

# A renormalizable $N = 1/2$ SYM theory with interacting matter

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## Abstract

We consider nonanticommutative SYM theories with chiral matter in the adjoint representation of the  $SU(\mathcal{N}) \otimes U(1)$  gauge group. In a superspace setup and manifest background covariant approach we investigate the one-loop renormalization of the theory when a cubic superpotential is present. The structure of the divergent terms reveals that the theory simply obtained from the ordinary one by trading products for star products is not renormalizable. Moreover, because of the different renormalization undergone by the abelian field compared to the non-abelian ones, the superpotential seems to be incompatible with the requests of renormalizability, gauge and  $N = 1/2$  invariance. However, by a suitable modification of the quadratic action for the  $U(1)$  (anti)chiral superfields and the addition of extra couplings, we find an action which is one-loop renormalizable and manifestly  $N = 1/2$  supersymmetric and supergauge invariant. We conclude that interacting matter can be safely introduced in NAC gauge theories, in contrast with previous results.

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# 1 Introduction

Non(anti)commutative field theories emerge naturally as low energy limits of strings in a background where a constant Neveu–Schwarz two form and/or a Ramond–Ramond two–form are turned on [1, 2, 3, 4]. In the supersymmetric case, the appearance of the RR flux  $\mathcal{F}_{\alpha\beta}$  modifies the superspace geometry through the appearance of a nontrivial anticommutator  $\{\theta_\alpha, \theta_\beta\} = \mathcal{F}_{\alpha\beta}$  [5, 6, 7, 3, 4, 8]. The effect on field theories defined in nonanticommutative (NAC) superspace is that the multiplication among superfields is no longer commutative but described by the so-called  $*$ –product. As a result, supersymmetry is in general partially broken from  $N = 1$  to  $N = 1/2$ . For extended supersymmetries suitable deformations can be realized which break less supersymmetry [9].

In the recent past quantum properties of NAC theories have been investigated. In particular, renormalizability is one of the more interesting issues since the partial breaking of supersymmetry could affect the ordinary boson-fermion cancellation leading to a worsening of the UV behavior of the theory. For the deformed WZ model loop calculations have been performed both in superspace [10, 11, 12] and in components [13]. They reveal that renormalizability is lost already at one loop but it can be restored by adding new couplings in the classical action depending on the deformation parameter. This modification is then sufficient to make the theory renormalizable at all orders [14]. The same analysis has been carried on also for deformed  $SU(\mathcal{N}) \otimes U(1)$  gauge theories in interaction with massive quantum chiral matter in the adjoint representation. In [15] we have found a general action for the pure gauge sector which is  $N = 1/2$  supergauge invariant and one-loop renormalizable. It differs from the one obtained from the ordinary action where we trade products for  $*$ –products by the addition of new couplings depending on the deformation parameter. They span the spectrum of all possible couplings allowed by supergauge invariance. Our results are confirmed by a similar analysis done in components [16]. General arguments in support of renormalizability for  $N = 1/2$  gauge theories coupled to non–interacting matter can be found in [17].

It is important to stress that in all theories investigated so far UV divergences are always logarithmic. This suggests that under NAC deformations supersymmetry is in general softly broken.

The previous results for super–Yang–Mills theories concern primarily the gauge sector. However, for a complete proof of the quantum consistency of the theories one should analyze also the matter sector. A preliminary discussion on theories with non-interacting massive matter can be found in [15], whereas a systematic attempt in this direction has been carried on recently in [18].

Working in components in the WZ gauge, the authors of [18] have investigated the structure of one–loop divergences in all sectors of the theory. When massive matter is present in the fundamental and/or in the adjoint representation of  $SU(\mathcal{N}) \otimes U(1)$ , both the gauge and matter sectors can be made finite by a suitable generalization of the classical action which contains new deformation-dependent couplings in addition to the ones obtained by generalizing products to  $*$ –products.

For chiral matter in the adjoint representation one can also add a superpotential

term. In [18] a deformed SYM theory with cubic superpotential has been investigated. The authors have found that the cancellation of one-loop divergences in the matter sector requires a modification of the classical superpotential which breaks supersymmetry completely. This is due to the following mechanism: According to the non-renormalization theorem, the renormalization of the chiral coupling is induced by the renormalization of the (anti)chiral superfields. On the other hand, abelian and non-abelian fields renormalize differently, so that in a  $SU(\mathcal{N}) \otimes U(1)$  theory a generalization of the superpotential which assigns different couplings to the abelian and non-abelian sectors is necessary in order to render the theory renormalizable. While in the ordinary case this is consistent with supersymmetry, in the NAC case one can easily realize that the generalized superpotential is no longer supersymmetric. Therefore, it seems that in NAC SYM theories interacting chiral matter can be consistently added at quantum level only at the price to give up supersymmetry completely. We will call it the “superpotential problem”.

In particular, it follows that  $N = 4$  SYM does not seem to possess a renormalizable  $N = 1/2$  deformation. On the other hand, string theory would provide a natural interpretation of this theory as the low energy dynamics of a set of D3-branes in a constant graviphoton background. Therefore, the absence of  $N = 1/2$  generalizations of  $N = 4$  SYM should be understood also from a string theory point of view.

As a first step it is then important to investigate whether the negative results of Ref. [18] find definitive confirmation or they can be overpassed. To this end, in a superspace setup we reconsider the problem of quantizing NAC SYM theories with a cubic superpotential. We start from the natural  $*$ -generalization of the ordinary superspace action for  $N = 1$  SYM with cubic superpotential for a single (anti)chiral field. First of all, we rephrase the conclusions of [18] in superspace language by arguing that the request for the theory to be renormalizable and supergauge invariant would force the appearance of terms in the action which would be manifestly non-supersymmetric.

Successively, we prove that a suitable generalization of the action can be found which solves the superpotential problem. It is obtained by assigning a different coupling constant to the quadratic term for the abelian matter superfields. The modification is done in a manifestly  $N = 1/2$  supersymmetric and supergauge invariant way and has a double effect: On one side, the kinetic terms for the abelian and non-abelian superfields appear with a different normalization. The relative coupling can then be chosen so to absorb part of the divergences and tune the renormalization of the abelian fields with the one for the non-abelians. In so doing, a renormalizable,  $N = 1/2$  and gauge invariant cubic superpotential can be added. On the other side, it changes the gauge-matter coupling in vertices where abelian (anti)chirals are present. As a crucial consequence, the evaluation of one-loop diagrams reveals that only  $N = 1/2$  susy and supergauge invariant divergent structures get produced. Therefore, a one-loop renormalizable action is obtained by adding all possible  $N = 1/2$  supergauge invariant couplings allowed by dimensional analysis. Its explicit expression is given in eq. (5.23).

In this paper we construct the renormalizable action by performing a dimensional and diagrammatic analysis of divergences, without entering the details of the calculations. The paper is organized as follows: In Section 2 we review the NAC superspace and the

generalization of the background field method already discussed in [15]. In Sections 3 and 4 we discuss the quantization of the NAC SYM model obtained by promoting ordinary products to be  $*$ -products in the action for  $SU(\mathcal{N}) \times U(1)$  gauge superfields coupled to chiral matter in the adjoint representation, in the presence of a cubic superpotential. We formulate the superpotential problem in the language of superspace and propose our solution which requires introducing a different coupling constant in front of the abelian quadratic action. In Section 5 we perform a general selection of all possible divergent structures which might appear at loop level and propose the most general one-loop renormalizable gauge-invariant action. Finally, in Section 6 we prove that all divergences can be multiplicatively renormalized while preserving gauge-invariance. Finally, Section 7 contains some conclusions and perspectives. An Appendix follows where we derive the Feynman rules necessary for perturbative calculations.

## 2 The general setting

We consider the  $N = (\frac{1}{2}, 0)$  NAC superspace spanned by nonanticommutative coordinates  $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$  satisfying

$$\{\theta^\alpha, \theta^\beta\} = 2\mathcal{F}^{\alpha\beta} \quad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0 \quad [x^{\alpha\dot{\alpha}}, x^{\beta\dot{\beta}}] = [x^{\alpha\dot{\alpha}}, \theta^\beta] = [x^{\alpha\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}] = 0 \quad (2.1)$$

where  $\mathcal{F}^{\alpha\beta}$  is a  $2 \times 2$  symmetric, constant matrix. This algebra is consistent only in euclidean signature where the chiral and antichiral sectors are totally independent and not related by complex conjugation.

The class of smooth superfunctions on the NAC superspace is endowed with the NAC but associative product

$$\phi * \psi \equiv \phi e^{-\overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta} \psi = \phi\psi - \phi \overleftarrow{\partial}_\alpha \mathcal{F}^{\alpha\beta} \overrightarrow{\partial}_\beta \psi - \frac{1}{2} \mathcal{F}^2 \partial^2 \phi \partial^2 \psi \quad (2.2)$$

where  $\mathcal{F}^2 \equiv \mathcal{F}^{\alpha\beta} \mathcal{F}_{\alpha\beta}$ . (Anti)chiral superfields can be consistently defined by the constraints  $\overline{D}_{\dot{\alpha}} * \phi = D_\alpha * \bar{\phi} = 0$ <sup>1</sup>.

Supersymmetric Yang-Mills theories in NAC superspace have been extensively discussed in Ref. [15]. As in the ordinary case, they are defined in terms of a scalar prepotential  $V \equiv V_A T^A$  in the adjoint representation of the gauge group. Being the theory in euclidean signature,  $V$  has to be *pure imaginary*,  $V^\dagger = -V$ .

The supergauge transformations for  $V$  are

$$e_*^V \rightarrow e_*^{V'} = e_*^{i\bar{\Lambda}} * e_*^V * e_*^{-i\Lambda} \quad (2.3)$$

where  $\Lambda, \bar{\Lambda}$  are chiral and antichiral superfields, respectively.

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<sup>1</sup>We use chiral representation [19] for supercharges and covariant derivatives. In particular, we define  $D_\alpha = \partial_\alpha + i\bar{\theta}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}$  and  $\overline{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}}$ .

Supergauge covariant derivatives in superspace can be defined in the so-called *gauge chiral* or *gauge antichiral* representation [19]. As discussed in [15] in the NAC case the two descriptions are no longer equivalent, especially when the construction of supergauge invariant actions is under concern. It turns out that the *gauge antichiral* representation is definitely preferable. We then define supergauge covariant derivatives as

$$\nabla_A \equiv (\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\alpha\dot{\alpha}}) = (D_\alpha, e_*^V * \bar{D}_{\dot{\alpha}} e_*^{-V}, -i\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}_*) \quad (2.4)$$

They can be expressed in terms of ordinary superspace derivatives and a set of connections,  $\nabla_A \equiv D_A - i\Gamma_A$ , where

$$\Gamma_\alpha = 0 \quad , \quad \bar{\Gamma}_{\dot{\alpha}} = ie_*^V * \bar{D}_{\dot{\alpha}} e_*^{-V} \quad , \quad \bar{\Gamma}_{\alpha\dot{\alpha}} = -iD_\alpha \bar{\Gamma}_{\dot{\alpha}} \quad (2.5)$$

The field strengths are then defined as  $*$ -commutators of supergauge covariant derivatives

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{2}[\nabla^\alpha, \bar{\nabla}_{\alpha\dot{\alpha}}]_* \quad , \quad \widetilde{W}_\alpha = -\frac{1}{2}[\bar{\nabla}^{\dot{\alpha}}, \bar{\nabla}_{\alpha\dot{\alpha}}]_* \quad (2.6)$$

and satisfy the Bianchi's identities  $\nabla^\alpha * \widetilde{W}_\alpha + \bar{\nabla}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} = 0$ . In terms of gauge connections they are given by

$$\bar{W}_{\dot{\alpha}} = \frac{i}{2}D^\alpha \bar{\Gamma}_{\alpha\dot{\alpha}} = D^2 \bar{\Gamma}_{\dot{\alpha}} \quad , \quad \widetilde{W}_\alpha = \frac{i}{2}\partial_\alpha^{\dot{\alpha}} \bar{\Gamma}_{\dot{\alpha}} + \frac{i}{2}[\bar{\nabla}^{\dot{\alpha}}, \bar{\Gamma}_{\alpha\dot{\alpha}}]_* \quad (2.7)$$

Covariantly (anti)chiral superfields can be defined according to  $\bar{\nabla}_{\dot{\alpha}} * \Phi = 0$  and  $\nabla_\alpha * \bar{\Phi} = 0$ , respectively.

At classical level, a NAC SYM theory with interacting chiral matter in the adjoint representation of  $SU(\mathcal{N}) \otimes U(1)$  can be described by the following action [15]

$$\begin{aligned} S &= \frac{1}{2g^2} \int d^4x d^2\bar{\theta} \operatorname{Tr}(\bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}}) \\ &+ \frac{1}{2g_0^2} \int d^4x d^4\theta \left[ \operatorname{Tr}(\bar{\Gamma}^{\dot{\alpha}}) * \operatorname{Tr}(\bar{W}_{\dot{\alpha}}) + 4i\mathcal{F}^{\rho\gamma}\bar{\theta}^2 \operatorname{Tr}(\partial_{\rho\dot{\rho}}\bar{\Gamma}^{\dot{\alpha}}) * \operatorname{Tr}(\bar{W}_{\dot{\alpha}} * \bar{\Gamma}_\gamma^{\dot{\rho}}) \right] \\ &+ \int d^4x d^4\theta \operatorname{Tr}(\bar{\Phi} * \Phi) \\ &+ h \int d^4x d^2\theta \operatorname{Tr}(\Phi * \Phi * \Phi) + \bar{h} \int d^4x d^2\bar{\theta} \operatorname{Tr}(\bar{\Phi} * \bar{\Phi} * \bar{\Phi}) \end{aligned} \quad (2.8)$$

where  $\Phi \equiv e_*^V * \phi * e_*^{-V}$ ,  $\bar{\Phi} = \bar{\phi}$  are covariantly (anti)chiral superfields expressed in terms of ordinary (anti)chirals. Therefore, the quadratic matter action contains nontrivial couplings between gauge and chiral superfields.

The action is invariant under the infinitesimal supergauge transformations

$$\begin{aligned} \delta\Phi &= i[\bar{\Lambda}, \Phi]_* \quad , \quad \delta\bar{\Phi} = i[\bar{\Lambda}, \bar{\Phi}]_* \\ \delta\bar{\Gamma}_{\alpha\dot{\alpha}} &= [\bar{\nabla}_{\alpha\dot{\alpha}}, \bar{\Lambda}]_* \quad , \quad \delta\bar{W}_{\dot{\alpha}} = i[\bar{\Lambda}, \bar{W}_{\dot{\alpha}}]_* \end{aligned} \quad (2.9)$$

As discussed in [15] the term proportional to  $\bar{\theta}^2$  in (2.8) is necessary in order to restore gauge invariance of  $\int \text{Tr}(\bar{\Gamma}^\alpha) \text{Tr}(\bar{W}_\alpha)$ .

