and the result that very small amounts of equity may (in some circumstances) be less efficient than none at all,²⁵ method (ii) seems to be called for.

Of course the results are subject to the concern that banks may want to use equity stakes to

²³ Of course on an *absolute* scale, the severity of the high quality project adverse selection problem still leads to an overall increase in efficiency from having IB hold some equity.

²⁴ See Santos (1999) for numbers on country-specific equity participation restrictions [e.g. US - 0%, Belgium - 5%, Canada - 10%, Portugal - 25%, Norway - 50%, Germany - unrestricted].

²⁵ This would be the case when the moral hazard problem is severe (i.e. X and/or $(P_h - P_l)$ are large) and the adverse selection problem is also severe (i.e. $aQ(b_h - b_l)$ is also large).

inefficiently increase the riskiness of their deposit insured portfolios. Studies by John, et al. (1994) and Santos (1999) show that the issue may not be too important (or even altogether absent!) for minority equity stakes. Other than that, it only remains to state that the *corporate finance* results of this paper should be viewed as complementary to the debate, which is currently being held mostly from the deposit taking/capital adequacy side of the bank's business. In particular, this paper points at the types of equity participation which should be considered as the most likely candidates for deregulation.

Comparative Statics

More precise comparative statics about the *benefits* of allowing equity participation by banks and about the size of the optimal equity stake can be developed. The D in equation 13 is the difference between EM's incentives to exert effort under debt-and-equity and EM's incentives under debt-only. Since effort is efficient, it is a measure of how desirable equity ownership by banks is.

Proposition 5: *The benefits of having IB own an (optimal) equity stake are*

- (i) *increasing in the bargaining power of IB a*
- (ii) *increasing in the amount of **risky** funds required at $t = 1$, $[I - (C - i)]$.*
- (iii) *increasing in the informational advantage b_h that IB has over OM following high effort.*

Proof: See Appendix

This proposition reiterates that there may be situations (or countries) where bank equity participation is more important than in others. In particular, if in Germany and Japan the accounting systems are not as transparent as in the US, then banks are likely to hold an informational advantage about 'their' firms. In this case, equity participation is particularly important. Similarly, less competitive banking systems (as e.g. in the UK, where the

Cruickshank report²⁶ has recently condemned the monopolistic and collusive business lending practices of the main banks) would confer more bargaining power onto banks, and one way around the holdup problem may be moderate amounts of bank equity participation.

Finally, it is obvious that there is no need to have IB hold equity if the firm has no non-contractible growth opportunities which require more investment than the firm generates internally. Also, if all cashflows and future investment needs can be contracted upon (i.e. described in a manner that can be enforced by courts), then the firm could just raise all the money it needs at $t = 0$, or alternatively specify the terms of future finance at $t = 0$, thus making it impossible for IB to extract rents in the future. Alternatively, if the firm were to generate enough cash to finance future projects out of internal capital (this may be true for mature companies), then there would again be no hold-up problem. However, it would be interesting to investigate the trade-off between forcing the firm to pay out its cash in order to prevent inefficient waste of cash by managers (as suggested in Jensen (1986)) on the one hand, and the incentive distortions generated by having the firm be short on internal cash as in this paper on the other hand. This is beyond the scope of the current paper.

As outlined in proposition 4, the optimal level of equity participation is either zero or γ^* . If γ^* is optimal (i.e. if the hold-up problem is severe), then the comparative statics of γ^* are immediate:

Proposition 6: *If non-zero equity participation is optimal, then the level is*

- (i) *Increasing in the funding needed at $t = 1$, $(I + i - C)$*
- (ii) *Decreasing in the severity of the moral hazard problem, $P_h X$*

Proof: See Appendix.

4. The Model and the Evidence

²⁶ See the Financial Times on Monday, March 20, 2000.

This section examines the empirical literature with respect to both the assumptions and the conclusions of the current paper.

