Quantum Model for Conjoint Recognition
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Abstract
In a conjoint memory recognition task, a person is presented a list of target items to remember. Afterwards, a test probe is presented which is sampled from one of three mutually exclusive and exhaustive categories: one is a target from the set of previously presented targets; a second is a non target but meaningfully related to a target; and a third is a non target and unrelated to any target. The episodic overestimate effect refers to the fact that the probability of accepting a probe when asked if it is a target plus the probability of accepting a probe when asked if it is a related non target is greater than the probability of accepting a probe when asked if it is either a target or a non related target. Logically these two probabilities should be identical. Previously these results were explained by a dual process theory. This article presents an alternative quantum memory recognition model for this effect that addresses some problematic issues that arise with the dual process explanation.

Quantum Models of Cognition
Beginning with seminal ideas by Aerts and Aerts (1994), Atmanspacher et al. (2002) and Khrennikov (1999), a growing number of scientists have begun to explore the use of quantum formalisms to understand what appear to be paradoxical findings in various areas of human cognition and decision making. A variety of quantum theoretical applications have appeared including applications to conceptual judgments, human inference, decision making behavior, and human memory (see Bruza, Busemeyer, & Gabora, 2009).

The purpose of this paper is to present a new application of quantum theory to a puzzling phenomenon observed in human memory recognition called the episodic over-distribution effect, which was discovered by Brainerd and Reyna (2008). First we describe the phenomena and explain why it is a puzzle, and then we present a quantum solution to the puzzle.
Figure 1a and 1b. Episodic over distribution (EOD) effects from trained targets and related probes.

Logically, a probe $x$ comes from $T$ or $G$ but not both, implying that $\Pr(\text{VorG}|T) = \Pr(V|T) + \Pr(G|T)$. The difference, $EOD(T) = \Pr(V|T)+\Pr(G|T) – \Pr(\text{VorG}|T)$ is an episodic over distribution effect. Figure 1a shows the observed EOD effects obtained from 116 different experimental conditions reported by Brainerd and Reyna (2008). All but 12 (10%) of the 116 studies produced this effect, and the mean value of the EOD equals .18 (after removing the bias effect described below).

Next consider the memory test trials that employ a test probe belonging to the related set $R$. If the verbatim question is asked, then the probability of accepting the probe is formally defined by $\Pr(V|R)$; but in the original article it is denoted $p(RV)$. If the gist question is asked, then the probability of accepting the probe is formally defined by $\Pr(G|R)$; but in the original article it is denoted $p(RG)$. Finally, if the verbatim or gist question is asked, then the probability of accepting the probe is formally defined by $\Pr(V\text{orG}|R)$; but in the original article this is denoted $p(RDV)$. Brainerd and Reyna (2008) didn’t report these probabilities, but Brainerd et al. (1999) reported statistical tests which indicated that $\Pr(VI|U) < \Pr(\text{GI}|U) < \Pr(\text{VG}|U)$. Furthermore, four out of the six studies exhibited episodic over distribution (EOD) effects, and the mean across the six studies reported in Table 6 of Brainerd et al. (1999) shows that $EOD(U) = \Pr(VI|U)+\Pr(\text{GI}|U) – \Pr(\text{VG}|U) = .04$ (which is comparable to $EOD(T) = .05$ found with the target probes in these six studies). The EOD effect in this case is even smaller than that for the related non target probe, but it is still present.

Another important finding was that the probabilities for the verbatim and gist instructions often summed to a value greater than unity. The sum $\Pr(V|T)+\Pr(G|T)$ exceeded one for 20 of the 116 (17%) data sets involving true targets, and the sum $\Pr(V|R)+\Pr(G|R)$ exceeded one for 12 of the 165 (7%) related non targets.

