Understanding Femtocell-Overlaid Cellular Networks

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“The” Cellular Trend

- Over 100%/year growth in data traffic to continue indefinitely
- AT&T saw 5000% increase in 3 years to Nov. 2009
- Revenues increasing slowly, linear at best

Choices facing the operators:
1. Strictly meter or restrict data usage, charge in proportion to bits consumed
2. Decrease $/bit exponentially
3. Lose money and/or watch network collapse

Decreasing $/bit exponentially appears to be our only choice
Good News First

- We’ve been doing this for many years now
- Cooper’s Law [Martin Cooper of Arraycomm, paraphrased]:
  
  “The data rate available to a wireless device doubles roughly every 30 months”

- This has held for over 100 years (since Marconi)
- 1,000,000x increase since late 1950’s, well more than half of this (about 1600x) driven by smaller cells
  - This dwarfs all other factors, including good per link communication engineering (i.e. getting closer to Shannon limit)
Bumping up against the Shannon Limit

**Rule of Thumb.** Shannon sum capacity for a SISO cellular system, treating interference as noise, is about 2 bps/Hz spatially averaged over the cell [AndBacGan10].

- Unless you “cheat” by only serving cell interior users, best possible long-term throughput is about 2 bps/Hz
- In theory, can increase with base station cooperation, MIMO, interference cancellation or alignment.
  - But these have not proven to scale well or provide order of magnitude gains in practice
  - Still worth pursuing, but expect a modest one-time boost, in grand scheme of things (see Cooper’s Law)
What about more spectrum?

- Network deployments are driven by peak demands in time and space
  - The money lives where people do
  - Cognitive radio/whitespace proponents often claim there is lots of “unused spectrum”. But where and when?
  - Very little free spectrum in LA, SF, NYC, etc. especially during peak hours

- At best, broadband-enabled spectrum may increase 2-3x via FCC action
  - This would be helpful
  - But again, not scalable in long term unless some magic discovered for 10-200+ GHz
Why Femtocells? Why Now?

- Low CapEx: 100-1000x lower than macro BS
- Low OpEx: 3rd party backhaul, “tower” and power
- Several converging trends make femto possible
  - Unprecedented device integration
  - Backhaul prevalence and speed
  - Cutthroat competition in BS market
  - VoIP no longer seen as sub-standard
  - Lessons learned from decade of WiFi development

- Advantages vs. Wi-Fi offload
  - Licensed spectrum; Unlicensed ever more crowded
  - Centralized MAC and coordination possible (in theory)
  - Efficient setup/teardown and handoffs (in theory)
Modeling Femtocells: The Need for Novel Models
What cellular actually looks (and will look) like

- Traditional grid model
- Actual 4G network today
- Completely random BSs

Zoom view with femtocells
Zoom w/ picos & femtos
The need for random spatial models

- Clearly, a fixed BS model is absurd for multi-tier networks
  - Need a statistical model
- Analogous to single user fading channels
  - Anyone actually ever seen a Rayleigh fading (or lognormal) channel?
  - Nevertheless, even dubious random channel models enable robust design for unpredictable variations
Starting point: the Poisson Point Process

- Rayleigh fading essentials
  - Complex Gaussian I&Q from CLT following from $U[0,2\pi]$ AOA
  - Highly tractable (exponential PDF)
- Poisson also is “max entropy”
  - All BS locations are i.i.d.
  - Exponential PDF for # BS’s/Area
  - Avg. # of BSs per area $A$ is $\lambda A$
  - Models irregular placements (good)
  - Does not model “repulsion” (bad)
- Idealized? Sure. But so is grid.
  - PPP gives lower bound to reality (worst-case) whereas grid provides upper bound (best-case)

Poisson models SINR CDF about as well as grid
One-tier downlink model

\[ p_c(T, \lambda, \alpha) = \mathbb{P}(\text{SINR} > T) = \mathbb{P}\left(\frac{hR^{-\alpha}}{\sigma^2 + I} > T\right) \]

WLOG, aggregate interference can be quantified for MS at the origin as

\[ I = \sum_{i \in \Phi_b \setminus b_o} g_i R_i^{-\alpha} \]
SINR can be found in very compact form

**Theorem:** When the fading power between any two nodes is exponentially distributed with mean $1/\mu$, the coverage probability is

$$p_c(T, \lambda, \alpha) = \pi \lambda \int_0^\infty e^{-\pi \lambda v (1+\rho(T,\alpha)) - \mu T \sigma^2 v^{\alpha/2}} dv,$$

where

$$\rho(T, \alpha) = T^{2/\alpha} \int_{T^{-2/\alpha}}^\infty \frac{1}{1 + u^{\alpha/2}} du.$$