The model in this paper assumes that (i) firm-bank relationships are valuable (see Flath (1993), Kester (1991), Hoshi, Kashyap and Scharfstein (1990) and Prowse (1990) for evidence from Japan; see Harm (1996) and Allen and Gale (1995) for evidence from Germany; see Berger and Udell (1995) for the US, see also James (1987) for some US evidence that bank loans are special and Lummer and McConnell (1989) for evidence that bank monitoring and relationships are what *makes* bank loans special), (ii) banks have privileged information about firms (see Allen and Gale (1995) and Flath (1993)), (iii) many firms (especially smaller firms and firms in many non-US economies) rely almost entirely on a small number of banks (often one) for the vast majority of their external funds (see Petersen and Rajan (1994) for small US firms, Ongena and Smith (1998) and Farinha and Santos (2000) for an international comparison - however, see Detragiache, Garella, and Guiso (2000) for evidence that Italy may be quite different). Ongena and Smith (1999b) provide an overview over the empirical literature on firm-bank relationships. Lastly, Edwards and Fischer (1994) find that in Germany “... the ending of a dominant house bank relationship between a firm and a bank was generally interpreted by other banks as a negative signal of the firm's prospects”, which is consistent with the existence of a lemons problem.

In terms of the predictions of the paper, we can examine the research on equity ownership by banks (and its relationship to firm performance), as well as some studies which do cross-country comparisons.

Consistent with the predictions of this paper, Kim (1991) finds that Japanese bank equity holdings increase in the proportion of financing it provides to the firm (i.e. the firm's **main** bank is more likely to hold equity) and holdings are larger if the firm faces many growth opportunities.²⁷ James (1995) finds that banks hold more substantial equity stakes following financial distress (this is the time when banks are allowed to hold equity under US law) when the firm has growth opportunities (i.e. they will need more *outside* finance in the near future).

²⁷ Kim's sample stems from a period (1964-1970) where Japanese firms had very little recourse to any financing other than bank debt.

In terms of performance, Pushner (1995) finds evidence in Japanese data consistent with a positive effect of bank equity participation on efficiency. While Morck, Nakamura, and Shivdasani (1999) find a slight negative relationship at low levels of equity, they do find significant evidence pointing towards a *convex* relationship between equity participation and value. This convexity is clearly consistent with the results of this paper. Finally, Gorton and Schmid (1999) find that German data is consistent with the predictions of this paper in 1974. However, by the late 1980's, the significance of their findings seems to disappear. Gorton and Schmid (1999) cite as main reasons for this reduced significance that (i) German banking has become more competitive and (ii) German firms have increased access to non-bank finance. Both of these are very much in line with the arguments in this paper, which predicts that diminished bank bargaining power and diminished reliance on bank debt would reduce the importance of bank equity participation. Lastly, Gorton and Schmid (1999) blame some of their difficulties in getting significant results on the fact that "economic theory provides little guidance as to the details of [the] nonlinearities [in the relationship between equity participation and performance]". Maybe the current paper can guide future empirical efforts.

5. Conclusion and Extensions

This paper argues that equity ownership by banks can help alleviate a hold-up problem between firms and their banks. It applies in particular to firms which rely heavily on financing from a small number of well informed banks. Significant improvements in efficiency are achieved even for *small, minority equity stakes* owned by banks. The paper speaks to regulators in countries like the US, where equity participation is restricted by law. In addition, it helps banks in other countries, where lenders must determine the *optimal* level of equity participation over the life of the firm. Finally, the paper is informative for transition economies, which are in the process of designing regulatory systems, as well as countries where banks are unable (or unwilling - see Dittus (1996)) to hold *controlling* equity stakes.

In as much as real world contracts may be more complicated than simple debt and equity contracts, this paper points to an important consideration in evaluating the effects of all contracts: the nature of the bank's claim on the firm's cashflows determines the relative bargaining power of the bank and the firm when future financing needs arise. Where

particular relationships are desirable (say the bank should be strong in order to encourage costly monitoring, or the firm should be strong in order to encourage firm specific investments by its managers and employers), this paper outlines desirable features of the contracts governing the lending relationship.

It would be interesting to further develop the effect that inter-bank competition has on the results of this paper.²⁸ Some authors even argue that adverse selection (a feature of the holdup problem) can have an impact on inter-bank competition itself.²⁹ Hence, the relationship between competition and the contractual solution to the holdup problem is likely to be quite involved.

Finally, there is further need to examine the nature of the firm-bank contract with respect to monitoring. In particular, this paper and many others assume that the inside bank has some intrinsic informational advantage over the outside financial markets. Presumably, this advantage arises from monitoring the firm. Of course, some of this monitoring is desirable, and there is clearly a trade-off between how much information the bank *should* have to keep the firm running efficiently and how much it *should not* have in order to alleviate the hold-up problem discussed in this paper. In as much as the form of the contract between the firm and the bank (e.g. debt or debt-and-equity) also directly affects the incentives for the bank to acquire an informational advantage to begin with,³⁰ this could be incorporated into the present analysis. In particular, it is likely that equity ownership makes the acquisition of information more desirable for the bank, and this would directly impact on the optimal amount of equity that banks should hold.