**Memory Strength Model**

Not only are these EOD effects inconsistent with a logical analysis of the task, they are also inconsistent with a unidimensional memory strength model of recognition memory (e.g. Wixted, 2007). According to the memory strength model, each test probe produces some feeling of familiarity that has a strength value denoted by a random variable $f$. If the strength of the familiarity is below a cutoff $c_R$, then it is categorized as an unrelated probe; if it falls above $c_R$ but falls below a higher cutoff $c_T$ then it is categorized as a related non target; and if it falls above the upper cutoff $c_T$, then it is categorized as a true target. This model satisfies the mutually exclusive principle because the strength must fall into one of three intervals ($f < c_R$, $c_R \leq f \leq c_T$, $f > c_T$) and so it cannot produce episodic over estimation either. For a probe of type $x \in \{T,R,U\}$, this model predicts that

$$\Pr(\text{V|x}) = \Pr(f_x > c_T),$$

$$\Pr(\text{G|x}) = \Pr(c_R \leq f_x \leq c_T) \text{ and}$$

$$\Pr(\text{VG|x}) = \Pr(f_x \geq c_T) = \Pr(c_R \leq f_x \leq c_T) + \Pr(f_x > c_T).$$

The latter always predicts that $EOD(x) = 0$.  

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**Classic Probability Model**

It seems reasonable to interpret the probabilities elicited by the 'verbatim or gist' instruction as the probability of the union of the two events V = 'categorize as verbatim', G = 'categorize as gist.' It also seems reasonable to allow the possibility that the participants failed to follow the mutually exclusive instructions, so that they were willing to implicitly categorize a probe as either verbatim or gist item or both. This implies the classic probability model

$$\Pr(V \cup G|x) = \Pr(V|x) + \Pr(G|x) - \Pr(V \cap G|x) \quad (2)$$

Under this assumption, episodic over distribution provides a measure of the joint probability that a probe is categorized as both verbatim and gist:

$$\Pr(V \cap G|x) = \Pr(V|x) + \Pr(G|x) - \Pr(V \cup G|x) = \text{EOD}(x).$$

This model also allows \(\Pr(G|x) + \Pr(V|x) > 1\), which is occasionally found.

This simple explanation can be empirically tested by obtaining a second estimate of the joint probability from the data. The participants were informed that the verbatim and gist properties were mutually exclusive, \(\Pr(V \cap G|x) = 0\) so it is safe to assume that \(\Pr(G \cap V|x) = \Pr(G|x) \Pr(V|x)\) which is the bound produced by assuming statistical independence. Given the instructions, the joint probability should fall far below the independence bound. Therefore, if this simple explanation is correct, then the EOD effect should certainly be bounded by the product of the individual probabilities, \(\text{EOD}(x) < \Pr(G|x) \Pr(V|x)\).

A more striking violation of the classic probability model can be seen in Figure 1. If the EOD effect equals the joint probability that a probe \(x\) is verbatim and gist, and \(\Pr(V \cup G|x) = \Pr(V \cap G|x)\) equals the disjunctive probability that the probe \(x\) is verbatim or gist, then according to classic probability theory, the probability of the intersection, measured by the EOD, must be less than the probability of the union, measured by \(\Pr(V \cap G|x)\). Figure 1a shows that when \(p(TVG) < .20\), the EOD effect for targets in \(T\) sometimes exceeds \(p(TVG)\). This finding also appears again in Figure 1b, which shows that the EOD effect for related non targets in \(R\) often exceeds \(p(RDVG)\) for \(p(RDVG) < .20\). Therefore, contrary to the assumptions of a classic probability model, the EOD effect exceeds \(\Pr(V \cap G|x)\) when the latter is small.

**Dual Process Model.**

Jacoby (1991) proposed a dual process model of memory recognition which was later extended by Brainerd et al. (1999), and the latter is presented here. This is a Markov model that posits three states. First we focus on test probes that are true targets from \(T\). In this case, the person may correctly recollect the target (denoted by state \(S\)) with probability \(p_S\), or fail to recollect but instead enter a familiar state \(F\) with probability \(p_F\), or the person may not recollect and not be familiar. According to this model, the probabilities of responses to questions for a probe that is a true target item from \(T\) are given by

$$\Pr(V|T) = p_S + (1 - p_S)p_F$$
$$\Pr(G|T) = (1 - p_S)p_F$$
$$\Pr(V \cup G|T) = p_S + (1 - p_S)p_F$$

Figure 2 shows that this independence bound is systematically violated. The data in Figure 2 are from Table 2 of Brainerd et al. (1999) for the related non targets, which happen to produce the largest EOD effects in this set of six studies. All points lying above the unit line are violations of the independence bound. Large violations tend to occur when the product is large.