$T = $ SINR threshold; $\lambda = $ BS density; $\alpha = $ PL exponent; $\sigma^2 = $ noise variance
Proof of Theorem

\[ p_c(T, \lambda, \alpha) = \mathbb{P}[\text{SINR} > T] \]

Conditioning on R:

\[ p_c(T, \lambda, \alpha) = \int_{R>0} \mathbb{P}(h > TR^\alpha (\sigma^2 + I)) f(R)dR \]

Since \( h \) is exponential(\( 1/\mu \)):

\[ p_c(T, \lambda, \alpha) = \int_{R>0} \mathbb{E}e^{-\mu TR^\alpha (\sigma^2 + I)} f(R)dR \]

\[ = \int_{R>0} \mathcal{L}_I(\mu T R^\alpha) e^{-\mu T R^\alpha \sigma^2} f(R)dR \]

Laplace transform of interference, with interferers outside a disc of radius R.

\[ F_r(r) = 1 - e^{-\pi \lambda R^2} \]

\[ f(R) = \frac{dF_r(r)}{dR} = 2\pi \lambda R e^{-\pi \lambda R^2} \]

\[ \mathbb{E}e^{-ax} = \int f(x)e^{-ax} dx = \mathcal{L}_x(a) \]
Manipulating Laplace Transform

$$\mathcal{L}_I(s) = \mathbb{E}[\exp(-s \sum g_i R_i^{-\alpha})], \text{ where } s = \mu TR^\alpha$$

**Probability Generating Functional of a PPP**

$$\mathbb{E} \left[ \prod_{x \in \Phi} f(x) \right] = \exp \left( -\lambda \int_{\mathbb{R}^2} (1 - f(x)) dx \right)$$

$$= \mathbb{E} \left[ \prod_{i \in \Phi \setminus \{b_o\}} \mathcal{L}_{g_i}(sR_i^{-\alpha}) \right] = \mathbb{E} \left[ \prod_{i \in \Phi \setminus \{b_o\}} \frac{\mu}{\mu + sR_i^{-\alpha}} \right]$$

Using the PGFL of a PPP

$$\mathcal{L}_I(s) = \exp \left( -2\pi \lambda \int_{\mathbb{R}} \left( 1 - \frac{\mu}{\mu + sx^{-\alpha}} \right) x dx \right)$$
Using the substitution, $u = x^2 R^{-2} T^{-2/\alpha}$ the LT can be simplified to

$$\mathcal{L}_I(\mu TR^\alpha) = \exp \left( -\pi R^2 \lambda \rho(T, \alpha) \right)$$

where

$$\rho(T, \alpha) = T^{2/\alpha} \int_{-2/\alpha}^{\infty} \frac{1}{1 + \mu^{\alpha/2}} du$$

Recall:

$$p_c(T, \lambda, \alpha) = \int_{R>0} \mathcal{L}_I(\mu TR^\alpha) e^{-\mu TR^\alpha \sigma^2} f(R) dR$$

Thus:

$$p_c(T, \lambda, \alpha) = 2\pi \lambda \int_{R>0} e^{-\lambda \pi R^2 (1 + \rho(T, \alpha))} e^{-\mu TR^\alpha \sigma^2} R dR$$

Using the substitution $R^2 \rightarrow v$ we obtain the result:

$$p_c(T, \lambda, \alpha) = \pi \lambda \int_0^\infty e^{-\pi \lambda v (1 + \rho(T, \alpha)) - \mu T \sigma^2 v^{\alpha/2}} dv,$$
Simplified coverage probability

1. Coverage probability in quasi-closed form for $\alpha = 4$

$$p_c(T, \lambda, 4) = \frac{\pi^{\frac{3}{2}} \lambda}{\sqrt{T/\text{SNR}}} \exp \left( \frac{(\lambda \pi \beta(T, 4))^2}{4T/\text{SNR}} \right) Q \left( \frac{\lambda \pi \beta(T, 4)}{\sqrt{2T/\text{SNR}}} \right).$$

where $\text{SNR} = (\mu \sigma^2)^{-1}$.

$$\beta(T, 4) = 1 + \sqrt{T}(\pi/2 - \arctan(1/\sqrt{T})).$$

2. Coverage probability with no noise (any $\alpha$):

$$p_c(T, \lambda, \alpha) = \frac{1}{\beta(T, \alpha)}.$$

Surprisingly simple expressions!
Moving to two-tier: Novel modeling aspects of femtocells