²⁸ See Boot and Thakor (2000) and Petersen and Rajan (1995) for work on interbank competition and relationship lending.

²⁹ See Dell'Ariccia, Friedman, and Marquez (1999).

³⁰ See Boot and Thakor (1993) for the impact of contractual form on the incentive to monitor.

Appendix

Sketch of the proof of Lemma 1 (*the maintained assumption is that, given the symmetry of information between IB and EM, they will bargain successfully - see e.g. Osborne and Rubinstein (1990). If this is not true, then bargaining might fail randomly, but the qualitative results of the paper remain unchanged*):

By Contradiction: Suppose OM does not infer that there was a bad signal. Then OM must believe that there is a positive probability that EM and IB received a good signal about the second period project. Call this positive probability $n > 0$. In this case, OM will agree to financing terms which allow it to break even given this assumption.

Case 1: $0 < n < 1$ (i.e. OM does not anticipate to face *only* firms which have received a good signal). In this case, OM will make strictly positive profits on loans to firms which have received a good signal (because, for $n < 1$, OM will base its terms of credit on a probability of success which is strictly lower than that of a firm with a good signal). Hence, if they indeed receive a good signal, EM and IB are better off bargaining over funds from IB rather than granting some rents to OM. Hence, it can not be that any firms which have received a good signal actually approach OM for funds and consequently $0 < n < 1$ is not an equilibrium.

Case 2; $v = 1$ (i.e. OM anticipates facing *only* firms which have received a good signal). In this case, OM will provide funds at an expected loss to firms which have received a bad signal, and hence all such firms will approach OM for funds. Again, in this case $n = 1$ is not an equilibrium.

It remains to be shown that $n = 0$ is indeed an equilibrium. In this case, OM will earn strictly positive rents if it finances firms which have received a good signal. Hence, if EM and IB have received a good signal, they will bargaining over funds rather than granting some rents to OM. Consequently, OM will indeed never encounter a firm that has received a good signal. On the other hand, firms which have received a bad signal are indifferent between approaching OM and not approaching OM. Hence, $n = 0$ is not an irrational equilibrium belief.

QED

Proof of Lemma 2: Equation 6 shows the share of the firm's payoffs accruing to EM if she were to choose high effort. On the other hand, if EM were to shirk, she would expect to earn

$$U_s = p(C + P_h X - I - i) + (1 - p)(C + P_l X - I - Z_l), \quad (1a)$$

where Z_l is the return that IB earns if the low quality project comes along. This is given by

$$Z_l = d + \mathbf{a}\Theta R_l, \quad (2a)$$

where d is as in equation 4 (the effort decision is not observed and hence does not affect d). R_l (similar to R_h in equation 3) is the amount over which EM and IB bargain, namely the expected rents that *would* go to OM in case of a low quality project if OM *were* approached for funds following a good signal

$$R_l = -I + [\Pi_l + Q_l(1 - \Pi_l)]D_l + [1 - \{\Pi_l + Q_l(1 - \Pi_l)\}][C - d] = Q_l(D_l - [C - d]). \quad (3a)$$

The second equality in equation 3a holds, because as before the terms of OM's debt contract would be set by the break-even condition (assuming being faced with the expected bad type firm), which is

$$I = (1 - Q_l)\Pi_l D_l + [1 - (1 - Q_l)\Pi_l][C - d]. \quad (4a)$$

Of course, following a good signal, IB and EM would bargain over R_l , rather than giving it to OM, and IB would obtain a share \mathbf{a} of this amount (this is the second term in equation 2a). Rearranging equation 4a and substituting into equation 3a to solve for R_l gives

$$R_l = Q_l(I + d - C) \frac{1}{(1 - Q_l)\Pi_l}. \quad (5a)$$

Using the definition of d from equation 4, we can now solve for Z_l from equation 2a as

$$Z_l = i - (I + i - C) \frac{\mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a}\Theta\mathbf{b}_h + 1} \quad (6a)$$

Using the definition of U_s in equation 1a, we get that

$$U_s = (C + P_h X - I - i) - (1 - p)(P_h - P_l)X + (1 - p)(I + i - C) \frac{\mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a}\Theta\mathbf{b}_h + 1} \quad (7a)$$

The last step is to compute the difference between U_e from equation 6 and U_s from equation 7a to obtain

$$U_e - U_s = (1 - p)(P_h - P_l)X - (1 - p)(I + i - C) \frac{\mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a}\Theta\mathbf{b}_h + 1} - k \quad (8a)$$

This expression is positive if and only if the condition in the lemma is true.