**Figure 2:** EOD plotted as a function of product of probabilities for related non targets presented in Brainerd et al. (1999) Table 2

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incorrectly predicts that $\Pr(VorG|T) = \Pr(V|T)$ when in fact $\Pr(VorG|T) > \Pr(V|T)$ (see Table 2, Brainerd et al., 1999).

To overcome the problem mentioned above, Buchner et al. (1998) added bias parameters which are estimated from the trials when the probe is taken from the unrelated set $U$. The bias parameter for the verbatim instruction equals $\beta_V = \Pr(V|U)$, the bias for the gist instruction equals $\beta_G = \Pr(G|U)$, and the bias for the $V$ or $G$ instruction equals $\beta_{VG} = \Pr(VorG|U)$.

These biases are added to Equation 3a and they apply to the case when the probe is not recollected and is unfamiliar:

$$\Pr(V|T) = p_s + (1 - p_s)p_r + (1 - p_s)(1 - p_r)\beta_V \quad (3b)$$

$$\Pr(G|T) = (1 - p_s)p_r + (1 - p_s)(1 - p_r)\beta_G$$

$$\Pr(VorG|T) = p_s + (1 - p_s)p_r + (1 - p_s)(1 - p_r)\beta_{VG}.$$

The bias corrected probabilities are then defined by

$$p'(TV) = \Pr(V|T) - (1 - p_s)(1 - p_r)\beta_V,$$

$$p'(TG) = \Pr(G|T) - (1 - p_s)(1 - p_r)\beta_G,$$

$$p'(TVG) = \Pr(VorG|T) - (1 - p_s)(1 - p_r)\beta_{VG}.$$

This extended model predicts that

$$p'(TV) = p'(TVG) \text{ and } EOD'(T) = p'(TG) + p'(TV) - p'(TVG) = (1 - p_s)p_r = p'(TG).$$

Brainerd and Reyna (2008) tested this prediction by regressing the bias corrected EOD on the estimate of $(1 - p_s)p_r$. This produced a regression coefficient equal to .81 ($R^2 = .36$), which was not significantly different from the predicted coefficient equal to unity. It is important to note, however, that some studies produced large EOD effects even when $(1 - p_s)p_r$ was estimated to be zero (see Figure 5 of Brainerd & Reyna, 2008).

In an analogous manner, the dual process model posits that a related non target probe may generate one of three states in the person – a state $S$ in which the probe is correctly categorized as a related non target with probability $q_s$, or fail to correctly categorize the probe as a related probe but instead enter a familiar state $F$ with probability $q_f$, or the person may not correctly categorize the probe as a related non target and not be familiar with it either. The probabilities to questions for a probe that is a related non target are given by

$$\Pr(GIR) = q_s + (1 - q_s)q_f + (1 - q_s)(1 - q_f)\beta_G \quad (3c)$$

$$\Pr(VIR) = (1 - q_s)q_f + (1 - q_s)(1 - q_f)\beta_V$$

$$\Pr(VorGIR) = q_s + (1 - q_s)q_f + (1 - q_s)(1 - q_f)\beta_{VG}.$$

The same acceptance rates for non target and unrelated probes taken from set $U$ are again used to estimate the extra bias terms.

Although the dual process model provides a good description of the findings from the conjoint memory recognition paradigm, several important issues remain unresolved. First, the model provides no explanation for the EOD effect that remains even for test probes taken from the unrelated non target set $U$. In this case, there is no possibility for recollection or familiarity, and some unknown bias must be evoked to explain the EOD effect. Second, large EOD effects occur in some studies in which the model predicts zero effects. The latter is a dramatic failure of the model.

**Quantum Recognition Model.**

The episodic over distribution effect can be interpreted as a type of interference effect that naturally arises in quantum probability systems (Khrennikov, 2010). Here we apply a quantum model that was previously developed to explain human probability judgment errors (Busemeyer, Pothos, Franco, 2009). Other quantum models of human memory have been also been proposed by Bruza et al. (2009) and Franco (2009).