- Random (i.e. unknown) femtocell locations
  - Femtocell deployments in most cases unplanned
  - Even more random than macro BS locations
  - PPP model seems quite appropriate
- Transmit powers of femtocell and macrocell BS’s are quite different: $P_f \ll P_c$
- 3rd party backhaul (~1-10 Mbps) may be bottleneck
- Femtocells often indoors: $\beta = 5$-$10$ dB of isolation
- Amount and timeliness of possible femto-femto and femto-macro coordination is TBD; possibly major factor
Downlink SIR Model: Femtocell MS

SIR at a femtocell user (at origin) d meters from its femto BS, open access:

\[
SIR_f = \frac{P_f d^{-\alpha}}{\beta I_{c \rightarrow f} + \beta^2 I_{f \rightarrow f}}
\]

\[
I_{c \rightarrow f} = \sum_{X_i \in \Phi_c(\lambda_c) \cap \bar{b}(0,r_o)} P_c |X_i|^{-\alpha}
\]

where \( r_o = d \left( \frac{P_c}{P_f} \right)^{\frac{1}{\alpha}} \)

\[
I_{f \rightarrow f} = \sum_{Y_i \in \Phi_f(\lambda_f) \cap \bar{b}(0,d)} P_f |Y_i|^{-\alpha}
\]

Note that these apparent exclusion regions do not degrade Poisson model; Rather they are necessary conditions (or femto MS would connect to stronger BS)
Uplink SIR Model: Femtocell BS (at origin)

\[
\text{SIR}_f = \frac{P_f}{\beta I_{c \rightarrow f} + \beta^2 I_{f \rightarrow f}}
\]

Depends on access model; main source of interference

Closed access, uplink PC with target \( P_r \):

\[
I_{c \rightarrow f} = \sum_{X_j \in \Pi(\lambda_u)} \left( \frac{h_{0,j}}{h_{i,j}} d_{ij} \right)^\alpha X_j | -\alpha P_r
\]

\( d_{ij} \) depends on BS locations (a separate PPP)

Open access is similar, but now cellular interferers are guaranteed to be outside the femtocell coverage area, or else they would connect to it.
Example insights from this approach

- Closed-access two-tier CDMA basically does not work
  [ChaAnd09-CDMA]
  - Near-far problem cannot be mitigated with power control
  - Higher-layer interference management is needed, e.g. time-hopping, femto-only freq. channel, aggressive handoffs (i.e. open access)

- In a static OFDMA allocation, most spectrum allocated to macrocell for optimum throughput [ChaAnd09-OFDMA]
  - Femtocells do not need much spectrum since SINR is high, and very few users to split the pie between

- With multiple antennas, multiuser transmission (SDMA) decreases achievable rate/coverage [ChaKouAnd09]
  - An SINR-based random spatial model indicates something important than information theory cannot
Conclusions

- Femtocells are here to stay, there is no other way
- Random spatial models are essential tools for understanding femtocell-overlaid cellular networks
- Don’t be afraid of the chaos: an entire branch of applied probability is there to help (stochastic geometry)
- Tons of work left to do:
  - Virtually all cellular knowledge and conventional wisdom from last 2-3 decades needs to be revisited in next 3-5 years
  - Random models other than Poisson should be considered
  - Industry-academic collaboration is needed to develop a spectrum of models that tradeoff between “highly tractable” and “highly realistic”

**Cellular is in major transition: many new research problems!**
References/Upcoming Special Issues


Upcoming Special Issues on Femtocells/Heterogeneous Networks:

- IEEE J. on Selected Areas in Comm. (JSAC), March 1, 2011 – J. Andrews (lead)
- IEEE J. on Selected Areas in Signal Processing, June 10, 2011 – R. Heath (lead)
How much data must we handle?

- Consider 10K people/km²
  - Medium density neighborhood in SF, LA, Chicago
  - Low density in NYC or major European/Asian cities
- Assume 1 Mbps/person DL peak demand (simultaneous)
  - 100 kbps/person 24-hour average results just from current Netflix consumption [Sahai10]
- Thus, require 10 Gbps/km²
  - 1 km² corresponds to BS range of about 600m
  - Even at 2 bps/Hz, this requires 5 GHz for the downlink
  - Not including overhead, frequency reuse, fairness, uplink!
- If base stations have range of 100m, density increases by factor of ~30, perhaps can make do with 500 MHz

The only option is to make cells smaller – and smarter