QED

Proof of Proposition 1: It is efficient for EM to exert effort by **A1**. Hence, efficiency is increased if it is more likely that EM chooses to exert effort. This is the case if the LHS of EM's incentive compatibility constraint in equation 7 increases. The benchmark case is where IB has no private information (either Q , the probability of a high signal, or both \mathbf{b}_h and \mathbf{b}_l , the

measures of informativeness of the signals, are zero) or no bargaining power (\mathbf{a} is zero). In that case the LHS of equation 7 becomes

$$(P_h - P_l) X . \quad (9a)$$

If, on the other hand, IB has bargaining power and private information, then the LHS of equation 7 is smaller than 9a if and only if $\mathbf{b}_h > \mathbf{b}_l$.

QED

Proof of Lemma 3: Unfortunately, there are now two cases.

Case 1, $g > g^*$: This means that second period lending is risk free. To see this, note that if first period debt d is such that

$$d < d^* \equiv (C - I) \quad (10a)$$

then at $t = 1$ OM could be promised a riskfree cashflow in return for providing I . In that case, EM has the outside option to raise funds at minimum (i.e. break-even) cost from OM and hence IB has no bargaining power. In order for IB to now break even, it must be that

$$i = d + \mathbf{g}P_h(C + X - I - d) < d^* + \mathbf{g}P_h(C + X - I - d^*) = (C - I) + \mathbf{g}P_h X \quad (11a)$$

or indeed

$$\mathbf{g} > \frac{(I + i - C)}{P_h X} = \mathbf{g}^* . \quad (12a)$$

Hence, for $\mathbf{g} > \mathbf{g}^*$, second period debt is indeed riskfree. Consequently, there is no effect from the adverse selection problem. Hence, the incentive compatibility constraint with outside equity reduce to the standard moral hazard one,

$$(1 - \mathbf{g})(P_h - P_l) X \geq \frac{k}{(1 - p)} . \quad (13a)$$

Case ii, $g < g^*$: This means that second period debt cannot be riskfree, and the analysis is similar to that of Lemma 2.

EM's utility from shirking is now given by

$$\widehat{U}_s = p(C + P_h X - I - i) + (1 - p)(C + P_h X - I - \widehat{Z}_l), \quad (14a)$$

where \widehat{Z}_l is the total return to IB if EM shirks. This is given by

$$\widehat{Z}_l = \widehat{d} + \Theta \mathbf{a} \widehat{R}_l + \mathbf{g}P_l(X + C - \widehat{d} - \widehat{D}_l^*), \quad (15a)$$

where \widehat{R}_l are the rents from the hold up due to the adverse selection problem. \widehat{D}_l^* is the break even face value of debt, in the absence of adverse selection, issued at $t = 1$ when the low

quality project is available,³¹ which is determined in

$$I = P_l \widehat{D}_l^* + (1 - P_l)(C - \widehat{d}) = P_l \left(\widehat{D}_l^* - (C - \widehat{d}) \right) + (C - \widehat{d}). \quad (16a)$$

Since we still have that OM will want to break even with a bad firm, we have

$$I = (1 - Q_l) \Pi_l \widehat{D}_l + [1 - (1 - Q_l) \Pi_l] [C - \widehat{d}] = (1 - Q_l) \Pi_l \left(\widehat{D}_l - [C - \widehat{d}] \right) + (C - \widehat{d}) \quad (17a)$$

Combining equations 16a and 17a to rewrite equation 15a yields

$$\widehat{Z}_l = \widehat{d} + \Theta \mathbf{a} \widehat{R}_l + \mathbf{g} P_l X + \mathbf{g} (1 - Q_l) \Pi_l (C - \widehat{d} - \widehat{D}_l) \quad (18a)$$

Similar to the debt-only case, the rents \widehat{R}_l are given by

$$\widehat{R}_l = -I + [\Pi_l + Q_l (1 - \Pi_l)] \widehat{D}_l + [1 - \{\Pi_l + Q_l (1 - \Pi_l)\}] [C - \widehat{d}] = Q_l \left(\widehat{D}_l - [C - \widehat{d}] \right) \quad (19a)$$