The first postulate of the quantum model is that a person’s memory is represented by a state vector, denoted $|\psi\rangle$, within a large but finite dimensional vector space $H$. Each dimension (basis vector) of the vector space represents a feature pattern that could be used to describe a past episode or event. A feature pattern is interpreted as a combination of many individual features. For example, experiencing the word ‘poodle’ on ‘list 1’ during rehearsal would represent a simple combination of two features (poodle, list 1). More complex combinations can be used to form a feature pattern, and so the dimensionality of the vector space can be large, but the finite dimension of the vector space puts limits on the capacity for describing past events or episodes. The use of feature vectors to represent memory is consistent with other memory recognition models (e.g., Hintzman, 1988; Shiffrin & Steyvers, 1997).

The state vector $|\psi\rangle$ represents beliefs about past events or episodes. Each coordinate value assigned by $|\psi\rangle$ to a particular dimension (basis vector) is called the probability amplitude (a complex number) for that feature pattern. The squared magnitude of the amplitude equals the probability that a feature pattern describing an event will be retrieved by the person. The state vector $|\psi\rangle$ has unit length so that the sum of the squared magnitudes equals one.

The second postulate is that an answer to a question about a test probe is represented by a subspace $S$ of the vector space $H$. Intuitively, a subspace is described by a subset of the complete set of feature patterns that are used to describe the entire vector space. Technically, a subspace is spanned by a subset of a complete set of orthogonal basis vectors that span the entire vector space.

When the person is given the verbatim instruction, the answer yes to the probe from set $T$ is represented by a subspace denoted $S_T$, and the answer no is represented by the orthogonal complement $S_T^\perp$. Alternatively, when the person is given the gist instruction, the answer yes to a probe from set $T$ is represented by a different subspace $S_{TG}$, and the orthogonal complement $S_{TG}^\perp$ is used for no. When given a test probe from the related non target set $R$ under the gist instruction, then the subspace for yes is $S_{RG}$ and the
subspace for no is $S_{U}$. When given a test probe from the unrelated non target set $U$ under the verbatim instruction, then the subspace for yes is denoted $S_{Y}$, and the subspace for no is $S_{N}$. Subspaces for the remaining questions are denoted using a similar notation.

Corresponding to each subspace $S$ is a projector, denoted $M_{S}$, which is an idempotent linear operator ($M_{S}M_{S} = M_{S}$) that maps an arbitrary state vector $|\psi\rangle$ onto the subspace $S$ to produce the projection $M_{S}|\psi\rangle$. Intuitively, the projector $M_{S}$ serves as a retrieval cue, and the projection $M_{S}|\psi\rangle$ is the retrieved image. The projector for the no answer to this question is the projector for the orthogonal complement, $(I - M_{S})$, where $I$ is the identity operator.

Our third postulate is that the probability of answering yes to a question equals the squared length of the orthogonal projection of the state $|\psi\rangle$ onto the subspace $S$ representing the answer yes to the question. Formally, the probability of answering yes equals $||M_{S}|\psi\rangle||^{2}$. Note that this probability can also be expressed as an inner product $||M_{S}|\psi\rangle||^{2} = \langle \psi | M_{S} | \psi \rangle$. For example, the probability of saying yes to a probe from set $x \in \{T,R,U\}$ under the verbatim instruction equals $||M_{S}|\psi\rangle||^{2}$, whereas the probability of yes for the same probe under the gist instruction equals $||M_{S}|\psi\rangle||^{2}$. The probability of saying no to a probe from set $x$ under the verbatim instruction equals $||I - M_{S}||\psi\rangle||^{2}$. Our fourth postulate concerns the revision of the state following an answer to a question. Suppose the person decides that the answer to the gist question about a probe from set $x \in \{T,R,U\}$ is yes. Then the initial state $|\psi\rangle$ undergoes a revision into a new state according to Lüder’s rule:

$$|\psi_{rev}\rangle = M_{S}|\psi\rangle / ||M_{S}|\psi\rangle||,$$

which is the normalized projection onto the yes answer to the gist question. If a later question is asked about the verbatim property for this same probe, then the probability of saying yes to the verbatim question (conditioned on saying yes to the gist question) equals $||M_{S}M_{S}|\psi\rangle||^{2}$.