The transformation law for  $\bar{W}_\alpha$  can be rewritten as

$$\delta \bar{W}_\alpha^A = \frac{i}{2} d_{ABC} [\bar{\Lambda}^B, \bar{W}_\alpha^C]_* - \frac{1}{2} f_{ABC} \{ \bar{\Lambda}^B, \bar{W}_\alpha^C \}_* \quad (2.10)$$

where  $A, B, C$  are  $SU(\mathcal{N}) \otimes U(1)$  indices. The first term is non-vanishing only in the NAC case and mixes nontrivially  $U(1)$  and  $SU(\mathcal{N})$  fields. In particular, the abelian field strength  $\bar{W}_\alpha^0$  is no longer a singlet but transforms under  $SU(\mathcal{N})$  into a linear combination of both  $U(1)$  and  $SU(\mathcal{N})$  fields.

In superspace, a convenient procedure for performing perturbative calculations for super Yang–Mills theories is the background field method [20, 19]. It consists of a non-linear quantum–background splitting on the gauge superfields which leads to separate background and quantum gauge invariances. Gauge fixing is then chosen which breaks the quantum invariance while keeping manifest invariance with respect to the background gauge transformations. Therefore, at any given order in the loop expansion the contributions to the effective action are expressed directly in terms of covariant derivatives and field strengths (without explicit dependence on the prepotential  $V$ ).

The generalization of the background field method to NAC SYM theories with chiral matter in a *real* representation of the gauge group has been performed in [15]. Here we summarize the main ingredients referring the reader to that paper for details.

We perform the splitting of the Euclidean prepotential  $e_*^V \rightarrow e_*^V * e_*^U$  where  $U$  is the background prepotential and  $V$  its quantum counterpart. Consequently, the covariant derivatives (2.4) become

$$\nabla_\alpha = \bar{\nabla}_\alpha = D_\alpha \quad , \quad \bar{\nabla}_\alpha = e_*^V * \bar{\nabla}_\alpha * e_*^{-V} = e_*^V * (e_*^U * \bar{D}_\alpha * e_*^{-U}) * e_*^{-V} \quad (2.11)$$

Covariantly (anti)chiral superfields in the adjoint representation are expressed in terms of background covariantly (anti)chiral objects as

$$\bar{\Phi} = \bar{\Phi} \quad , \quad \Phi = e_*^V * \Phi * e_*^{-V} = e_*^V * (e_*^U * \phi * e_*^{-U}) * e_*^{-V} \quad (2.12)$$

and then splitted as  $\Phi \rightarrow \bar{\Phi} + \Phi_q$  and  $\bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q$ , where  $\bar{\Phi}, \bar{\Phi}$  are background fields and  $\Phi_q, \bar{\Phi}_q$  their quantum fluctuations.

We perform quantum-background splitting in the action (2.8) and extract the Feynman rules necessary for one-loop calculations.

### Gauge sector

As in the ordinary case, the invariance under quantum gauge transformations [20, 19, 15] is broken by choosing gauge–fixing functions as  $f = \bar{\nabla}^2 * V, \bar{f} = \nabla^2 * V$ , while preserving manifest invariance of the effective action and correlation functions under background gauge transformations [20, 19, 15]. In Ref. [15] the gauge–fixing procedure for  $SU(\mathcal{N}) \otimes U(1)$  NAC gauge theories has been discussed in detail. As a result, extracting the

quadratic part in the quantum  $V$  fields from  $\int d^4x d^2\bar{\theta} [\frac{1}{2g^2} \text{Tr}(\overline{W}^{\dot{\alpha}} \overline{W}_{\dot{\alpha}}) + \frac{1}{2g_0^2} \text{Tr}(\overline{W}^{\dot{\alpha}}) \text{Tr}(\overline{W}_{\dot{\alpha}})]$  and adding the gauge-fixing action

$$S_{GF} = -\frac{1}{g^2\alpha} \int d^4x d^4\theta \text{Tr} \left[ (\overline{\nabla}^2 * V)(\nabla^2 * V) \right] \quad (2.13)$$

in Feynman gauge we find

$$S \rightarrow -\frac{1}{2g^2} \int d^4x d^4\theta \left[ V^a * \hat{\square} * V^a + V^0 * \left( \nabla^2 * \overline{\nabla}^2 + \overline{\nabla}^2 * \nabla^2 - \frac{g_0^2 + g^2}{g_0^2} \overline{\nabla}^{\dot{\alpha}} * \nabla^2 * \overline{\nabla}_{\dot{\alpha}} \right) * V^0 \right] \quad (2.14)$$

where the label  $a$  runs over  $SU(\mathcal{N})$  indices and we have defined

$$\hat{\square} = \square_{cov} - i\widetilde{\mathbf{W}}^\alpha * \nabla_\alpha - i\overline{\mathbf{W}}^{\dot{\alpha}} * \overline{\nabla}_{\dot{\alpha}} \quad , \quad \square_{cov} = \frac{1}{2} \overline{\nabla}^{\alpha\dot{\alpha}} * \overline{\nabla}_{\alpha\dot{\alpha}} \quad (2.15)$$

Introducing also the covariant operator

$$\widetilde{\square} = \nabla^2 * \overline{\nabla}^2 + \overline{\nabla}^2 * \nabla^2 - \overline{\nabla}^{\dot{\alpha}} * \nabla^2 * \overline{\nabla}_{\dot{\alpha}} = \square_{cov} - i\widetilde{\mathbf{W}}^\alpha * \nabla_\alpha + \frac{i}{2} (\overline{\nabla}^{\dot{\alpha}} * \overline{W}_{\dot{\alpha}}) \quad (2.16)$$

perturbative contributions can be written in terms of background covariant propagators

$$\begin{aligned} \langle V^a V^b \rangle &= g^2 \left( \frac{1}{\hat{\square}} \right)^{ab} \\ \langle V^0 V^0 \rangle &= g^2 \left\{ \frac{1}{\widetilde{\square}} \left[ 1 + \left( \frac{g^2}{g^2 + g_0^2} \right) \overline{\nabla}^{\dot{\alpha}} * \nabla^2 * \overline{\nabla}_{\dot{\alpha}} * \frac{1}{\widetilde{\square}} \right] \right\}^{00} \end{aligned} \quad (2.17)$$

Their expansion in powers of the background fields provides the ordinary  $\frac{1}{\square}$  propagator for abelian and non-abelian superfields plus pure-gauge interaction vertices (see Appendix A).

Further vertices come from the background field expansion of the  $\bar{\theta}^2$  term in the second line of action (2.8). Their explicit expressions can be found in Appendix E of Ref. [15].

The ghost action associated to the gauge-fixing (2.13) is given in terms of background covariantly (anti)chiral FP and NK ghost superfields as

$$S_{gh} = \int d^4x d^4\theta \left[ \bar{c}' c - c' \bar{c} + \dots + \bar{b} b \right] \quad (2.18)$$

### Chiral sector

We now discuss the quantization of the matter action in (2.8) when  $\Phi, \bar{\Phi}$  are full covariantly (anti)chiral superfields. With obvious modifications the results hold also for the background covariantly chiral ghosts in (2.18).

We first express the full covariantly (anti)chiral superfields in terms of background covariantly (anti)chiral superfields according to (2.12). Expanding in powers of  $V$  we have (we use the notation  $\Phi_*^3 \equiv \Phi * \Phi * \Phi$ )

$$S_0 + S_{int} = \int d^4x d^4\theta \bar{\Phi} * \Phi + \int d^4x d^4\theta \left( \bar{\Phi}[V, \Phi]_* + \frac{1}{2} \bar{\Phi}[V, [V, \Phi]_*]_* + \dots \right) + h \int d^4x d^2\theta \text{Tr}(\Phi_*^3) + \bar{h} \int d^4x d^2\bar{\theta} \text{Tr}(\bar{\Phi}_*^3) \quad (2.19)$$

where the Trace over group indices has been omitted since the quantization procedure works independently of the color structure. After the shift  $\Phi \rightarrow \Phi + \Phi_q$ ,  $\bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q$  only terms with two quantum superfields need be considered for one-loop calculations.

Quantization is accomplished by adding source terms

$$S_j = \int d^4x d^2\theta j * \Phi_q + \int d^4x d^2\bar{\theta} \bar{\Phi}_q * \bar{j} = \int d^4x d^4\theta \left( j * \frac{1}{\square_+} * \nabla^2 \Phi_q + \bar{\Phi}_q * \frac{1}{\square_-} * \bar{\nabla}^2 * \bar{j} \right) \quad (2.20)$$

where, for any (anti)chiral superfield, we have defined

$$\begin{aligned} \bar{\nabla}^2 * \nabla^2 * \Phi &= \square_+ * \Phi & \square_+ &= \square_{cov} - i\widetilde{W}^\alpha * \nabla_\alpha - \frac{i}{2}(\nabla^\alpha * \widetilde{W}_\alpha) \\ \nabla^2 * \bar{\nabla}^2 * \bar{\Phi} &= \square_- * \bar{\Phi} & \square_- &= \square_{cov} - i\widetilde{W}^{\dot{\alpha}} * \bar{\nabla}_{\dot{\alpha}} - \frac{i}{2}(\bar{\nabla}^{\dot{\alpha}} * \widetilde{W}_{\dot{\alpha}}) \end{aligned} \quad (2.21)$$

and performing the gaussian integral in

$$Z = \int \mathcal{D}\Phi_q \mathcal{D}\bar{\Phi}_q e^{S_{int}(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{j}})} e^{\int d^4x d^4\theta (\bar{\Phi}_q * \Phi_q + j * \frac{1}{\square_+} * \nabla^2 \Phi_q + \bar{\Phi}_q * \frac{1}{\square_-} * \bar{\nabla}^2 * \bar{j})} \quad (2.22)$$

The Feynman rules can then be read from

$$Z = \Delta e^{S_{int}(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{j}})} e^{-\int d^4x d^4\theta j * \frac{1}{\square_-} * \bar{j}} \quad (2.23)$$

where  $\Delta \equiv \int \mathcal{D}\Phi_q \mathcal{D}\bar{\Phi}_q e^{S_0}$ . In particular, we obtain the covariant scalar propagator

$$\langle \Phi^A \bar{\Phi}^B \rangle = - \left( \frac{1}{\square_-} \right)^{AB} \quad (2.24)$$

At one-loop, from the matter sector we have two different contributions to the effective action. A first contribution to the gauge effective action comes from the perturbative evaluation of  $\Delta$ . This can be worked out by using the doubling trick procedure introduced in [19] for ordinary SYM theories and generalized to NAC theories in [15]. The corresponding Feynman rules are collected in Ref. [15]. A second contribution comes from the perturbative expansion of  $e^{S_{int}}$  from which we can read gauge-chiral vertices. Further interaction vertices arise from the expansion of  $1/\square_-$  in powers of the background fields (see Appendix A).



### 3 One-loop divergences: The gauge sector

In Ref. [15] we computed divergent contributions to the pure gauge sector of the NAC  $SU(\mathcal{N}) \otimes U(1)$  SYM theory. It turned out that the classical action (2.8) is not renormalizable since further divergent configurations arise at one-loop which are  $N = 1/2$  supersymmetric and supergauge invariant. However, we proved that it is possible to deform the classical action in such a way as to produce a one-loop renormalizable theory. The manner in which we proceeded is to start *ab initio* with a deformed action containing all possible terms allowed by gauge invariance, R-symmetry, and dimensional analysis. We computed all one-loop divergences produced by the new action and determined a one-loop renormalizable action depending on a number of arbitrary coupling constants. Computing the  $\beta$ -functions we found that they allow for specific restrictions on these constants. In particular, two different choices for minimal deformed actions are allowed which are one-loop renormalizable

$$\begin{aligned}
S_{gauge}^{(1)} = & \frac{1}{2g^2} \int d^4x d^4\theta \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \right) \\
& + \frac{1}{2g_0^2 \mathcal{N}} \int d^4x d^4\theta \left[ \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} \right) * \operatorname{Tr} \left( \bar{W}_{\dot{\alpha}} \right) \right. \\
& \quad + 4i \mathcal{F}^{\rho\gamma} \bar{\theta}^2 \operatorname{Tr} \left( \partial_{\rho\dot{\rho}} \bar{\Gamma}^{\dot{\alpha}} \right) * \operatorname{Tr} \left( \bar{W}_{\dot{\alpha}} * \bar{\Gamma}_{\gamma}^{\dot{\rho}} \right) \\
& \quad \left. - \mathcal{F}^2 \bar{\theta}^2 \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \right) \operatorname{Tr} \left( \bar{W}^{\dot{\beta}} * \bar{W}_{\dot{\beta}} \right) \right] \\
& + \frac{1}{l^2} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\beta}} * \bar{W}_{\dot{\beta}} \right)
\end{aligned} \tag{3.1}$$

or

$$\begin{aligned}
S_{gauge}^{(2)} = & \frac{1}{2g^2} \int d^4x d^4\theta \left[ \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \right) \right. \\
& \quad \left. + \mathcal{F}^2 \bar{\theta}^2 \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \right) * \operatorname{Tr} \left( \bar{W}^{\dot{\beta}} * \bar{W}_{\dot{\beta}} \right) \right] \\
& + \frac{1}{2g_0^2 \mathcal{N}} \int d^4x d^4\theta \left[ \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} \right) * \operatorname{Tr} \left( \bar{W}_{\dot{\alpha}} \right) \right. \\
& \quad \left. + 4i \mathcal{F}^{\rho\gamma} \bar{\theta}^2 \operatorname{Tr} \left( \partial_{\rho\dot{\rho}} \bar{\Gamma}^{\dot{\alpha}} \right) * \operatorname{Tr} \left( \bar{W}_{\dot{\alpha}} * \bar{\Gamma}_{\gamma}^{\dot{\rho}} \right) \right] \\
& + \frac{1}{l^2} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \operatorname{Tr} \left( \bar{\Gamma}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\beta}} * \bar{W}_{\dot{\beta}} \right)
\end{aligned} \tag{3.2}$$

In both cases the theory contains three independent coupling constants. While  $g, g_0$  are the  $SU(\mathcal{N}) \otimes U(1)$  couplings already present in the ordinary theory, the appearance of the third coupling  $l$  is strictly related to the NAC deformation we have performed. We note that  $g, g_0$  must be different reflecting the fact that, as in the ordinary case,  $SU(\mathcal{N})$  and  $U(1)$  fields renormalize differently.

## 4 One–loop divergences: The matter sector

We now study the structure of one–loop divergent contributions to the matter sector with particular attention to the superpotential problem.

We begin with the classical action

$$S_{matter} = \int d^4x d^4\theta \operatorname{Tr}(\bar{\Phi} * \Phi) + h \int d^4x d^2\theta \operatorname{Tr}(\Phi_*^3) + \bar{h} \int d^4x d^2\bar{\theta} \operatorname{Tr}(\bar{\Phi}_*^3) \quad (4.1)$$

for covariantly (anti)chiral superfields. Applying background field method we evaluate one–loop diagrams with external matter.