From equation 17a we can solve for $\left(\widehat{D}_l - [C - \widehat{d}] \right)$ as

$$\left(\widehat{D}_l - [C - \widehat{d}] \right) = (I + \widehat{d} - C) \frac{1}{(1 - Q_l) \Pi_l} \quad (20a)$$

Combining equations 19a and 20a into 18a gives

$$\widehat{Z}_l = (I - C)(\Theta \mathbf{a} \mathbf{b}_l - \mathbf{g}) + \mathbf{g} P_l X + \widehat{d} (\Theta \mathbf{a} \mathbf{b}_l + (1 - \mathbf{g})) \quad (21a)$$

Of course, \widehat{d} was set when IB expected EM to take the equilibrium action - effort. Hence, they are determined analogously to d in the case without equity. A parallel argument to that case (see equation 5) leads to

$$\widehat{d} = i - (I + i - C) \frac{\mathbf{a} \Theta \mathbf{b}_h - \mathbf{g}}{\mathbf{a} \Theta \mathbf{b}_h + (1 - \mathbf{g})} - \mathbf{g} P_h X \frac{1}{\mathbf{a} \Theta \mathbf{b}_h + (1 - \mathbf{g})} \quad (22a)$$

Using 22a in 21a we get that

$$\widehat{Z}_l = i - \mathbf{g} (P_h - P_l) X + \mathbf{g} P_h X \frac{\mathbf{a} \Theta (\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a} \Theta \mathbf{b}_h + (1 - \mathbf{g})} - (I + i - C) \left[\frac{\mathbf{a} \Theta (\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a} \Theta \mathbf{b}_h + (1 - \mathbf{g})} \right] \quad (23a)$$

Substituting 23a into 14a results in

$$\begin{aligned} \widehat{U}_s = & (C + P_h X - I - i) - (1 - p)(1 - \mathbf{g})(P_h - P_l) X \\ & + (1 - p) \left((I + i - C - \mathbf{g} P_h X) \left[\frac{\mathbf{a} \Theta (\mathbf{b}_h - \mathbf{b}_l)}{\mathbf{a} \Theta \mathbf{b}_h + (1 - \mathbf{g})} \right] \right) \end{aligned} \quad (24a)$$

Since, in equilibrium IB and OM just break even, we still have that

³¹ This specification allow the interpretation of \widehat{R}_l as the 'rents due to the hold up problem'.

$$\widehat{U}_e = U_e = (C + P_h X - I - i) - k \quad (25a)$$

Hence, we obtain that EM will exert effort iff $\widehat{U}_e \geq \widehat{U}_s$, or

$$(1-g)(P_h - P_l)X - (I + i - C - gP_h X) \left[\frac{a\Theta(\mathbf{b}_h - \mathbf{b}_l)}{a\Theta\mathbf{b}_h + (1-g)} \right] > \frac{k}{(1-p)} \quad (26a)$$

QED

Proof of Proposition 2: Equity participation improves efficiency, if EM has no incentives to exert effort under a debt-only capital structure (equation 7 in lemma 2 is not satisfied), but EM does exert effort under a debt-and-equity capital structure (equations 9 or 10 in lemma 3 are satisfied). This happens (for some k) if the LHS of equation 7 is less than the LHS of equations 9 or 10. But this difference is exactly \mathbf{D} in the proposition.

QED

Proof of Proposition 3: It is easy to see that the expression in equation 13 can never be positive if \mathbf{a} or \mathbf{Q} are zero, as in that case the left hand side reduces to $-g(P_h - P_l)X < 0$. Similarly, $\mathbf{b}_h = \mathbf{b}_l = 0$ and $\mathbf{b}_h \neq \mathbf{b}_l$ lead to the same result.