On the basis of the fourth postulate we can derive formulas for the probabilities of sequences of events. If the gist question is asked first and the verbatim question is asked second, then the probability of saying yes to the gist question and then saying yes to the verbatim question equals (see Nielsen & Chuang, 2000, p. 86)

$$Pr(G \text{ and then } V(x)) = \frac{||M_{S}|\psi\rangle||^{2} - ||M_{S}|\psi_{rev}\rangle||^{2}}{||M_{S}|\psi\rangle||^{2}}.$$  

If these two questions are mutually exclusive, then $M_{S}M_{S} = M_{S}M_{V} = 0$.

If the gist question is asked first and the verbatim question is asked second in a sequence, then there are three mutually exclusive ways to satisfy yes to gist or then yes to verbatim: (a) yes to gist followed by yes to verbatim with probability $||M_{S}M_{S}|\psi\rangle||^{2}$, (b) yes to gist and then no to verbatim with probability $||I - M_{S}||M_{S}|\psi\rangle||^{2}$, (c) no to gist and then yes to verbatim with probability $||M_{S}(I - M_{S})|\psi\rangle||^{2}$. Therefore the probability of first saying yes to gist or later saying yes to verbatim equals the sum of these three ways that the disjunction can occur:

$$||M_{S}M_{S}|\psi\rangle||^{2} + ||I - M_{S}||M_{S}|\psi\rangle||^{2} + ||M_{S}(I - M_{S})|\psi\rangle||^{2}.$$

Note that

$$||M_{S}M_{S}|\psi\rangle||^{2} = ||M_{S}|\psi\rangle||^{2} = ||M_{S} + (I - M_{S})||M_{S}|\psi\rangle||^{2} = ||M_{S}V|M_{S}G|\psi\rangle||^{2} + ||(I - M_{S})V|M_{S}G|\psi\rangle||^{2},$$

and so we arrive at the formula

$$Pr(G \text{ or then } V(x)) = ||M_{S}|\psi\rangle||^{2} + ||M_{S}(I - M_{S})|\psi\rangle||^{2}.$$  

We can expand the second term in Equation 4b into a form that is more useful for later comparisons:

$$||M_{S}V|M_{S}G|\psi\rangle||^{2} = \langle \psi | (I - M_{S})M_{S}M_{S}V|M_{S}G|\psi \rangle = \langle \psi | (I - M_{S})M_{S}M_{S}V|M_{S}\psi \rangle + \langle \psi | M_{S}M_{S}V|M_{S}\psi \rangle - \langle \psi | M_{S}M_{S}V|M_{S}\psi \rangle - \langle \psi | M_{S}M_{S}V|M_{S}\psi \rangle = ||M_{S}V|M_{S}G|\psi\rangle||^{2} + ||M_{S}V|M_{S}G|\psi\rangle||^{2} - \langle \psi | M_{S}M_{S}V|M_{S}\psi \rangle - \langle \psi | M_{S}M_{S}|\psi \rangle. \quad (4c)$$

Now we can re-express

$$Pr(G \text{ or then } V(x)) = ||M_{S}|\psi\rangle||^{2} + ||M_{S}V|M_{S}G|\psi\rangle||^{2} + ||M_{S}V|M_{S}G|\psi\rangle||^{2} - \langle \psi | M_{S}M_{S}V|M_{S}\psi \rangle - \langle \psi | M_{S}M_{S}|\psi \rangle. \quad (4c)$$

Our fifth postulate concerns the commutative relations between the projectors for the verbatim and gist instructions. In general, the feature patterns used to describe the verbatim instruction are different than those used to describe the gist instruction. The verbatim instruction may orient the person towards the use of detailed surface features such as perceptual features, while the gist instruction may orient the person towards the use of deeper semantic features. Technically, the set of orthogonal basis vectors used to define the verbatim subspace is different than the set of orthogonal basis vectors used to define the gist subspace.