### 4.1 The quadratic action

Divergent diagrams contributing to the two–point function are given in Fig. 1 where the internal lines correspond to ordinary  $1/\square$  propagators (straight lines correspond to chiral propagators, whereas waved lines correspond to vectors). It turns out that divergent contributions to the quadratic term come only from vertices not including the deformation parameter. Therefore, they coincide with the ones of the underformed theory and are given by

$$\mathcal{S} \int d^4x d^4\theta \left[ (9h\bar{h} - 2g^2) \mathcal{N} \operatorname{Tr}(\bar{\Phi} * \Phi) + (9h\bar{h} + 2g^2) \operatorname{Tr}\bar{\Phi} * \operatorname{Tr}\Phi \right] \quad (4.2)$$

where  $\mathcal{S}$  is the self–energy divergent integral which in dimensional regularization is ( $d = 4 - 2\epsilon$ )

$$\mathcal{S} \equiv \int d^d q \frac{1}{q^2(q-p)^2} = \frac{1}{(4\pi)^2} \frac{1}{\epsilon} + \mathcal{O}(1) \quad (4.3)$$

We note that a new trace structure appears reflecting the fact that  $SU(\mathcal{N})$  and  $U(1)$  superfields acquire different contributions. In fact, considering only the kinetic term, the previous result reads (using eq. (2.12))

$$\mathcal{S} \int d^4x d^4\theta \left[ (9h\bar{h} - 2g^2) \mathcal{N} \bar{\phi}^a \phi^a + 18h\bar{h} \mathcal{N} \bar{\phi}^0 \phi^0 \right] \quad (4.4)$$

In particular, corrections to the abelian kinetic term coming from the gauge–chiral loop cancel in agreement with the calculation done in components [18].

In the ordinary case, the appearance of the double–trace term is harmless since it is supergauge invariant. In the NAC case this is no longer true since its variation is

$$\delta \operatorname{Tr}\bar{\Phi} * \operatorname{Tr}\Phi = 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \left[ \operatorname{Tr}(\partial_{\alpha}^{\dot{\alpha}} \bar{\Lambda} * \partial_{\beta\dot{\alpha}} \bar{\Phi}) * \operatorname{Tr}\Phi + \operatorname{Tr}\bar{\Phi} * \operatorname{Tr}(\partial_{\alpha}^{\dot{\alpha}} \bar{\Lambda} * \partial_{\beta\dot{\alpha}} \Phi) \right] \quad (4.5)$$

and does not vanish when integrated on superspace coordinates.

On general grounds, it is easy to see that there are two possible gauge completions for  $\int \operatorname{Tr}\bar{\Phi} * \operatorname{Tr}\Phi$ . In fact, the following expressions (both for background covariantly and full covariantly (anti)chiral superfields)

$$\operatorname{Tr}\bar{\Phi} * \operatorname{Tr}\Phi + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \operatorname{Tr}(\bar{\Gamma}_{\alpha}^{\dot{\alpha}} * \bar{\Phi}) * \operatorname{Tr}(\partial_{\beta\dot{\alpha}} \Phi) + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \operatorname{Tr}(\bar{\Gamma}_{\alpha}^{\dot{\alpha}} * \Phi) * \operatorname{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \quad (4.6)$$

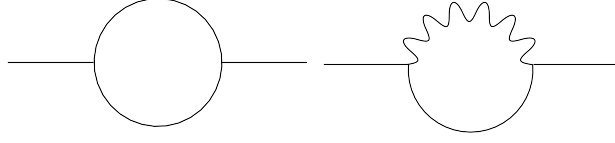


Figure 1: One-loop two-point functions with chiral external fields.

and

$$\begin{aligned}
\text{Tr}\bar{\Phi} * \text{Tr}\Phi & - 2i\mathcal{F}^{\alpha\beta}\bar{\theta}^2 \text{Tr} \left[ \bar{\Gamma}_\alpha^{\dot{\alpha}} * \left( \partial_{\beta\dot{\alpha}}\bar{\Phi} - \frac{i}{2}[\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Phi}]_* \right) \right] * \text{Tr}(\Phi) \\
& - 2i\mathcal{F}^{\alpha\beta}\bar{\theta}^2 \text{Tr} \left[ \bar{\Gamma}_\alpha^{\dot{\alpha}} * \left( \partial_{\beta\dot{\alpha}}\Phi - \frac{i}{2}[\bar{\Gamma}_{\beta\dot{\alpha}}, \Phi]_* \right) \right] * \text{Tr}(\bar{\Phi}) \quad (4.7)
\end{aligned}$$

are both gauge invariant when integrated. While the first expression involves only gauge-chiral cubic terms in addition to the quadratic term, the second one involves also quartic couplings. Therefore, we have to investigate whether at one-loop the theory develops further divergent terms cubic and/or quartic in the background fields which provide the gauge completion of  $\int \text{Tr}\Phi * \text{Tr}\bar{\Phi}$ .

Divergences proportional to gauge-chiral cubic terms are still obtained from diagrams in Fig. 1 where the internal lines correspond to covariant  $1/\square_-$  and  $1/\hat{\square}$  propagators expanded up to quadratic order in the background gauge superfields (see eqs. (A.6, A.23)). Summing the contributions coming from both diagrams in Fig. 1 we obtain

$$\begin{aligned}
(9h\bar{h} + 2g^2) \mathcal{S} \int d^4x d^4\theta \ 2i \mathcal{F}^{\alpha\beta}\bar{\theta}^2 \left[ \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) + \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Phi}) \right] \\
- 2i(9h\bar{h} - 2g^2) \mathcal{S} \int d^4x d^4\theta \ \mathcal{F}^{\alpha\beta}\bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}) * \text{Tr}(\Phi * \bar{\Phi}) \quad (4.8)
\end{aligned}$$

The first line is exactly the gauge completion of (4.2) according to (4.6). In addition, a second divergent term appears in the second line. Since it is gauge invariant it is allowed by super Ward identities.

We should not expect divergent four-point functions proportional to  $\bar{\Gamma}_{\alpha\dot{\alpha}}$  connections since there is no need to saturate gauge-variation of two-point divergences. In fact, from a direct inspection one can realize that only structures of the form

$$\mathcal{F}^2 \int d^4x d^4\theta \ \bar{\theta}^2 \Phi * \bar{\Phi} * \bar{W}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}} \quad (4.9)$$

can be divergent. For any kind of trace structure all these terms are gauge-invariant and do not interfere with the previous structures.

To summarize, the evaluation of one-loop divergences reveals that the action (4.1) we started with is not renormalizable because of the appearance of new one-loop structures not originally present.

At this stage it is easy to generalize the classical action to a renormalizable one in a gauge invariant way: It is sufficient to start with a classical quadratic action of the form

$$\int d^4x d^4\theta \left\{ \text{Tr}(\bar{\Phi} * \Phi) \right. \tag{4.10}$$

$$\left. + \left[ \text{Tr}\bar{\Phi} * \text{Tr}\Phi + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Phi}) \right] \right\}$$

supplemented by the gauge invariant terms appearing in the second line of (4.8) and in (4.9).

We stress once again that the divergent contributions (4.2) to the quadratic action would be present also in the ordinary, not deformed theory. Therefore, also in that case we would be forced to generalize the classical quadratic action to contain a double-trace part, in order to make the theory renormalizable. The crucial difference is that the double-trace term would be gauge invariant and no gauge completion would be required.

As already mentioned, in the NAC case the double trace quadratic action has in principle two possible gauge completions. From direct inspection, the theory seems to prefer the gauge invariant structure (4.6) rather than (4.7).

## 4.2 The superpotential problem

As we now describe, when a chiral superpotential is turned on the generalization (4.10) for the quadratic matter action is not sufficient to make the theory renormalizable.

Since the nonrenormalization theorem for chiral integrals works also in the NAC case [11, 12], the cubic superpotential in (4.1) does not get corrected by new diagrams proportional to  $\Phi^3$  and/or  $\bar{\Phi}^3$ . As in the ordinary case, the renormalization of the chiral coupling constant is induced by the wave-function renormalization under the requirement that  $Z_h Z_\Phi^{-3/2} = 1$  (a similar relation holds for the antichiral coupling). On the other hand,  $SU(\mathcal{N})$  and  $U(1)$  chiral superfields renormalize differently, so should do the corresponding chiral couplings. Therefore, a cubic superpotential as the one in (4.1) which assigns the same coupling to the  $SU(\mathcal{N})$ ,  $U(1)$  and mixed interaction vertices is inconsistent with the request of renormalizability. We note that this problem is not peculiar of the NAC deformation being present already in the ordinary case.

The way out is once again the generalization of the classical action to include different couplings for different cubic vertices. Exploiting the fact that in Euclidean space  $Z_{\bar{\Phi}}$  is not necessarily equal to  $Z_\Phi$ , we can trigger the renormalization in such a way that for instance all the renormalization asymmetry between non-abelian and abelian fields is confined to the antichiral sector. As a consequence, we can consistently choose the ordinary  $h \int d^4x d^2\theta \text{Tr}(\Phi_*^3)$  superpotential in the chiral sector, but generalize the one for the antichiral sector to

$$\int d^4x d^2\bar{\theta} \left[ \bar{h}_1 \text{Tr}(\bar{\Phi}_*^3) + \bar{h}_2 \text{Tr}\bar{\Phi} * \text{Tr}(\bar{\Phi}_*^2) + \bar{h}_3 (\text{Tr}\bar{\Phi})_*^3 \right] \tag{4.11}$$

However, while in the ordinary case the different structures are separately gauge invariant, in the NAC case the addition of the  $\bar{h}_2, \bar{h}_3$  terms breaks gauge invariance. In fact, due

to the lack of  $\theta$ -integration, the traces are no longer cyclic and  $\delta \int (\text{Tr} \bar{\Phi} * \text{Tr} (\bar{\Phi}_*^2))$  and  $\delta \int (\text{Tr} \bar{\Phi}_*^3)$  are non-vanishing.

The gauge completion of these terms reads

$$\begin{aligned} & \bar{h}_2 \int d^4 x d^2 \bar{\theta} \left\{ \text{Tr} \left( \bar{\Phi} - 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \bar{\Gamma}_\alpha^{\dot{\alpha}} * \left\{ \partial_{\beta\dot{\alpha}} \bar{\Phi} - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Phi}]_* \right\} \right) * \text{Tr} (\bar{\Phi}_*^2) \right. \\ & \left. + \text{Tr} \bar{\Phi} * \text{Tr} \left( \bar{\Phi}_*^2 - 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \bar{\Gamma}_\alpha^{\dot{\alpha}} * \left\{ \partial_{\beta\dot{\alpha}} \bar{\Phi}_*^2 - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Phi}_*^2]_* \right\} \right) \right\} \end{aligned} \quad (4.12)$$

and

$$\bar{h}_3 \int d^4 x d^2 \bar{\theta} \text{Tr} \left( \bar{\Phi} - 6i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \bar{\Gamma}_\alpha^{\dot{\alpha}} * \left\{ \partial_{\beta\dot{\alpha}} \bar{\Phi} - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Phi}]_* \right\} \right) * \text{Tr} (\bar{\Phi}) * \text{Tr} (\bar{\Phi}) \quad (4.13)$$

respectively.

The terms proportional to  $\bar{\Gamma}_{\alpha\dot{\alpha}}$  in the previous expressions break supersymmetry completely since they are given by non-antichiral expressions integrated over an antichiral measure. Therefore, one-loop renormalizability, gauge invariance and  $N = 1/2$  supersymmetry seem to be incompatible. This is the translation in superspace language of the negative result already found in components [18].

### 4.3 The solution to the superpotential problem

Fortunately, generalizing the superpotential to contain more than one coupling constant does not seem to be the only possibility for constructing a renormalizable action. In fact, an alternative procedure exists for treating the diverse renormalization of the abelian fields in a consistent way. The idea is to start with a classical quadratic action of the form (4.10) but with a new coupling in front of the double-trace term

$$\begin{aligned} & \int d^4 x d^4 \theta \left\{ \text{Tr} (\bar{\Phi} * \Phi) + \frac{\kappa - 1}{\mathcal{N}} \left[ \text{Tr} \bar{\Phi} * \text{Tr} \Phi \right. \right. \\ & \left. \left. + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr} (\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr} (\partial_{\beta\dot{\alpha}} \Phi) + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr} (\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr} (\partial_{\beta\dot{\alpha}} \bar{\Phi}) \right] \right\} \end{aligned} \quad (4.14)$$

and tune the renormalization of  $\kappa$  with the wave-function renormalization in order to make  $SU(\mathcal{N})$  and  $U(1)$  superfields to renormalize in the same way. Consequently, a cubic superpotential of the form  $h \int \text{Tr} \bar{\Phi}_*^3 + \bar{h} \int \text{Tr} \Phi_*^3$  can be safely added, with no need of further terms like the ones in (4.11).

As discussed in details in Appendix A, the background field method can be easily generalized to the action (4.14) by performing a change of variables  $\Phi_q \rightarrow \Phi'_q = (\Phi_q^a, \kappa_1 \Phi_q^0)$  and  $\bar{\Phi}_q \rightarrow \bar{\Phi}'_q = (\bar{\Phi}_q^a, \kappa_2 \bar{\Phi}_q^0)$ ,  $\kappa_1 \kappa_2 = \kappa$ , in the functional integral. The net result is a rescaling of the covariant propagators according to eqs. (A.18-A.21). Expanding the propagators in powers of the background gauge fields (see Appendix A) this is equivalent to a rescaling of the abelian propagator

$$\langle \bar{\phi}^0 \phi^0 \rangle = \frac{1}{\kappa} \frac{1}{\square_0} \quad (4.15)$$

and a rescaling of all gauge–chiral interaction vertices involving abelian superfields. Precisely, vertices containing  $\Phi^0, \bar{\Phi}^0$  acquire an extra coupling constant  $1/\kappa_1, 1/\kappa_2$ , respectively.

It is important to note that in the covariant propagators the  $\kappa_1, \kappa_2$  couplings appear only in terms proportional to the deformation parameter. Therefore, the dependence on these two couplings would disappear in the ordinary  $N = 1$  supersymmetric case. In that case, as it is well known, the rescaling (4.15) of the abelian propagator would be the only effect of choosing a modified quadratic lagrangian for the abelian superfields.

To summarize, we begin with a NAC classical gauge theory whose gauge sector is still described by (3.1) or (3.2), whereas the matter action is given by (4.14) supplemented by the single–trace cubic superpotential. However, as appears from one–loop calculations, extra couplings need be considered which are consistent with  $N = 1/2$  supersymmetry and supergauge invariance. In the next Section we will select all possible couplings which can be added at classical level.

## 5 The most general gauge invariant action

Before entering the study of renormalization properties, we will select all possible divergent structures which could come out at quantum level on the basis of dimensional analysis and global symmetries of the theory.

### 5.1 Dimensional analysis and global symmetries

The most general divergent term which may arise at quantum level has the form

$$\int d^4x d^4\theta \bar{\theta}^{\bar{\tau}} \mathcal{F}^\alpha \Lambda^\beta D^\gamma \bar{D}^{\bar{\gamma}} \partial^\delta \bar{\Gamma}^{\bar{\sigma}} \Phi^n \bar{\Phi}^m h^r \bar{h}^s \quad (5.1)$$

where all the exponents are non negative integers. Of course, powers of the gauge coupling  $g$  can appear. However, its presence is irrelevant for our discussion, being  $g$  adimensional and with zero R–symmetry charge. Therefore, in what follows we will neglect it.