QED

Proof of Proposition 4: The benefits of equity participation are given by \mathbf{D} in proposition 3. As equation 14 shows, these benefits are declining for equity levels greater than \mathbf{g}^* . Before that, \mathbf{D} is convex:

The first derivative of \mathbf{D} with respect to \mathbf{g} is given by

$$\frac{d\Delta}{d\mathbf{g}} = -(P_h - P_l)X + a\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{(C + P_h X - I - i) + a\Theta\mathbf{b}_h P_h X}{(a\Theta\mathbf{b}_h + (1-g))^2}. \quad (27a)$$

The second derivative is given by

$$\frac{d^2\Delta}{d\mathbf{g}^2} = 2a\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{(C + P_h X - I - i) + a\Theta\mathbf{b}_h P_h X}{(a\Theta\mathbf{b}_h + (1-g))^3} > 0. \quad (28a)$$

Consequently, the maximum of \mathbf{D} can only be at $\mathbf{g} = 0$ or at $\mathbf{g} = \mathbf{g}^*$. Also, at $\mathbf{g} = 0$, we obviously have that $\mathbf{D} = 0$. The proposition then comes down to establishing under what conditions \mathbf{D} at $\mathbf{g} = \mathbf{g}^*$ is larger than zero:

$$\Delta(\mathbf{g} = \mathbf{g}^*) = -\mathbf{g}^*(P_h - P_l)X + \mathbf{g}^* a\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{[a\Theta\mathbf{b}_h + 1]P_h X - (I + i - C)}{[a\Theta\mathbf{b}_h + 1][a\Theta\mathbf{b}_h + (1-\mathbf{g}^*)]}. \quad (29a)$$

Substituting for \mathbf{g}^* leads to

$$\begin{aligned}\Delta(\mathbf{g} = \mathbf{g}^*) &= (I+i-C) \left[\frac{P_l}{P_h} - 1 + \mathbf{a}\Theta(\mathbf{b}_h - \mathbf{b}_l) \frac{[\mathbf{a}\Theta\mathbf{b}_h + 1]P_h X - (I+i-C)}{[\mathbf{a}\Theta\mathbf{b}_h + 1][(\mathbf{a}\Theta\mathbf{b}_h + 1)P_h X - (I+i-C)]} \right] \\ &= (I+i-C) \left[\frac{P_l}{P_h} - \frac{\mathbf{a}\Theta\mathbf{b}_l + 1}{\mathbf{a}\Theta\mathbf{b}_h + 1} \right]\end{aligned}\quad (30a)$$

This expression is less than zero when the condition in part (i) of the proposition holds, which implies that $\mathbf{g} = 0$ is optimal. In the converse case (part (ii) of the proposition), $\mathbf{g} = \mathbf{g}^*$ is optimal.

Finally, part (iii) follows from the fact that sometimes \mathbf{D} is initially decreasing (i.e. equity is initially inefficient), but at the maximum (\mathbf{g}^*) \mathbf{D} is larger than zero. This initial decrease happens if the first derivative of \mathbf{D} in equation 27a is negative at $\mathbf{g} = 0$. This is exactly the condition in the proposition.

QED

Proof of Proposition 5: From proposition 4 we know that IB will hold either zero equity or \mathbf{g}^* . Hence, it suffices to examine \mathbf{D} at \mathbf{g}^* , which is given in equation 30a.

Part (i) - the derivative of \mathbf{D} at \mathbf{g}^* with respect to α is given by

$$\left. \frac{d\Delta}{d\alpha} \right|_{\mathbf{g}=\mathbf{g}^*} = \frac{\Theta(\mathbf{b}_h - \mathbf{b}_l)(I+i-C)}{(\mathbf{a}\Theta\mathbf{b}_h + 1)^2} > 0 \quad (31a)$$

Part (ii) - the derivative of \mathbf{D} at \mathbf{g}^* with respect to $(I+i-C)$ is given by

$$\left. \frac{d\Delta}{d(I+i-C)} \right|_{\mathbf{g}=\mathbf{g}^*} = \left[\frac{P_l}{P_h} - \frac{\mathbf{a}\Theta\mathbf{b}_l + 1}{\mathbf{a}\Theta\mathbf{b}_h + 1} \right]. \quad (32a)$$

From proposition 4 we know that the expression in equation 32a is positive whenever IB holds any equity at all.

Part (iii) - the derivative of \mathbf{D} at \mathbf{g}^* with respect to \mathbf{b}_h is given by

$$\left. \frac{d\Delta}{d\mathbf{b}_h} \right|_{\mathbf{g}=\mathbf{g}^*} = \frac{\mathbf{a}\Theta(\mathbf{a}\Theta\mathbf{b}_l + 1)(I+i-C)}{(\mathbf{a}\Theta\mathbf{b}_h + 1)^2} > 0 \quad (33a)$$

QED

Proof of Proposition 6: Immediate from the definition of γ^* (equation 11)

QED.

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