There are three fundamentally different ways to represent the relations between the projectors. The first way is to assume that the subspaces are orthogonal. Intuitively, the feature patterns used to describe the verbatim instruction are orthogonal to all the basis vectors used to describe the gist instruction. More succinctly, this implies $M_{S}M_{G} = 0$. This is in fact how the sets of probes, $T$ and $R$, were formed. However, this representation predicts no EOD effects as in Equation 1.

The second way is to assume that the projectors are compatible. Intuitively, this means that a common set of feature patterns are used to describe both the verbatim and gist instructions. In this case, the order that questions are asked does not make any difference. Technically, the
projectors are compatible if $M_x^c M_G = M_G M_x^c$. However, if the projectors are compatible, then
\[
\langle \psi | M_x^c M_G | \psi \rangle = \langle \psi | M_G M_x^c | \psi \rangle
\]
\[
= \langle \psi | M_x^c M_G M_G M_x^c | \psi \rangle = \langle \psi | M_G M_x^c M_G M_x^c | \psi \rangle
\]
\[
= \langle \psi | M_x^c M_G | \psi \rangle^2.
\]
Thus when the projectors are compatible we obtain
\[
Pr(G \text{ and then } V|x) = \langle \psi | M_x^c M_G | \psi \rangle \langle \psi | M_G M_x^c | \psi \rangle = Pr(V \text{ and then } G|x).
\]
Furthermore, Equation 4b reduces to the same form as the classic probability model:
\[
Pr(G \text{ or then } V|x) = \langle \psi | M_x^c M_G | \psi \rangle \langle \psi | M_G M_x^c | \psi \rangle = Pr(V \text{ or then } G|x).
\]
Given the instructions stating that the verbal and gist probes are mutually exclusive, we expect that $\langle \psi | M_x^c M_G | \psi \rangle^2 < \langle \psi | M_x^c M_G | \psi \rangle^2 - \langle \psi | M_G M_x^c | \psi \rangle^2$. Therefore this assumption once again predicts that the EOD effect is bounded by the product of individual probabilities. But recall that this bound is violated as shown in Figure 2 of this paper.

The third way is to assume that the projectors are incompatible. Intuitively, people do not use a common set of feature patterns to describe verbatim and gist instructions, but these features are not mutually exclusive either. In this case, the subspace for the verbatim instruction does not share the same basis vectors as the subspace for the gist instruction, but they are not orthogonal either. This implies that $M_x^c M_G \neq 0$, and moreover, $M_x^c M_G \neq M_G M_x^c$, so that these two projectors do not commute. Thus the order of application does matter, which is consistent with the finding that the order that information is retrieved from memory is important and can affect the final probabilities of answers (Storm, Bjork, & Bjork, 2007). In particular, for the conjoint recognition paradigm, the quantum model predicts that the probability of saying yes to the verbatim question followed by yes to the gist question equals $\langle \psi | M_x^c M_G | \psi \rangle^2$, but the probability of the reverse order equals $\langle \psi | M_G M_x^c | \psi \rangle^2$. These two probability sequences will not necessarily be equal because the two projectors do not commute in general. As we will later show, according to quantum theory, this is the key reason for the episodic overestimation effect.

**Quantum Explanation for the Episodic Over Distribution Effect.**

The key to understanding the episodic overestimation effect lies in the theoretical analysis of the ‘verbatim or gist’ instruction. According to the quantum model, this disjunctive question entails the use of two projectors. If the projectors for these two questions are incompatible, then they must be applied sequentially. Gist is processed more quickly (Brainerd et al., 1999), and so we assume the person first queries the gist and if this fails to produce acceptance, then the person queries the verbatim property. Using Equation 4c, we obtain the following prediction
\[
\text{EOD}(x) = Pr(V|x) + Pr(G|x) - Pr(G \text{ or then } V|x)
\]
\[
= \langle \psi | M_x^c M_G | \psi \rangle^2 + \langle \psi | M_G M_x^c | \psi \rangle^2 - \langle \psi | M_G M_x^c | \psi \rangle^2
\]
\[
= \langle \psi | (I-M_G) M_x^c M_G | \psi \rangle^2 + \langle \psi | (I-M_G) M_G M_x^c | \psi \rangle^2.
\]
The term $\langle \psi | M_G M_x^c | \psi \rangle^2$ is the inner product between the projection $M_G | \psi \rangle$ on to the verbatim subspace and the projection $M_x^c | \psi \rangle$ on to the gist subspace. The other term $\langle \psi | (I-M_G) M_x^c M_G | \psi \rangle^2$ is the complex conjugate $\langle \psi | M_G M_x^c | \psi \rangle^2$.