We make the following simplifications:

- We can choose the connections to be the bosonic  $\bar{\Gamma}^{\alpha\dot{\alpha}}$ . In fact, thanks to the relation  $\bar{\Gamma}^{\alpha\dot{\alpha}} = -iD_\alpha \bar{\Gamma}_{\dot{\alpha}}$ , switching from bosonic to fermionic connections would amount to shifting  $\gamma \rightarrow \gamma + \bar{\sigma}$ .
- The parameter  $\bar{\tau}$  takes the values 0, 1, 2. However, we can fix it to be 2 by writing  $\bar{\theta}^{\dot{\alpha}} = \bar{D}^{\dot{\alpha}} \bar{\theta}^2 \rightarrow \bar{\theta}^2 \bar{D}^{\dot{\alpha}}$  and  $-1 = \bar{D}^2 \bar{\theta}^2 \rightarrow \bar{\theta}^2 \bar{D}^2$  where we think of integrating by parts the antichiral derivatives.
- Assuming that the NAC deformation is a soft supersymmetry breaking mechanism we set  $\beta = 0$ .

- At one-loop, the  $\Phi^3$  vertex provides a single power of the  $h$  coupling and one external  $\Phi$ -field. Taking into account that further external chirals can come from gauge-chiral vertices, we have the constraint  $r \leq n$ . Similarly, for the antichiral vertex it must be  $s \leq m$ .

Therefore, the general structure for divergences can be reduced to the following form

$$\int d^4x d^4\theta \bar{\theta}^2 \mathcal{F}^\alpha \nabla^\gamma \bar{D}^{\bar{\gamma}} \partial^\delta \bar{\Gamma}^{\bar{\sigma}} \Phi^n \bar{\Phi}^m h^r \bar{h}^s \quad r \leq n \quad , \quad s \leq m \quad (5.2)$$

where the number of  $\nabla$ -derivatives should not exceed  $(\bar{\sigma} + 2(n - 1))$  in order to avoid the integrand to be a total  $\nabla$ -derivative. Further constraints on the exponents come from imposing the global symmetries as listed in Table 1, in addition to the request for the integrand to have mass dimension 2. Moreover, we need impose the number of dotted and undotted indices to be even from the requirement that they contract among themselves to generate a supersymmetry singlet. Finally, we impose  $\alpha \geq 1$  to allow for a non-trivial dependence on the nonanticommutative parameter.

With the charge assignments given in Table 1 the set of constraints read

$$\begin{aligned} \text{Dimensions:} & \quad -3 - \alpha + \frac{\gamma}{2} + \frac{\bar{\gamma}}{2} + \delta + \bar{\sigma} + n + m = 0 \\ \text{R-charge:} & \quad 2 - 2\alpha + \gamma - \bar{\gamma} - n + m + r - s = 0 \\ \text{Index contraction:} & \quad 2\alpha + \gamma + \delta + \bar{\sigma} = 2l + 4 \\ & \quad \bar{\gamma} + \delta + \bar{\sigma} = 2l' \\ \text{Derivatives:} & \quad \gamma \leq \bar{\sigma} + 2n - 2 \\ \text{\(\Phi\)-symmetry:} & \quad n - m + 3(s - r) = 0 \\ \text{One-loop rules:} & \quad r \leq n \\ & \quad s \leq m \end{aligned} \quad (5.3)$$

where  $l, l' \geq 0$  are integer numbers.

Combining the first two equations we get

$$8 - 4l' = 3n + m - r + s \geq 3n + m - r \geq 2n + m \geq 0 \quad (5.4)$$

from which we derive the conditions

$$l' \leq 2 \quad 2n + m \leq 8 - 4l' \quad (5.5)$$

A simple constraint on  $l$  can be obtained from merging the third, the fourth and the sixth equations in (5.3)

$$\begin{aligned} 2(l - l') + 4 &= 2\alpha + \gamma - \bar{\gamma} \leq 2\alpha + \bar{\sigma} + 2n - 2 - \bar{\gamma} \\ &= 2\alpha + 2l' - \delta + 2n - 2\bar{\gamma} - 2 \\ \Rightarrow l &\leq \alpha + 2l' - 3 - \frac{1}{2}\delta + n - \bar{\gamma} \end{aligned} \quad (5.6)$$

	dim	R-charge	$\Phi$ -charge
$\bar{\Gamma}^{\alpha\dot{\alpha}}$	1	0	0
$D_\alpha \equiv \nabla_\alpha$	1/2	1	0
$\bar{D}_{\dot{\alpha}}$	1/2	-1	0
$\theta$	-1/2	1	0
$\partial_{\alpha\dot{\alpha}}$	1	0	0
$\mathcal{F}^{\rho\gamma}$	-1	-2	0
$\Phi$	1	-1	1
$h$	0	1	-3
$\bar{\Phi}$	1	1	-1
$\bar{h}$	0	-1	3

Table 1: Dimensions, R and  $\Phi$ -charge assignments of  $N = 1/2$  operators.

Then, using the first constraint and the previous bound we find

$$2n + m = 3 + \alpha + n - \frac{1}{2}\gamma - \frac{1}{2}\bar{\gamma} - 2l' + \bar{\gamma} \leq 8 - 4l' \quad (5.7)$$

which, after a bit of trivial algebra, provides a constraint on  $\alpha$

$$1 \leq \alpha \leq 4 - l' - \bar{\gamma} \quad (5.8)$$

Finally, using this condition we can constrain  $l$  even more and obtain

$$0 \leq l \leq 5 - \delta - l' - 2\bar{\gamma} \quad (5.9)$$

Now we are ready to list divergent contributions. We assign values 0, 1, 2 to  $l'$  according to (5.5), and we fix  $\delta$ ,  $\bar{\sigma}$  and  $\bar{\gamma}$ , which are bounded by  $l'$  itself. Then we can vary  $l$  into the range given by (5.9) and  $\alpha$  in the range (5.8), while the value of  $\gamma$  follows immediately from the third equation in (5.3). Finally, the remaining parameters  $(n, m, r, s)$  are varied according with the set of equations (5.3).

A detailed investigation reveals that, independently of their particular trace structure, the only allowed terms are (for the moment we forget about  $*$ -products)

1. Matter sector. These structures are obtained by setting  $\bar{\sigma} = 0$  when  $l' = 0, 1$  and correspond to

$$\bullet \bar{h}(h\bar{h})^r \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi \bar{\Phi}^4 \quad r = 0, 1 \quad (5.10)$$

$$\bullet (h\bar{h})^r \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi (\nabla^2 \Phi) \bar{\Phi}^2 \quad r \leq 2 \quad (5.11)$$

$$\bullet h \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi (\nabla^2 \Phi)^2 \quad (5.12)$$

$$\bullet h \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 (\nabla_\alpha \Phi) (\nabla_\beta \Phi) \Phi \quad (5.13)$$



Powers of the gauge coupling  $g$  are also allowed. The first three terms are non-vanishing whatever the color structure is. In the abelian case they correspond to the actual structures which arise at one and two loops in the ungauged NAC WZ model [12, 14, 13]. The last term, instead, is nontrivial only when  $\nabla_\alpha\Phi$  and  $\nabla_\beta\Phi$  have different color index. Therefore, it is present only when gauging the WZ model with a non-abelian group.

2. Mixed sector. All structures selected correspond to the case  $l' = 1$  and are given by

$$\bullet \quad (h\bar{h})^r \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \partial_\beta^{\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}} \Phi \bar{\Phi} \quad (5.14)$$

$$\bullet \quad (h\bar{h})^r \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}_\beta^{\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}} \Phi \bar{\Phi} \quad (5.15)$$

$$\bullet \quad \bar{h} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{\alpha\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}} \bar{\Phi} \bar{\Phi} \bar{\Phi} \quad (5.16)$$

$$\bullet \quad (h\bar{h})^r \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \Phi \bar{\Phi} \quad (5.17)$$

where in (5.14) the space-time derivative can act on any of the three fields.

At one-loop, we can only have  $r = 0, 1$ . When  $r = 0$  a  $g^2$  factor is present and corresponds to contributions generated by mixed gauge-chiral vertices. When  $r = 1$  we have divergent terms generated by pure (anti)chiral vertices.

3. Gauge sector. This case corresponds to  $l' = 2$  because of the bound  $2n + m \leq 8 - 4l' = 0$  which implies  $n = m = 0$ , i.e. no external (anti)chiral fields. The structures we find are exactly the ones found in [15].

The previous analysis can be generalized to the case  $\beta \neq 0$  in (5.1) allowing for positive powers of the UV cut-off. It is not difficult to see that for any positive value of  $\beta$  non-trivial structures which satisfy all the constraints cannot be constructed. This proves that even in the presence of interacting matter supersymmetry is softly broken.

## 5.2 Gauge invariance

The previous structures have been selected without requiring supergauge invariance. We expect that imposing it as a further constraint, only particular linear combinations of the previous terms with specific color structures will survive.

In the matter sector, thanks to the presence of the  $\bar{\theta}^2$  factor, the (anti)chiral interaction terms (5.10–5.12) are gauge-invariant, independently of their color structure. The term (5.13) is non-vanishing only when it is single-trace and it is gauge invariant.

Focusing on the mixed sector, it is easy to see that the general terms (5.16, 5.17) are always gauge invariant, independently of their trace structure.

Terms (5.14, 5.15), instead, give rise to different gauge invariant combinations depending on their trace structure. The only invariant single-trace operator which can arise at

one-loop is

$$\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\{\Phi,\bar{\Phi}\}-\frac{i}{2}[\bar{\Gamma}_{\beta\dot{\alpha}},\bar{\Gamma}_\alpha^{\dot{\alpha}}]_*\{\Phi,\bar{\Phi}\}\right) \quad (5.18)$$

where the explicitly indicated \*-product is the only non-trivial \*-product which appears. Looking at double-trace operators, we already know that structures of the form (5.14, 5.15) combine with the double-trace 2pt function in order to make it gauge invariant (see eqs. (4.6, 4.7)). Further gauge invariant combinations from (5.14, 5.15) are

$$\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\Phi-\frac{i}{2}[\bar{\Gamma}_{\beta\dot{\alpha}},\bar{\Gamma}_\alpha^{\dot{\alpha}}]_*\Phi\right)\text{Tr}(\bar{\Phi}) \quad (5.19)$$

$$\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\bar{\Phi}-\frac{i}{2}[\bar{\Gamma}_{\beta\dot{\alpha}},\bar{\Gamma}_\alpha^{\dot{\alpha}}]_*\bar{\Phi}\right)\text{Tr}(\Phi) \quad (5.20)$$

$$\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\right)\text{Tr}(\Phi\bar{\Phi}) \quad (5.21)$$

while there is no way to saturate the gauge variation of  $\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\bar{\Gamma}_\alpha^{\dot{\alpha}}\right)\text{Tr}((\partial_{\beta\dot{\alpha}}\Phi)\bar{\Phi})$  or, similarly, of the term obtained by exchanging  $\Phi \leftrightarrow \bar{\Phi}$ . Indeed, only the combination  $\left[\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\bar{\Gamma}_\alpha^{\dot{\alpha}}\right)\text{Tr}((\partial_{\beta\dot{\alpha}}\Phi)\bar{\Phi})+\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\bar{\Gamma}_\alpha^{\dot{\alpha}}\right)\text{Tr}(\Phi(\partial_{\beta\dot{\alpha}}\bar{\Phi}))\right]$  is gauge invariant. However, integrating by parts, this reduces to (5.21).

Using similar arguments, we find that the only triple-trace gauge-invariant operator is

$$\mathcal{F}^{\alpha\beta}\bar{\theta}^2\text{Tr}\left(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\right)\text{Tr}(\Phi)\text{Tr}(\bar{\Phi}) \quad (5.22)$$

Finally, looking at the gauge sector, once we impose gauge invariance only terms corresponding to all NAC structures present in (3.1, 3.2) are selected.

### 5.3 The general action

We are now ready to propose the most general classical action for a NAC gauge theory with massless matter in the adjoint of  $SU(\mathcal{N}) \otimes U(1)$ . Introducing the greatest number of coupling constants compatible with gauge invariance, we write

$$S = S_{gauge} + S_{matter} + S_{\bar{\Gamma}} + S_{\bar{W}} \quad (5.23)$$

where  $S_{gauge}$  is given in (3.1) (or equivalently (3.2)),

$$\begin{aligned} S_{matter} = & \int d^4x d^4\theta \left\{ \text{Tr}(\bar{\Phi} * \Phi) + \frac{\kappa - 1}{\mathcal{N}} \left[ \text{Tr}\bar{\Phi} * \text{Tr}\Phi \right. \right. \\ & \left. \left. + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Phi}) \right] \right\} \\ & + h \int d^4x d^2\theta \text{Tr}\Phi_*^3 + \bar{h} \int d^4x d^2\bar{\theta} \text{Tr}\bar{\Phi}_*^3 + \tilde{h}_3 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}((\nabla_\alpha\Phi)(\nabla_\beta\bar{\Phi})\Phi) \\ & + \sum_{j=1}^3 h_3^{(j)} \mathcal{C}_j^{ABC} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi^A (\nabla^2\Phi^B) (\nabla^2\Phi^C) \end{aligned}$$

$$\begin{aligned}
& + \sum_{j=1}^{10} h_4^{(j)} \mathcal{D}_j^{ABCD} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi^A (\nabla^2 \Phi^B) \bar{\Phi}^C \bar{\Phi}^D \\
& + \sum_{j=1}^{12} h_5^{(j)} \mathcal{E}_j^{ABCDE} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \Phi^A \bar{\Phi}^B \bar{\Phi}^C \bar{\Phi}^D \bar{\Phi}^E
\end{aligned} \tag{5.24}$$

and  $S_{\bar{\Gamma}}, S_{\bar{W}}$  contain all possible gauge invariant mixed terms proportional to the bosonic connection

$$\begin{aligned}
S_{\bar{\Gamma}} & = t_1 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr} \left( \partial_{\beta\dot{\alpha}} \bar{\Gamma}_{\alpha}^{\dot{\alpha}} \right) \text{Tr} (\bar{\Phi} \Phi) \\
& + t_2 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr} \left( \partial_{\beta\dot{\alpha}} \bar{\Gamma}_{\alpha}^{\dot{\alpha}} \right) \text{Tr} \bar{\Phi} \text{Tr} \Phi \\
& + t_3 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr} \left( \left( \partial_{\beta\dot{\alpha}} \bar{\Gamma}_{\alpha}^{\dot{\alpha}} - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Gamma}_{\alpha}^{\dot{\alpha}}]_* \right) \{ \bar{\Phi}, \Phi \} \right) \\
& + t_4 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr} \left( \left( \partial_{\beta\dot{\alpha}} \bar{\Gamma}_{\alpha}^{\dot{\alpha}} - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Gamma}_{\alpha}^{\dot{\alpha}}]_* \right) \Phi \right) \text{Tr} \bar{\Phi} \\
& + t_5 \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr} \left( \left( \partial_{\beta\dot{\alpha}} \bar{\Gamma}_{\alpha}^{\dot{\alpha}} - \frac{i}{2} [\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Gamma}_{\alpha}^{\dot{\alpha}}]_* \right) \bar{\Phi} \right) \text{Tr} \Phi \\
& + \sum_{j=1}^{18} \tilde{t}_6^{(j)} \mathcal{G}_j^{ABCDE} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{\Gamma}^{A\alpha\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}}^B \bar{\Phi}^C \bar{\Phi}^D \bar{\Phi}^E
\end{aligned} \tag{5.25}$$

and to the field-strength

$$S_{\bar{W}} = \sum_{j=1}^{12} l_j \mathcal{H}_j^{ABCD} \mathcal{F}^2 \int d^4x d^4\theta \bar{\theta}^2 \bar{W}^{A\dot{\alpha}} \bar{W}_{\dot{\alpha}}^B \Phi^C \bar{\Phi}^D \tag{5.26}$$

We have introduced the following group tensors to take into account all possible color structures (we use the shorten notation  $\text{Tr}(T^A) = (A)$  for any group matrix)

$$\mathcal{C}_1^{ABC} = (ABC) \quad \mathcal{C}_2^{ABC} = (AB)(C) \quad \mathcal{C}_3^{ABC} = (A)(B)(C)$$

$$\begin{aligned}
\mathcal{D}_1^{ABCD} & = (ABCD) & \mathcal{D}_2^{ABCD} & = (ACBD) \\
\mathcal{D}_3^{ABCD} & = (A)(BCD) & \mathcal{D}_4^{ABCD} & = (C)(ABD) \\
\mathcal{D}_5^{ABCD} & = (AB)(CD) & \mathcal{D}_6^{ABCD} & = (AC)(BD) \\
\mathcal{D}_7^{ABCD} & = (AB)(C)(D) & \mathcal{D}_8^{ABCD} & = (AC)(B)(D) & \mathcal{D}_9^{ABCD} & = (A)(B)(CD) \\
\mathcal{D}_{10}^{ABCD} & = (A)(B)(C)(D)
\end{aligned}$$

$$\begin{aligned}
\mathcal{E}_1^{ABCDE} & = (ABCDE) & \mathcal{E}_2^{ABCDE} & = (ABCD)(E) & \mathcal{E}_3^{ABCDE} & = (BCDE)(A) \\
\mathcal{E}_4^{ABCDE} & = (ABC)(DE) & \mathcal{E}_5^{ABCDE} & = (BCD)(AE) & \mathcal{E}_6^{ABCDE} & = (ABC)(D)(E)
\end{aligned}$$

$$\begin{aligned}
\mathcal{E}_7^{ABCDE} &= (BCD)(A)(E) & \mathcal{E}_8^{ABCDE} &= (AB)(CD)(E) & \mathcal{E}_9^{ABCDE} &= (BC)(DE)(A) \\
\mathcal{E}_{10}^{ABCDE} &= (A)(BC)(D)(E) & \mathcal{E}_{11}^{ABCDE} &= (AB)(C)(D)(E) \\
\mathcal{E}_{12}^{ABCDE} &= (A)(B)(C)(D)(E)
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}_1^{ABCDE} &= (ABCDE) & \mathcal{G}_2^{ABCDE} &= (ACBDE) \\
\mathcal{G}_3^{ABCDE} &= (ABCD)(E) & \mathcal{G}_4^{ABCDE} &= (ACBD)(E) & \mathcal{G}_5^{ABCDE} &= (BCDE)(A) \\
\mathcal{G}_6^{ABCDE} &= (ABC)(DE) & \mathcal{G}_7^{ABCDE} &= (BCD)(AE) & \mathcal{G}_8^{ABCDE} &= (AB)(CDE) \\
\mathcal{G}_9^{ABCDE} &= (ABC)(D)(E) & \mathcal{G}_{10}^{ABCDE} &= (BCD)(A)(E) & \mathcal{G}_{11}^{ABCDE} &= (A)(B)(CDE) \\
\mathcal{G}_{12}^{ABCDE} &= (AB)(CD)(E) & \mathcal{G}_{13}^{ABCDE} &= (BC)(DE)(A) & \mathcal{G}_{14}^{ABCDE} &= (BC)(AD)(E) \\
\mathcal{G}_{15}^{ABCDE} &= (A)(BC)(D)(E) & \mathcal{G}_{16}^{ABCDE} &= (AB)(C)(D)(E) \\
\mathcal{G}_{17}^{ABCDE} &= (A)(B)(CD)(E) & \mathcal{G}_{18}^{ABCDE} &= (A)(B)(C)(D)(E)
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_1^{ABCD} &= (ABCD) & \mathcal{H}_2^{ABCD} &= (ACBD) \\
\mathcal{H}_3^{ABCD} &= (A)(BCD) & \mathcal{H}_4^{ABCD} &= (C)(ABD) & \mathcal{H}_5^{ABCD} &= (D)(ABC) \\
\mathcal{H}_6^{ABCD} &= (AB)(CD) & \mathcal{H}_7^{ABCD} &= (AC)(BD) \\
\mathcal{H}_8^{ABCD} &= (AB)(C)(D) & \mathcal{H}_9^{ABCD} &= (AC)(B)(D) & \mathcal{H}_{10}^{ABCD} &= (AD)(B)(C) \\
\mathcal{H}_{11}^{ABCD} &= (A)(B)(CD) & \mathcal{H}_{12}^{ABCD} &= (A)(B)(C)(D)
\end{aligned} \tag{5.27}$$

Whenever in the action the  $*$ -product is not explicitly indicated the products are indeed ordinary products. This happens in most terms above because of the presence of the  $\bar{\theta}^2$  factor.

## 6 One-loop renormalizability and gauge invariance

In this Section we will provide general arguments in support of the one-loop renormalizability of the action (5.23).

The action (5.23) has been obtained by including all possible divergent structures which can appear at one-loop. Therefore, one might be tempted to conclude that it is *a fortiori* renormalizable. However, some of these terms need enter particular linear combinations in order to insure gauge-invariance. Such terms are identified by couplings  $(\kappa - 1)$  and  $t_3, t_4, t_5$ . Therefore, proving one-loop renormalizability amounts to prove that quantum corrections maintain the correct gauge-invariant combinations. In what follows we will be mainly focused on these terms and find the conditions under which gauge invariance is maintained at quantum level.

In order to perform one-loop calculations we use the background-field method revised in Section 2 and applied to the general action (5.23). In Appendix A the necessary Feynman rules are collected.

When drawing possible divergent diagrams we make use of the following observations: First of all, from the dimensional analysis performed in Section 5, one-loop divergences

may be proportional to the non-anticommutation parameter  $\mathcal{F}$  at most quadratically. Therefore, we do not take into account diagrams which give higher powers of  $\mathcal{F}$ . Moreover, the structures we are mainly interested in (the ones associated to the couplings  $(\kappa - 1)$  and  $t_3, t_4, t_5$ ) are proportional to  $\mathcal{F}^{\alpha\beta}$ , so they cannot receive corrections from diagrams which contain vertices proportional to  $\mathcal{F}^2$ .

For each supergraph we perform  $\nabla$ -algebra [19, 20, 21] in order to reduce it to an ordinary momentum graph and read the background structures associated to the divergent integrals. We discuss renormalizability of the different sectors, separately.

## 6.1 Pure gauge sector

In the absence of a superpotential term, the one-loop effective action for the gauge sector has been already computed in [15].

With the addition of the cubic superpotential and the related modifications of the classical action, the gauge effective action could, *a priori*, get corrected because of two different reasons: The modification of the chiral propagators to include different couplings for the abelian superfields which might affect the evaluation of  $\Delta$  in (2.23), and the presence of new mixed gauge-chiral interaction vertices from  $S_{int}$  in (2.22) as coming from  $S_{\overline{\Gamma}}$  and  $S_{\overline{W}}$  and the second line of (5.24).

The former modification is harmless because of the reparametrization invariance of  $\Delta$  under the change of variables  $\Phi^A \rightarrow \Phi'^A \equiv (\Phi^a, \kappa_1 \Phi^0)$ ,  $\overline{\Phi}^A \rightarrow \overline{\Phi}'^A \equiv (\overline{\Phi}^a, \kappa_2 \overline{\Phi}^0)$ ,  $\kappa = \kappa_1 \kappa_2$

$$\begin{aligned} \Delta &= \int D\Phi D\overline{\Phi} \exp \int d^4x d^4\theta \left( \text{Tr} \overline{\Phi} \Phi + \frac{\kappa - 1}{\mathcal{N}} \text{Tr} \overline{\Phi} \text{Tr} \Phi \right) \\ &\sim \int D\Phi' D\overline{\Phi}' \exp \int d^4x d^4\theta \text{Tr} \overline{\Phi}' \Phi' \end{aligned} \quad (6.1)$$

The  $\kappa$ -independence of  $\Delta$  can be also checked by explicit calculations, noting that in its one-loop expansion abelian superfields never enter.

The other source of possible modifications for the gauge effective action is the appearance of new gauge-chiral vertices in  $S_{\overline{\Gamma}}$  and  $S_{\overline{W}}$ , eqs. (5.25) and (5.26), and second line of (5.24). In any case the new vertices produce tadpole-like diagrams when contracting the matter superfields leaving gauge fields as background fields. After  $\nabla$ -algebra, the tadpole provides the covariant propagator  $1/\square_{cov}$  which can be expanded as in (A.23) up to second order in  $\overline{\Gamma}$  producing divergent contributions. It is easy to prove that these divergences cancel exactly as in the ordinary case.

We conclude that the addition of a cubic superpotential and related modifications does not change the results in [15] for the divergent part of the one-loop gauge effective action. Therefore, if we start with a classical action as the one in (3.1) or (3.2) we can multiplicatively renormalize all the divergences of the gauge sector (see Ref. [15] for the detailed calculation).

## 6.2 Gauge–matter sector

We now study one–loop divergent contributions to the rest of the action, i.e.  $S_{matter} + S_{\overline{\Gamma}} + S_{\overline{W}}$  (see eqs. (5.24–5.26)). The contributions identified by the couplings  $(\kappa - 1)$  and  $t_3, t_4, t_5$ , whose gauge invariance is under discussion, belong to this sector. Therefore, we concentrate primarily on this kind of terms.

Divergent contributions come from diagrams in Fig. 2 where internal lines are covariant gauge and chiral propagators (see eqs. (A.1, A.18–A.21)). Expanding the propagators in powers of the background superfields we find two, three and four–point divergences, whereas higher powers give rise to finite contributions.

We analyze the diagrams separately.

### Diagram (2a)

Diagram (2a) is obtained by joining two vertices in Fig. (3a) by one chiral propagator  $1/\square_{-}$  and one vector propagator  $1/\hat{\square}$ . Expanding the propagators at the lowest order,  $1/\square_{-}, 1/\hat{\square} \sim 1/\square$ , we obtain the ordinary divergent quadratic term when the  $*$ –product at the vertices is neglected. Quadratic terms with a nontrivial dependence on  $\mathcal{F}$  are finite. Instead, divergent three–point functions exhibiting a linear dependence on  $\mathcal{F}$  come from the first order expansion of the propagators (see eqs.(A.6, A.23)). Their dependence on the NAC parameter comes either when expanding the  $*$ –product at the vertices or from  $\mathcal{F}^{\alpha\beta}$  terms in eqs. (A.6, A.23). Combining all contributions, diagram (2a) gives rise to

$$\Gamma_2^{(1)}(g) + \Gamma_3^{(1)}(g) + \Gamma_3^{\prime(1)}(g) + \Gamma_4^{(1)}(g) \quad (6.2)$$

where

$$\begin{aligned} \Gamma_2^{(1)}(g) + \Gamma_3^{(1)}(g) = 2g^2 \mathcal{S} \int d^4x d^4\theta \left[ -\mathcal{N} \text{Tr}(\overline{\Phi}\Phi) \right. \\ \left. + \text{Tr}\overline{\Phi} * \text{Tr}\Phi + 2i\overline{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\overline{\Gamma}_\alpha^{\dot{\alpha}} * \overline{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) + 2i\overline{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\overline{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\overline{\Phi}) \right] \end{aligned} \quad (6.3)$$

and

$$\Gamma_3^{\prime(1)}(g) = 4ig^2 \mathcal{S} \mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \overline{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\overline{\Gamma}_\alpha^{\dot{\alpha}}) \text{Tr}(\Phi\overline{\Phi}) \quad (6.4)$$

Four–point functions  $\Gamma_4^{(1)}(g)$  come from the second order expansion of the product of the two propagators. They are divergent but always proportional to  $\mathcal{F}^2$ , therefore automatically gauge–invariant.

We note that the divergences (6.3, 6.4) come in the right linear combinations for preserving gauge–invariance.

### Diagrams (2b, 2c)

With the aim of discussing gauge invariance, it is convenient to consider the sum of diagrams (2b) and (2c). Diagram (2b) is obtained by joining one  $h$  and one  $\bar{h}$  vertices in Fig. (3h, 3j) by two  $1/\square_{-}$  propagators, whereas diagram (2c) is generated from diagram (2b) by the insertion of an extra  $(\kappa - 1)$ –vertex in Figs. (3c, 3d). Expanding the chiral propagators at the lowest order,  $1/\square_{-} \sim 1/\square$  and neglecting the  $*$ –product at the vertices,

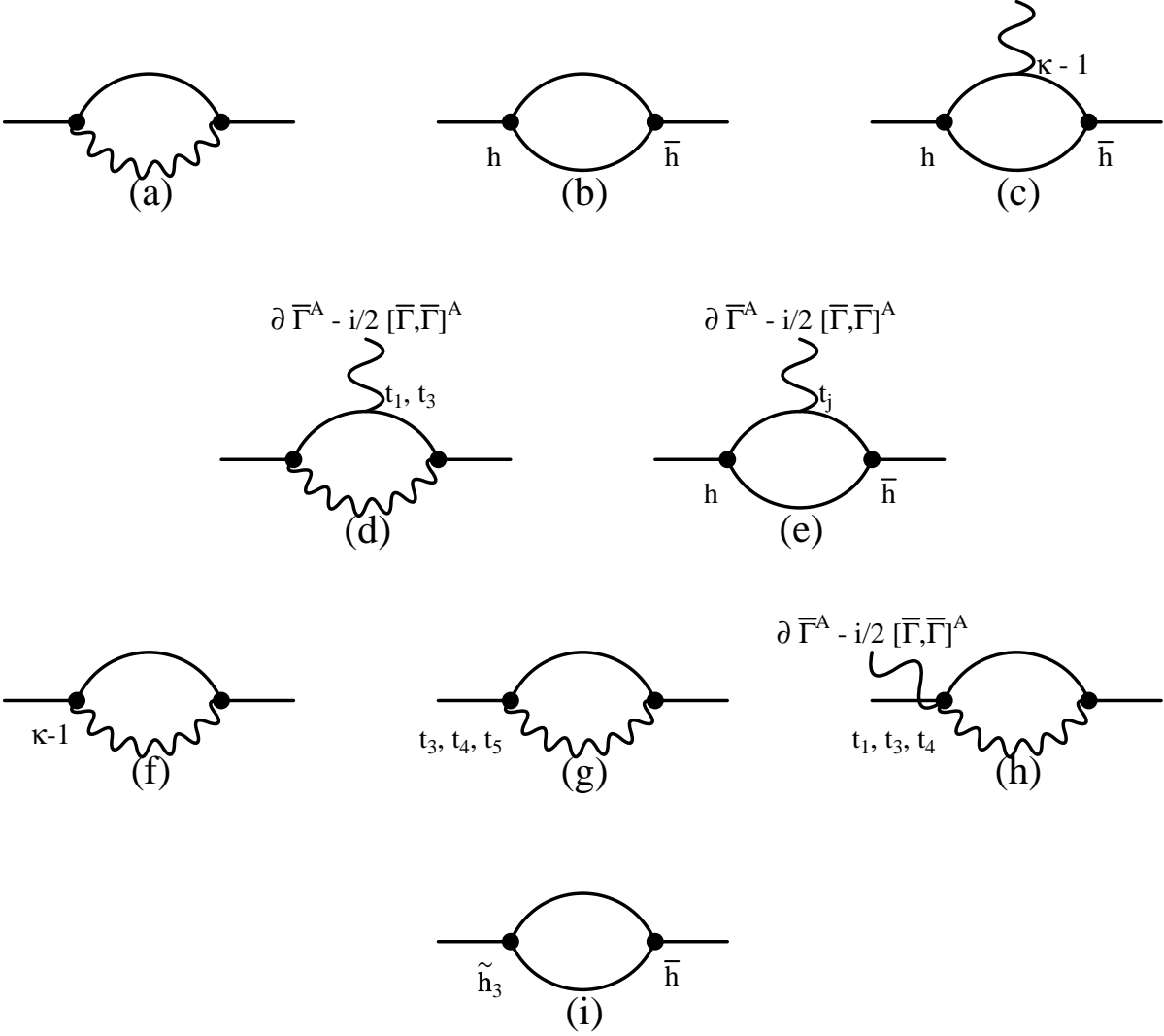


Figure 2: Master diagrams which, after the expansion of the covariant propagators, give rise to two, three and four-point divergent contributions.

from diagram (2b) we obtain the ordinary divergent quadratic term and from diagram (2c) a three-point divergent contribution linear in  $\mathcal{F}$ . Further three and four-point contributions come from the higher order expansion of the propagators in both diagrams. In diagram (2b) the linear dependence in the NAC parameter comes either from terms in the propagator expansion or from the  $*$ -product at the vertices.