Recall that $\langle \psi | M_x^c M_G M_G M_x^c | \psi \rangle = \langle \psi | M_G M_x^c M_G M_x^c | \psi \rangle^2$.

Thus we re-express the quantum prediction as
\[
\text{EOD}(x) = \langle \psi | (I-M_G) M_x^c M_G | \psi \rangle^2 + \langle \psi | (I-M_G) M_G M_x^c | \psi \rangle^2.
\]

with the interference defined as
\[
I_{\text{G}}(x) = \langle \psi | (I-M_G) M_x^c M_G | \psi \rangle + \langle \psi | (I-M_G) M_G M_x^c | \psi \rangle^2.
\]

Recall from analysis that any complex number $z$ can be written in terms of product of its magnitude and phase, $z = |z|e^{i\phi} = |z|cos(\phi) + isin(\phi)$, and the conjugate equals $z^* = |z|e^{-i\phi} = |z|cos(\phi) - isin(\phi)$, so that $z + z^* = |z|cos(\phi)$. Therefore we can rewrite the interference as follows:
\[
I_{\text{G}}(x) = 2\langle \psi | (I-M_G) M_x^c M_G | \psi \rangle cos(\phi).
\]

To explain the results, we require the interference to be positive. Intuitively, this means that there must be a positive correlation between the ‘gist’ and then verbal’ projection and the ‘not gist and then verbal’ projection.

The violation of the independence bound shown in Figure 2 requires that
Consider, for example, the results that produced the largest violation of the independence bound reported by Brainerd et al. (1999). This occurred in Experiment 3 with the unrelated non target probe under the no prime condition reported in Table 2 of Brainerd et al. (1999). The results reported for this condition are as follows: Pr(V|R) = .52, Pr(G|R) = .68, and Pr(VorG|R) = .70. For these results we obtain Pr(V|IR)Pr(G|IR) = .3536 < EOD(R) = .52 + .68 − .70 = .50. The excess over the independence bound equals .50 − .3536 = .1464. To explain these results we need to satisfy the constraint \(\|M_{V,G}\|\psi\|^2 < .3536\) and therefore \(I_{V,G} > .1464\). So that \(\|M_{V,G}\|\psi\|^2 + I_{V,G} = EOD(R) = .50\).

In order to explain those occasions when EOD(x) exceeds P(VorG|x) as found in Figure 1 of this paper, we must impose a stronger constraint:

\[I_{V,G}(x) > \frac{(\|M_{V}\|\psi\|^2 + \|M_{G}\|\psi\|^2)}{2} - \|M_{V,G}\|\psi\|^2.\]

This implies that

\[2(I_{V,G}(x) + \|M_{V,G}\|\psi\|^2) > \|M_{V}\|\psi\|^2 + \|M_{G}\|\psi\|^2\]

which in turn implies

\[I_{V,G}(x) > \|M_{V}\|\psi\|^2 + \|M_{G}\|\psi\|^2 - (I_{V,G}(x) + \|M_{V,G}\|\psi\|^2)\]

which finally implies

\[EOD(x) > P(VorG|x).\]

For example, if \(\|M_{V}\|\psi\|^2 + \|M_{G}\|\psi\|^2 = 1.0\) then \(I_{V,G} > .50 - \|M_{V,G}\|\psi\|^2\) will produce such an effect.

**Conclusions**

In the conjoint memory recognition task, the participant is informed that memory probes may come from either a set of previously presented targets or a set of non targets that are meaningfully related to one of targets, or from a set of unrelated non targets. Under verbatim instructions they are asked to accept only targets, under gist instructions they are asked to accept only related non targets, and under disjunctive instructions they are asked to accept verbatim or gist items. The important fact is that the person knows that the probe cannot be both in the verbatim category and the gist category. The episodic overestimate effect refers to the fact that the probability of accepting a probe when asked if it is a target plus the probability of accepting a probe when asked if it is either a target or a non related target. Logically these two probabilities should be identical because verbatim and gist categories are mutually exclusive, and therefore the two individual probabilities should sum to equal the probability of the union. Furthermore, this finding cannot be explained by a recognition memory model based on categorizing probes according to familiarity. Thus these results pose an interesting paradox for memory researchers.