Combining all contributions, the sum of the two diagrams gives rise to

$$\Gamma_2^{(1)}(h, \bar{h}) + \Gamma_3^{(1)}(h, \bar{h}) + \Gamma_3^{\prime(1)}(h, \bar{h}) + \Gamma_4^{(1)}(h, \bar{h}) \quad (6.5)$$

where

$$\begin{aligned} & \Gamma_2^{(1)}(h, \bar{h}) + \Gamma_3^{(1)}(h, \bar{h}) = \mathcal{S} \int d^4x d^4\theta \left\{ 9h\bar{h} \left( \mathcal{N} + 4\frac{1-\kappa}{\mathcal{N}\kappa} \right) \text{Tr}(\bar{\Phi} * \Phi) \right. \\ & + 9h\bar{h} \left( 1 + 2\left(\frac{1-\kappa}{\mathcal{N}\kappa}\right)^2 \right) \left[ \text{Tr}\bar{\Phi} * \text{Tr}\Phi + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) \right. \\ & \left. \left. + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Phi}) \right] \right\} \quad (6.6) \end{aligned}$$

$$\begin{aligned} \Gamma_3^{\prime(1)}(h, \bar{h}) &= \left[ \frac{54}{\mathcal{N}} - \frac{18}{\kappa\mathcal{N}} - \frac{18}{\kappa_1\mathcal{N}} - \frac{18}{\kappa_2\mathcal{N}} - 36\frac{1-\kappa}{\kappa\mathcal{N}} \right] i \\ & \quad \times h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}} \{\Phi, \bar{\Phi}\}) \\ & + \left[ -\frac{36}{\mathcal{N}^2} + \frac{36}{\kappa_1\mathcal{N}^2} + \frac{36}{\kappa\mathcal{N}^2} - \frac{36}{\kappa_1\kappa\mathcal{N}^2} \right] i \\ & \quad \times h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\bar{\Phi}) \text{Tr}\Phi \\ & + \left[ -\frac{36}{\mathcal{N}^2} + \frac{36}{\kappa_2\mathcal{N}^2} + \frac{36}{\kappa\mathcal{N}^2} - \frac{36}{\kappa_2\kappa\mathcal{N}^2} - 36\left(\frac{1-\kappa}{\kappa\mathcal{N}}\right)^2 \right] i \\ & \quad \times h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}\Phi) \text{Tr}\bar{\Phi} \\ & + \left[ -\frac{36}{\mathcal{N}^2} + \frac{36}{\kappa_1\mathcal{N}^2} + \frac{36}{\kappa_2\mathcal{N}^2} - \frac{36}{\kappa\mathcal{N}^2} - 72\left(\frac{1-\kappa}{\kappa\mathcal{N}}\right)^2 \right] i \\ & \quad \times h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}) \text{Tr}(\Phi\bar{\Phi}) \\ & + \left[ \frac{36}{\mathcal{N}^3} \frac{1-\kappa}{\kappa} \left( -1 + \frac{1}{\kappa_1} + \frac{1}{\kappa_2} - \frac{1}{\kappa} \right) - 72\left(\frac{1-\kappa}{\kappa\mathcal{N}}\right)^3 \right] i \\ & \quad \times h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Gamma}_\alpha^{\dot{\alpha}}) \text{Tr}\Phi \text{Tr}\bar{\Phi} \quad (6.7) \end{aligned}$$

and

$$\Gamma_4^{(1)}(h, \bar{h}) = -36 \left( \frac{1-\kappa}{\kappa\mathcal{N}} \right) h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}([\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Gamma}_\alpha^{\dot{\alpha}}] \{\Phi, \bar{\Phi}\})$$



$$-36 \left( \frac{1-\kappa}{\kappa\mathcal{N}} \right)^2 h\bar{h}\mathcal{S}\mathcal{F}^{\alpha\beta} \int d^4x d^4\theta \bar{\theta}^2 \text{Tr}([\bar{\Gamma}_{\beta\dot{\alpha}}, \bar{\Gamma}_{\alpha}^{\dot{\alpha}}]\Phi) \text{Tr}\bar{\Phi} \quad (6.8)$$

We note that  $\Gamma_2^{(1)}(h, \bar{h}) + \Gamma_3^{(1)}(h, \bar{h})$  gives a gauge-invariant correction to the quadratic action. On the other hand, in  $\Gamma_3^{(1)}(h, \bar{h})$  the first three lines are not gauge invariant. Possible gauge completions for these terms are contained in  $\Gamma_4^{(1)}(h, \bar{h})$  if the corresponding factors satisfy the following constraints

$$- \frac{i}{2} \left[ \frac{54}{\mathcal{N}} - \frac{18}{\kappa\mathcal{N}} - \frac{18}{\kappa_1\mathcal{N}} - \frac{18}{\kappa_2\mathcal{N}} - 36 \frac{1-\kappa}{\kappa\mathcal{N}} \right] i = -36 \left( \frac{1-\kappa}{\kappa\mathcal{N}} \right) \quad (6.9)$$

$$- \frac{i}{2} \left[ -\frac{36}{\mathcal{N}^2} + \frac{36}{\kappa_1\mathcal{N}^2} + \frac{36}{\kappa\mathcal{N}^2} - \frac{36}{\kappa_1\kappa\mathcal{N}^2} \right] i = 0 \quad (6.10)$$

$$- \frac{i}{2} \left[ -\frac{36}{\mathcal{N}^2} + \frac{36}{\kappa_2\mathcal{N}^2} + \frac{36}{\kappa\mathcal{N}^2} - \frac{36}{\kappa_2\kappa\mathcal{N}^2} - 36 \left( \frac{1-\kappa}{\kappa\mathcal{N}} \right)^2 \right] i = -36 \left( \frac{1-\kappa}{\kappa\mathcal{N}} \right)^2 \quad (6.11)$$

Having introduced two independent couplings  $\kappa_1, \kappa_2$  we have the freedom to fix them in order to satisfy this set of equations. It is easy to see that a non-trivial solution is given by

$$\kappa_1 = 1 \quad , \quad \kappa_2 = \kappa \quad (6.12)$$

with no further requests on  $\kappa$ . Therefore, these conditions provide the right prescription for computing (2b,2c)-type contributions to the effective action while preserving background gauge invariance.

Given the solution (6.12) and recalling eq. (A.11) we conclude that the extra coupling in front of the abelian quadratic action originates entirely from a rescaling of the antichiral superfields.

#### Diagrams (2d)

Diagrams of type (2d) are obtained by inserting in diagram (2a) one  $t_1$  or one  $t_3$  vertex (the insertion of  $t_2, t_4, t_5$  vertices would give diagrams with vanishing color factors). Expanding the propagators and considering only divergent terms linear in the deformation parameter, it is easy to see that the diagram with the insertion of one  $t_1$  vertex gives divergent contributions of the form  $t_1, t_2$  in  $S_{\bar{\Gamma}}$ , whereas the diagram with one  $t_3$  vertex contributes to the  $t_1, t_3, t_4, t_5$  structures. They all come out automatically in the right gauge-invariant combinations.

#### Diagrams (2e)

Diagrams of type (2e) are obtained by inserting in diagram (2b) one of the  $t_j$  vertices. Expanding the propagators and considering only divergent terms linear in the deformation parameter, from diagrams with  $t_1, t_2$  vertices gauge-invariant structures associated to  $t_1$  and  $t_2$  in  $S_{\bar{\Gamma}}$  arise. From diagrams with the insertion of vertices  $t_3, t_4, t_5$  the background structure proportional to  $\partial_{\beta}^{\dot{\alpha}}\bar{\Gamma}_{\alpha\dot{\alpha}}$  combines with the structure  $[\bar{\Gamma}_{\beta}^{\dot{\alpha}}, \bar{\Gamma}_{\alpha\dot{\alpha}}]$  to give gauge-invariant divergent contributions of the form  $t_1, \dots, t_5$ .

### Diagrams (2f)

This kind of diagrams are obtained by contracting the  $(\kappa - 1)$  vertices with a quantum gauge  $V$ -field (see Figs. (3e, 3f, 3g)) with the ordinary vertex in Fig. (3a). Expanding the covariant propagators it is easy to see that they are either vanishing or finite.

### Diagrams (2g)

This class of diagrams is constructed by contracting a  $t_3, t_4, t_5$ -vertex in Fig. (3p) with the ordinary vertex (3a) (diagrams with  $t_1$  and  $t_2$  vertices vanish for color reasons). Explicit calculations reveal that nontrivial cancellations occur, so that no divergent contributions arise proportional to  $t_4$  and  $t_5$ , whereas a non-vanishing term is generated by  $t_3$  which is automatically in the right linear combination to respect gauge invariance. Precisely, it corrects  $t_1, t_3, t_4, t_5$  couplings.

### Diagrams (2h)

These diagrams are obtained by contracting one vertex (3o) with the ordinary vertex (3a). In all cases divergences arise when expanding the propagators at lowest order (self-energy diagrams). They are automatically gauge invariant and correct the  $t_1, t_3, t_4, t_5$  couplings.

### Diagram (2i)

Finally, possible divergent contributions come from contracting the  $\tilde{h}_3$  vertex with the ordinary  $\bar{h}$ -vertex in Fig. (3j). They come from expanding the propagators up to the first order in  $\bar{\Gamma}$ . Even in this case non-trivial cancellations occur and the final result is the sum of non-vanishing, but gauge invariant contributions to the  $t_1, t_3, t_4, t_5$  couplings.

The list of diagrams we have analyzed includes all possible divergent diagrams linear in the deformation parameter. Any other divergence is necessarily proportional to  $\mathcal{F}^2$  and comes either from the expansion of the  $*$ -products in the previous diagrams or from new diagrams constructed from  $\mathcal{F}^2$ -vertices in (5.23). Since we know that any single  $\mathcal{F}^2$  term is automatically gauge-invariant and appears in the action with its own coupling, we can immediately conclude that the  $\mathcal{F}^2$  sector of the action is one-loop renormalizable.

In conclusion, we have provided evidence that the general action (5.23) is multiplicatively renormalizable. Its renormalization can be then performed by setting

$$\begin{aligned}
\Phi_B^a &= Z^{\frac{1}{2}} \Phi^a & , & & \bar{\Phi}_B^a &= \bar{Z}^{\frac{1}{2}} \bar{\Phi}^a \\
\Phi_B^0 &= Z^{\frac{1}{2}} \Phi^0 & , & & \bar{\Phi}_B^0 &= \bar{Z}^{\frac{1}{2}} \bar{\Phi}^0 \\
(\kappa - 1)_B &= Z_\kappa (\kappa - 1) \\
h_B &= Z_h h & , & & \bar{h}_B &= Z_{\bar{h}} \bar{h} \\
& & & & \tilde{h}_{3B} &= Z_{\tilde{h}_3} \tilde{h}_3 \\
h_{3B}^{(j)} &= Z_{h_3^{(j)}} h_3^{(j)} & , & & h_{4B}^{(j)} &= Z_{h_4^{(j)}} h_4^{(j)} & , & & h_{5B}^{(j)} &= Z_{h_5^{(j)}} h_5^{(j)} \\
t_{nB} &= Z_{t_n} t_n & & & n &= 1, \dots, 5
\end{aligned}$$

$$\begin{aligned}\tilde{h}_{6B}^{(j)} &= Z_{\tilde{h}_6^{(j)}} \tilde{h}_6^{(j)} \\ l_{nB} &= Z_{l_n} l_n \quad n = 1, \dots, 12\end{aligned}\tag{6.13}$$

where we have assigned the same renormalization function to the abelian and non-abelian scalar superfields.

We consider for instance the nontrivial renormalization of the quadratic matter action, first two lines of eq. (5.24). At one-loop, in terms of renormalized superfields, we can write

$$\begin{aligned}\Gamma_{1loop} \rightarrow \int d^4x d^4\theta \left\{ \left( (Z\bar{Z})^{\frac{1}{2}} - 1 + \frac{a}{\epsilon} \right) \text{Tr}(\bar{\Phi} * \Phi) + \right. \\ \left. \left( (Z\bar{Z})^{\frac{1}{2}} Z_\kappa - 1 + \frac{b}{\epsilon} \right) \frac{\kappa - 1}{\mathcal{N}} \left[ \text{Tr}\bar{\Phi} * \text{Tr}\Phi + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}}\Phi) \right. \right. \\ \left. \left. + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}}\bar{\Phi}) \right] \right\}\end{aligned}\tag{6.14}$$

where, from eqs. (6.3, 6.6) we read

$$\begin{aligned}a &= \frac{1}{(4\pi)^2} \left[ -2g^2\mathcal{N} + 9h\bar{h} \left( \mathcal{N} + 4\frac{1-\kappa}{\kappa\mathcal{N}} \right) \right] \\ b &= \frac{1}{(4\pi)^2} \frac{1}{\kappa - 1} \left[ 2g^2\mathcal{N} + 9h\bar{h} \left( 1 + 2\left( \frac{1-\kappa}{\kappa\mathcal{N}} \right)^2 \right) \right]\end{aligned}\tag{6.15}$$

In order to cancel divergences we can set

$$\begin{aligned}Z = \bar{Z} = 1 - \frac{1}{(4\pi)^2} \frac{1}{\epsilon} \left[ -2g^2\mathcal{N} + 9h\bar{h} \left( \mathcal{N} + 4\frac{1-\kappa}{\kappa\mathcal{N}} \right) \right] \\ Z_\kappa = 1 + \frac{1}{(4\pi)^2} \frac{1}{\epsilon} \left[ -2g^2\mathcal{N} \frac{\kappa}{\kappa - 1} + 9h\bar{h}\mathcal{N} \left( \frac{\kappa - 2}{\kappa - 1} \right) - 18\frac{h\bar{h}}{\kappa^2\mathcal{N}}(2\kappa^2 - \kappa - 1) \right]\end{aligned}\tag{6.16}$$

Different choices with  $Z \neq \bar{Z}$  are also allowed.

Renormalization of the rest of the couplings then follows, accordingly.

## 7 Conclusions

In this paper we have studied the problem of the renormalizability for nonanticommutative  $N = 1/2$  SYM theories in the presence of interacting matter. The introduction of a superpotential for (anti)chiral superfields complicates the investigation of the quantum properties of the gauge theory, not only from a technical point of view. In fact, at a first sight the non-trivial interplay between partial breaking of supersymmetry, gauge invariance of the action and renormalization procedure leads to drastic consequences for the theory: In NAC geometry only  $SU(\mathcal{N}) \otimes U(1)$  gauge theories are well defined and, as in the ordinary case, the renormalization of the kinetic term requires a different renormalization function for the  $SU(\mathcal{N})$  and  $U(1)$  wave-functions. Consequently, superpotential

terms proportional to the abelian fields need appear with different coupling constants. In superspace formalism this can be realized by generalizing the single-trace (anti)chiral interaction to contain different trace structures, each one with its own coupling. However, the addition of multi-trace terms, while completely harmless in the ordinary SYM theories, in the NAC case affects the theory in a non-trivial way. In fact, these terms are no longer gauge singlets and require suitable completions which break explicitly the residual  $N = 1/2$  supersymmetry.

The way-out we have proposed amounts to re-establish perfect equivalence between  $SU(\mathcal{N})$  and  $U(1)$  wave-function renormalizations by multiplying the abelian quadratic term by an extra coupling constant. As a consequence, a single-trace superpotential is allowed which respects  $N = 1/2$  supersymmetry and supergauge invariance. Basically, we have shifted the problem of deforming the action from the superpotential to the Kähler potential or, in other words, from an integral on chiral variables to an integral on the whole superspace. This has the nice effect to leave the residual  $N = 1/2$  supersymmetry unbroken. It is important to stress that in contradistinction with the ordinary case where rescaling the abelian kinetic term or suitably rescaling the superpotential couplings lead to equivalent theories, in the NAC case this is no longer true. In one case we obtain a consistent  $N = 1/2$  theory whereas in the other case we lose completely supersymmetry. The ultimate cause is the non-trivial NAC gauge transformations undergone by the abelian superfields.

Having solved the main problem of adding a matter cubic superpotential we have studied the most general divergent structures which could arise at loop level selecting them on the basis of dimensional considerations and global symmetries. We have then proposed the action (5.23) as the most general renormalizable gauge-invariant  $N = 1/2$  deformation of the ordinary SYM field theory with interacting matter. The next steps should be the complete study of one-loop renormalization, the computation of the  $\beta$ -functions and the implementation of the massive case. Moreover, strictly speaking our results hold only at one-loop. Higher loop calculations would be necessary to further confirm the good renormalization properties of our action.

Generalizing in an obvious way our construction to include more than one (anti)chiral superfields would lead to a consistent NAC generalization of the  $N = 4$  SYM. This would be an important step towards clarifying the stringy origin of NAC deformations and deformations of the AdS/CFT correspondence. In particular, it would be nice to investigate how robust properties of  $N = 4$  SYM like finiteness and integrability might be affected by NAC deformations.

Finally, our approach could be easily applied to the abelian three-field Wess-Zumino model studied in [22].

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## A Feynman rules for the general action (5.23)

In this Appendix we apply the NAC background field method to the action (5.23) and derive the Feynman rules necessary for calculations of Section 6.

### Gauge sector

We first concentrate on the gauge sector. As discussed in details in Ref. [15] and reviewed in Section 2, with the convenient choice of the gauge-fixing action (2.13), in Feynman gauge the covariant gauge propagators are

$$\begin{aligned}\langle V^a V^b \rangle &= g^2 \left( \frac{1}{\hat{\square}} \right)^{ab} \\ \langle V^0 V^0 \rangle &= g^2 \left\{ \frac{1}{\tilde{\square}} \left[ 1 + \left( \frac{g^2}{g^2 + g_0^2} \right) \nabla^{\dot{\alpha}} * \nabla^2 * \nabla_{\dot{\alpha}} * \frac{1}{\tilde{\square}} \right] \right\}^{00}\end{aligned}\quad (\text{A.1})$$

where  $\hat{\square}, \tilde{\square}$  have been defined in (2.15, 2.16) in terms of  $\square_{cov}$ . On a generic superfield in the adjoint representation of  $SU(\mathcal{N}) \otimes U(1)$  we have

$$\begin{aligned}(\square_{cov} * \phi)^A &= \left( \frac{1}{2} \nabla^{\alpha\dot{\alpha}} * \nabla_{\alpha\dot{\alpha}} * \phi \right)^A \\ &= \left( \square\phi - i[\bar{\Gamma}^{\alpha\dot{\alpha}}, \partial_{\alpha\dot{\alpha}}\phi]_* - \frac{i}{2}[(\partial^{\alpha\dot{\alpha}}\bar{\Gamma}_{\alpha\dot{\alpha}}), \phi]_* - \frac{1}{2}[\bar{\Gamma}^{\alpha\dot{\alpha}}, [\bar{\Gamma}_{\alpha\dot{\alpha}}, \phi]_*]_* \right)^A \\ &\equiv \square_{cov}^{AB} * \phi^B\end{aligned}\quad (\text{A.2})$$

Using the general NAC rule

$$[F, G]_*^A = \frac{1}{2} i f^{ABC} \{F^B, G^C\}_* + \frac{1}{2} d^{ABC} [F^B, G^C]_* \quad (\text{A.3})$$

valid for any couple of field functions in the adjoint representation of the gauge group, and expanding the  $*$ -product we find

$$\begin{aligned}\square_{cov}^{AB} &= \square \delta^{AB} + f^{ACB} \bar{\Gamma}^{C\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + i d^{ACB} \mathcal{F}^{\alpha\beta} (\partial_{\alpha} \bar{\Gamma}^{C\gamma\dot{\gamma}}) \partial_{\beta} \partial_{\gamma\dot{\gamma}} - \frac{1}{2} f^{ACB} \mathcal{F}^2 (\partial^2 \bar{\Gamma}^{C\alpha\dot{\alpha}}) \partial^2 \partial_{\alpha\dot{\alpha}} \\ &+ \dots\end{aligned}\quad (\text{A.4})$$

Only the first two terms in (A.2) have been explicitly indicated. The rest can be treated in a similar manner.

The  $\frac{1}{\hat{\square}}$  and  $\frac{1}{\tilde{\square}}$  propagators can be expanded in powers of the background fields. We formally write

$$\begin{aligned}\frac{1}{\hat{\square}} &= \frac{1}{\square_{cov}} + \frac{1}{\square_{cov}} * \left( i \tilde{W}^{\alpha} \nabla_{\alpha} + i \bar{W}^{\dot{\alpha}} * \nabla_{\dot{\alpha}} \right) * \frac{1}{\hat{\square}} \\ \frac{1}{\tilde{\square}} &= \frac{1}{\square_{cov}} + \frac{1}{\square_{cov}} * \left( i \tilde{W}^{\alpha} \nabla_{\alpha} - \frac{i}{2} (\nabla^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}}) \right) * \frac{1}{\tilde{\square}}\end{aligned}\quad (\text{A.5})$$

Expanding the right hand side we obtain terms proportional to  $\tilde{W}_\alpha, \bar{W}_{\dot{\alpha}}$  and terms proportional to the bosonic connections coming from  $1/\square_{cov}$ . As follows from dimensional considerations and confirmed by direct inspection, terms proportional to the field strengths never enter divergent diagrams as long as we focus on contributions linear in the NAC parameter. Therefore, at this stage we can neglect them. Using the expansion (A.4) we then find

$$\begin{aligned} & \left( \frac{1}{\widehat{\square}} \right)^{ab}, \left( \frac{1}{\widetilde{\square}} \right)^{00} \rightarrow \left( \frac{1}{\square_{cov}} \right)^{AB} \\ & \simeq \frac{1}{\square} \delta^{AB} - \frac{1}{\square} f^{ACB} \bar{\Gamma}^{C\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \frac{1}{\square} - \frac{1}{2} \frac{1}{\square} f^{ACD} f^{DEB} \bar{\Gamma}^{C\alpha\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}}^E \frac{1}{\square} \\ & \quad - \frac{1}{\square} i d^{ACB} \mathcal{F}^{\alpha\beta} (\partial_\alpha \bar{\Gamma}^{C\gamma\dot{\gamma}}) \partial_\beta \partial_{\gamma\dot{\gamma}} \frac{1}{\square} + \frac{1}{2} \frac{1}{\square} f^{ACB} \mathcal{F}^2 (\partial^2 \bar{\Gamma}^{C\alpha\dot{\alpha}}) \partial^2 \partial_{\alpha\dot{\alpha}} \frac{1}{\square} + \dots \end{aligned} \quad (\text{A.6})$$

In this expression we recognize the ordinary bare propagator  $1/\square$  plus a number of gauge interaction vertices.

Further interactions come from the expansion of the remaining terms in (3.1) or (3.2). Their explicit expression can be found in Appendix E of [15].

### Matter sector

We now derive propagators and interaction vertices for the action  $S_{matter} + S_{\bar{\Gamma}} + S_{\bar{W}}$  in (5.23). Since in this paper we are primarily interested in computing divergent contributions linear in the NAC parameter, we restrict our analysis to Feynman rules which contribute to this kind of terms. In particular, we do not take into account vertices proportional to  $\mathcal{F}^2$ .

We first concentrate on the calculation of the chiral propagators. As given in eq. (5.24) the full covariant scalar quadratic term is

$$\int d^4x d^4\theta \left\{ \text{Tr}(\bar{\Phi}\Phi) + \frac{\kappa-1}{\mathcal{N}} \text{Tr}\bar{\Phi}\text{Tr}\Phi \right\} \quad (\text{A.7})$$

which can be expanded in terms of the background covariantly (anti)chiral fields (2.12) as

$$\begin{aligned} & \int d^4x d^4\theta \left\{ \text{Tr}(\bar{\Phi} * e^V * \Phi * e^{-V}) + \frac{\kappa-1}{\mathcal{N}} \text{Tr}(\bar{\Phi}) \text{Tr}(e^V * \Phi * e^{-V}) \right\} \\ & = \int d^4x d^4\theta \left\{ \text{Tr} \left( \bar{\Phi}\Phi + \bar{\Phi}[V, \Phi]_* + \frac{1}{2} \bar{\Phi}[V, [V, \Phi]_*]_* + \dots \right) \right. \\ & \quad \left. + \frac{\kappa-1}{\mathcal{N}} \text{Tr}(\bar{\Phi}) \text{Tr} \left( \Phi + [V, \Phi]_* + \frac{1}{2} [V, [V, \Phi]_*]_* + \dots \right) \right\} \quad (\text{A.8}) \end{aligned}$$

We perform the quantum-background splitting

$$\Phi \rightarrow \Phi + \Phi_q, \quad \bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q \quad (\text{A.9})$$

and concentrate on the evaluation of the quadratic functional integral

$$\int \mathcal{D}\Phi_q \mathcal{D}\bar{\Phi}_q e^{\int d^4x d^4\theta \{ \text{Tr}(\bar{\Phi}_q \Phi_q) + \frac{\kappa-1}{\mathcal{N}} \text{Tr}\bar{\Phi}_q \text{Tr}\Phi_q \}} \quad (\text{A.10})$$

In order to deal with a simpler integral we make the change of variables

$$\Phi_q^A \rightarrow \Phi_q'^A = (\Phi_q^a, \kappa_1 \Phi_q^0) \quad , \quad \bar{\Phi}_q^A \rightarrow \bar{\Phi}_q'^A = (\bar{\Phi}_q^a, \kappa_2 \bar{\Phi}_q^0) \quad (\text{A.11})$$

where  $\kappa_1$  and  $\kappa_2$  are two arbitrary constants satisfying  $\kappa_1 \kappa_2 = \kappa$ . The functional integral (A.10) then takes the standard form

$$\int \mathcal{D}\Phi'_q \mathcal{D}\bar{\Phi}'_q e^{\int d^4x d^4\theta \text{Tr}\bar{\Phi}'_q \Phi'_q} \quad (\text{A.12})$$

We stress that the redefinition (A.11) in terms of two independent couplings is admissible because we are working in Euclidean space where chiral and antichiral fields are not related by complex conjugation.

Adding source terms

$$\begin{aligned} & \text{Tr} \int d^4x d^2\theta \ j \Phi'_q + \text{Tr} \int d^4x d^2\bar{\theta} \ \bar{\Phi}'_q \bar{j} \\ & = \text{Tr} \int d^4x d^4\theta \ \left( j * \frac{1}{\square_+} * \nabla^2 \Phi'_q + \bar{\Phi}'_q * \frac{1}{\square_-} * \bar{\nabla}^2 * \bar{j} \right) \end{aligned} \quad (\text{A.13})$$

with  $\square_{\pm}$  defined in (2.21), and taking into account the complete action the quantum partition function reads

$$\begin{aligned} \mathbf{Z}(j, \bar{j}) & = e^{S_{int}(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{j}})} \int \mathcal{D}\Phi'_q \mathcal{D}\bar{\Phi}'_q \exp \text{Tr} \int d^4x d^4\theta \ [ \bar{\Phi}'_q \Phi'_q \\ & \quad + j * \frac{1}{\square_+} * \nabla^2 \Phi'_q + \bar{\Phi}'_q * \frac{1}{\square_-} * \bar{\nabla}^2 * \bar{j} ] \end{aligned} \quad (\text{A.14})$$

Here  $S_{int}$  contains all gauge–scalar fields interaction vertices in (A.8) plus interactions coming from the rest of terms in  $S_{matter} + S_{\bar{\Gamma}} + S_{\bar{W}}$ .

We can perform the Gaussian integral in (A.15) by standard techniques, obtaining the NAC generalization of the usual superspace expression [19]

$$\mathbf{Z} = \Delta * e^{S_{int}(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{j}})} \exp \left( - \int d^4x d^4\theta \ j * \frac{1}{\square_-} * \bar{j} \right) \quad (\text{A.15})$$

where  $\Delta$  is the functional determinant

$$\Delta = \int \mathcal{D}\Phi'_q \mathcal{D}\bar{\Phi}'_q \exp \text{Tr} \int d^4x d^4\theta \ \bar{\Phi}'_q \Phi'_q \quad (\text{A.16})$$

which contributes to the gauge effective action [15].