One possible explanation is that participants do not follow instructions and allow implicit categorizations of probes as both gist and verbatim. Then the episodic over distribution effect equals the joint probability. But given the instructions (which state that this joint probability should be zero), this joint probability should be bounded below the product implied by statistical independence. However, strong violations of this statistical independence bound are also found.

In the past, the only satisfactory model for these findings was the dual process model. This model assumes that individuals categorize the probe into one of three states—a correct recollection state, a non recollection but familiar state, and a non recollection and non familiar state. The probe is accepted in either of the first two states. This model predicts that, after adjusting for response biases, the probability of accepting targets under the verbatim instruction equals the probability of accepting targets under the verbatim or gist instructions. This model violates the classic probability OR rule, but it provides a good fit to most of the data. The problem is that this model does not provide any explanation for the episodic over distribution effects that also occur for non related targets. As mentioned earlier on page 2, the average EOD(U) for non related targets equals .04 (averaged across the six studies in which it was reported), which is comparable to the average EOD(T) equal to .05 obtained with target probes (averaged across these same six studies). Also this model cannot explain some unusual conditions under which the episodic over distribution effect exceeds the probability of acceptance under the gist or verbatim instructions.

This paper presented a quantum memory recognition model to explain the episodic over distribution effect. The main contribution of the model is to provide a new way to understand the effect of the three different types of retrieval instructions: verbatim, gist, verbatim or gist. The model assumes that memory is represented by a vector in a high dimensional space. The verbatim instruction is represented as one subspace within this space, and the probability of accepting a probe as verbatim is determined by the squared length of the projection of the verbatim subspace. The gist instruction corresponds to a different subspace, and the probability of accepting a probe as gist equals the squared length of the state on the gist subspace. But these two subspaces are incompatible. They do not share the same basis vectors yet they are not orthogonal either. Given that the subspaces are incompatible, the disjunction question must be answered by using a sequence of two projections. To decide whether to accept a probe under the gist or verbatim question, first the person checks the gist, and if this is not accepted then the person checks the verbatim. Then the probability of accepting the probe equals the probability that the person accepts the gist plus the probability that the gist is not accepted but the person then accepts it as a verbatim. This model is capable of explaining (a) the episodic over distribution effect (including the results for the unrelated non targets), (b) the fact that the effect exceeds the statistical independence bound, and (c) the fact that the effect can exceed the probability of the disjunction. An advantage of the quantum model over the dual process model is that it explains the EOD effect for all three types of probes (targets, related non targets, and unrelated non targets).
using the same principles. The dual process model cannot explain the EOD effect for unrelated probes from set U.

A key question is why verbatim and gist questions result in non-commuting projections. The answer is that these two questions require using different sets of features for evaluating the questions. The gist question evokes a representation that relies on deeper semantic features whereas the verbatim question evokes a representation that relies of surface episodic details.

Psychologically, a major difference between the dual process model and the quantum model is the state of the memory system immediately before a choice (accept or reject probe) is made and the process for making this choice. According to the dual process model, the memory system is in precisely one of three states (either a recollect state, or a familiar state, or neither recollect or familiar) immediately before the choice, and the choice is simply a read out of the state that happens to be present when the question is asked. According to the quantum model, the memory system is not exactly in a verbatim or gist or neither type of state; but asking the question forces the system to jump or collapse on one of these answers. Asking the question forces the system to construct an answer, this process of constructing an answer is inherently indeterminate.

The quantum model presented here to explain memory recognition is formally the same as one previously developed to explain human probability judgment errors (Busemeyer, Pothos, Riccardo, 2009; Trueblood and Busemeyer, 2010). Thus the same model provides an integration of two quite different phenomena from two unrelated cognitive paradigms. This demonstrates the power of a quantum approach to integrate a broad range of findings using a common set of principles.

References