From the expression (A.15) we can read the covariant propagators for prime superfields

$$\langle \Phi_q'^A \bar{\Phi}_q'^B \rangle = - \left( \frac{1}{\square_-} \right)^{AB} \quad (\text{A.17})$$

which, in terms of the original  $\Phi, \bar{\Phi}$  superfields gives

$$\langle \Phi_q^a \bar{\Phi}_q^b \rangle = - \left( \frac{1}{\square_-} \right)^{ab} \quad (\text{A.18})$$

$$\langle \Phi^0 \bar{\Phi}^b \rangle = - \frac{1}{\kappa_2} \left( \frac{1}{\square_-} \right)^{0b} \quad (\text{A.19})$$

$$\langle \Phi^a \bar{\Phi}^0 \rangle = - \frac{1}{\kappa_1} \left( \frac{1}{\square_-} \right)^{a0} \quad (\text{A.20})$$

$$\langle \Phi^0 \bar{\Phi}^0 \rangle = - \frac{1}{\kappa} \left( \frac{1}{\square_-} \right)^{00} \quad (\text{A.21})$$

The expansion of the scalar covariant propagators can be performed following a prescription similar to the one used for the gauge propagator. We can formally write

$$\frac{1}{\square_-} = \frac{1}{\square_{cov}} + \frac{1}{\square_{cov}} * \left( i \bar{W}^{\dot{\alpha}} * \bar{\nabla}_{\dot{\alpha}} + \frac{i}{2} (\bar{\nabla}^{\dot{\alpha}} * \bar{W}_{\dot{\alpha}}) \right) * \frac{1}{\square_-} \quad (\text{A.22})$$

Since terms proportional to the field strengths never enter divergent diagrams linear in  $\mathcal{F}^{\alpha\beta}$ , we can neglect them and write

$$\begin{aligned} \left( \frac{1}{\square_-} \right)^{AB} &\rightarrow \left( \frac{1}{\square_{cov}} \right)^{AB} \\ &\simeq \frac{1}{\square} \delta^{AB} - \frac{1}{\square} f^{ACB} \bar{\Gamma}^{C\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \frac{1}{\square} - \frac{1}{2} \frac{1}{\square} f^{ACD} f^{DEB} \bar{\Gamma}^{C\alpha\dot{\alpha}} \bar{\Gamma}_{\alpha\dot{\alpha}}^E \frac{1}{\square} \\ &\quad - \frac{1}{\square} i d^{ACB} \mathcal{F}^{\alpha\beta} (\partial_{\alpha} \bar{\Gamma}^{C\gamma\dot{\gamma}}) \partial_{\beta} \partial_{\gamma\dot{\gamma}} \frac{1}{\square} + \frac{1}{2} \frac{1}{\square} f^{ACB} \mathcal{F}^2 (\partial^2 \bar{\Gamma}^{C\alpha\dot{\alpha}}) \partial^2 \partial_{\alpha\dot{\alpha}} \frac{1}{\square} + \dots \end{aligned} \quad (\text{A.23})$$

The first term is diagonal in the color indices and gives the ordinary bare propagator. The rest provides interaction vertices between scalars and gauge superfields.

From the expansion (A.23) it is clear that the mixed propagators (A.19, A.20) are always proportional to the NAC parameter, according to the fact that in the  $N = 1$  limit they need vanish. It follows that the dependence on the  $\kappa_1$  and  $\kappa_2$  couplings is peculiar of the NAC theory, whereas in the ordinary limit only their product  $\kappa$  survives.

Additional interaction terms are contained in  $S_{int}$  and arise from the background field expansion of the full action  $S_{matter} + S_{\bar{\Gamma}} + S_{\bar{W}}$ . We now describe the correct way to obtain such vertices concentrating only on the ones at most linear in  $\mathcal{F}^{\alpha\beta}$ .

We begin by considering  $S_{matter}$ . From the quadratic action  $\int d^4x d^4\theta \text{Tr} \bar{\Phi} \Phi$ , after the expansion (A.8) and the shift (A.9) we obtain (3a,3b)-type vertices in Fig. 3 where  $V$  is

quantum and  $\Phi$  and/or  $\bar{\Phi}$  are background. Expanding the  $*$ -products ordinary vertices plus vertices proportional to  $\mathcal{F}^{\alpha\beta}$  and  $\mathcal{F}^2$  arise.

We then consider the  $(\kappa - 1)$  terms in (5.24)

$$\begin{aligned} \frac{\kappa - 1}{\mathcal{N}} \int d^4x d^4\theta \left[ \text{Tr} \bar{\Phi} * \text{Tr} \Phi \right. \\ \left. + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \bar{\Phi}) * \text{Tr}(\partial_{\beta\dot{\alpha}} \Phi) + 2i\bar{\theta}^2 \mathcal{F}^{\alpha\beta} \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} * \Phi) * \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \right] \end{aligned} \quad (\text{A.24})$$

We expand the (anti)chiral superfields as

$$\Phi \rightarrow \Phi + \Phi_q + [V, \Phi + \Phi_q]_* + \frac{1}{2}[V, [V, \Phi + \Phi_q]_*]_* \quad , \quad \bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_q \quad (\text{A.25})$$

and, at the same order in  $V$ , the gauge connection as

$$\bar{\Gamma}_{\alpha\dot{\alpha}} \rightarrow \bar{\Gamma}_{\alpha\dot{\alpha}} - \nabla_\alpha [\bar{\nabla}_{\dot{\alpha}}, V]_* + \frac{1}{2}\nabla_\alpha [[\bar{\nabla}_{\dot{\alpha}}, V]_*, V]_* \quad (\text{A.26})$$

Collecting the various terms we generate (3c,3d)–vertices in Fig. 3 with background gauge connections and quantum matter plus (3e,3f,3g)–vertices with quantum gauge and  $\Phi$  or  $\bar{\Phi}$  background.

As a nontrivial example, we derive in details the contributions (3e,3f,3g). Forgetting for a while the superspace integration and the overall coupling constant and writing  $\partial_\alpha = \nabla_\alpha - i\bar{\theta}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$ , from the first term in (A.24) we have

$$\begin{aligned} \text{Tr}([V, \Phi]_*) \text{Tr} \bar{\Phi} &\rightarrow -\mathcal{F}^{\alpha\beta} \text{Tr}([\partial_\alpha V, \partial_\beta \Phi]) \text{Tr} \bar{\Phi} \\ &\rightarrow -2i\mathcal{F}^{\alpha\beta} \bar{\theta}^{\dot{\alpha}} \text{Tr}(V \nabla_\alpha \Phi) \text{Tr} \partial_{\beta\dot{\alpha}} \bar{\Phi} - 2\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}(\partial_\alpha^{\dot{\alpha}} V \Phi) \text{Tr} \partial_{\beta\dot{\alpha}} \bar{\Phi} \end{aligned} \quad (\text{A.27})$$

where superspace total derivatives have been neglected and  $\Phi, \bar{\Phi}$  stand for either quantum or background.

Using the expansion (A.26) the second term in (A.24) gives a contribution of the form

$$2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} \bar{\Phi}) \text{Tr}(\partial_{\beta\dot{\alpha}} \Phi) \rightarrow -2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}([\bar{\nabla}^{\dot{\alpha}}, V] \bar{\Phi}) \text{Tr}(\partial_{\beta\dot{\alpha}} \nabla_\alpha \Phi) \quad (\text{A.28})$$

Similarly, the third term in (A.24) gives

$$\begin{aligned} 2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) &\rightarrow 2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \left\{ \text{Tr}(\bar{\Gamma}_\alpha^{\dot{\alpha}} [V, \Phi]) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \right. \\ &\left. - \text{Tr}(\nabla_\alpha \bar{D}^{\dot{\alpha}} V \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) + \text{Tr}([V, \bar{\Gamma}_\alpha^{\dot{\alpha}}] \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) - i\text{Tr}([\bar{\Gamma}^{\dot{\alpha}}, \nabla_\alpha V] \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \right\} \\ &= 2\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}(\partial_\alpha^{\dot{\alpha}} V \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) + 2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}([\bar{\nabla}^{\dot{\alpha}}, \nabla_\alpha V] \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \end{aligned} \quad (\text{A.29})$$

Summing the three contributions a nontrivial cancellation occurs between the second term in (A.27) and the first term in (A.29) and we are left with

$$\begin{aligned} \frac{\kappa - 1}{\mathcal{N}} \int d^4x d^4\theta \left\{ -2i\mathcal{F}^{\alpha\beta} \bar{\theta}^{\dot{\alpha}} \text{Tr}(V \nabla_\alpha \Phi) \text{Tr} \partial_{\beta\dot{\alpha}} \bar{\Phi} - 2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}([\bar{\nabla}^{\dot{\alpha}}, V] \bar{\Phi}) \text{Tr}(\partial_{\beta\dot{\alpha}} \nabla_\alpha \Phi) \right. \\ \left. + 2i\mathcal{F}^{\alpha\beta} \bar{\theta}^2 \text{Tr}([\bar{\nabla}^{\dot{\alpha}}, \nabla_\alpha V] \Phi) \text{Tr}(\partial_{\beta\dot{\alpha}} \bar{\Phi}) \right\} \end{aligned} \quad (\text{A.30})$$

which correspond to the three vertices (3e, 3f, 3g).

The rest of terms in the  $S_{matter}$  can be easily treated by the shift (A.25). Neglecting  $\mathcal{F}^2$  contributions only the superpotential and the  $\tilde{h}_3$  term survive and lead to pure matter vertices of the form (3h, 3i, 3j, 3k, 3l) and the mixed vertex (3m).

We now turn to  $S_{\overline{\Gamma}}$  and briefly sketch the quantization of  $t_j$  vertices. At linear order in the NAC parameter we can forget the  $*$ -product in the commutators of  $t_3, t_4, t_5$  terms. We perform the shift (A.25) on the (anti)chirals and (A.26) on the connection. In particular, for the gauge invariant linear combination appearing in  $t_3, t_4, t_5$  terms we have

$$\partial_{\beta\dot{\alpha}}\overline{\Gamma}_{\alpha}^{\dot{\alpha}} - \frac{i}{2}[\overline{\Gamma}_{\beta\dot{\alpha}}, \overline{\Gamma}_{\alpha}^{\dot{\alpha}}] \longrightarrow \partial_{\beta\dot{\alpha}}\overline{\Gamma}_{\alpha}^{\dot{\alpha}} - \frac{i}{2}[\overline{\Gamma}_{\beta\dot{\alpha}}, \overline{\Gamma}_{\alpha}^{\dot{\alpha}}] - \overline{\nabla}_{\beta\dot{\alpha}}\overline{\nabla}_{\alpha}\overline{\nabla}^{\dot{\alpha}} V \quad (\text{A.31})$$

Collecting only the contributions which may contribute at one-loop we produce the (3n) vertex in Fig. 3 where matter is quantum and (3o, 3p) vertices where  $\Phi$  or  $\overline{\Phi}$  are quantum. We note that they all exhibit a gauge-invariant background dependence.

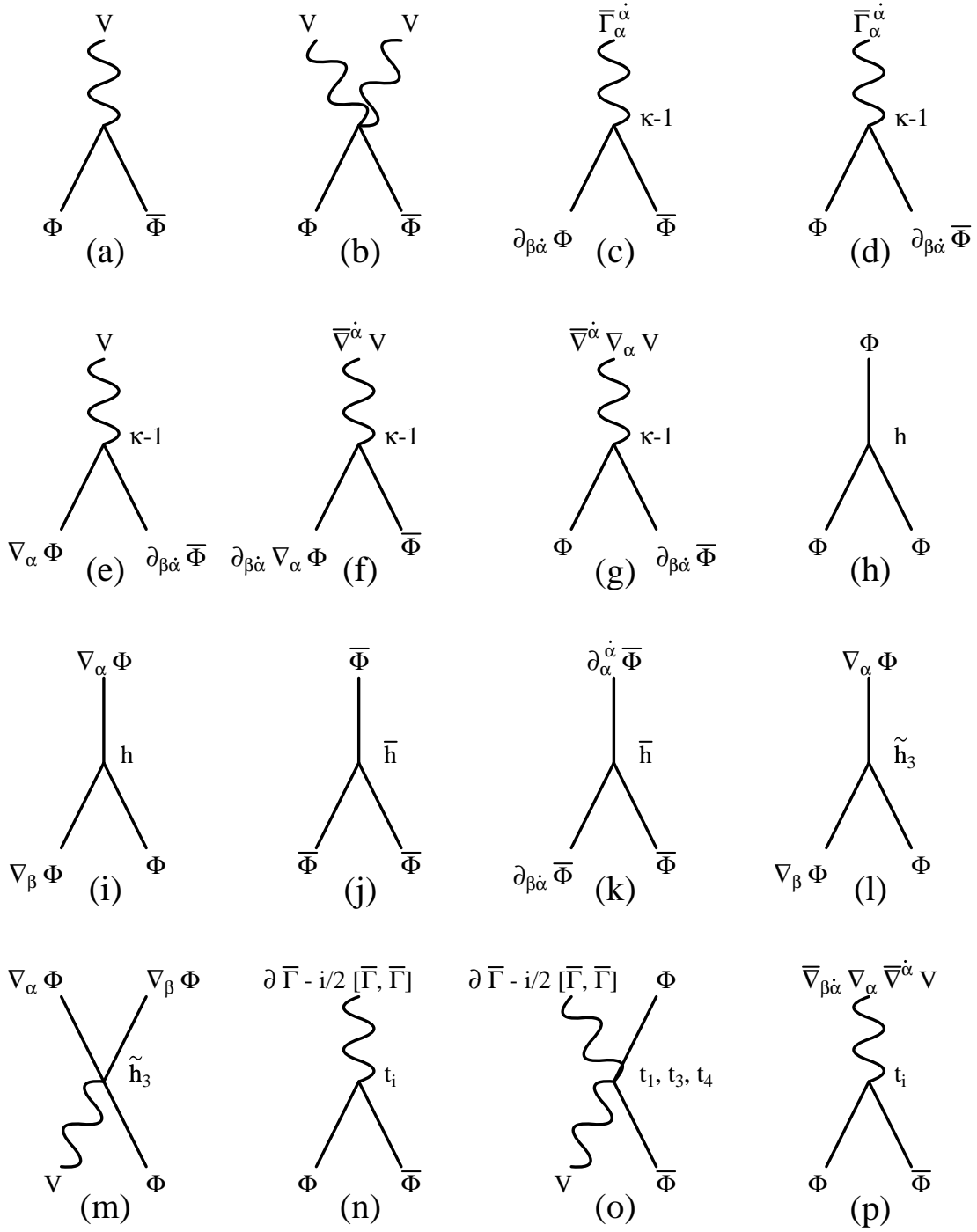


Figure 3: Vertices from the action (5.23) at most linear in the NAC parameter  $\mathcal{F}^{\alpha\beta}$ . The (a,b,h,j)-vertices are order zero in  $\bar{\theta}$ , the (e)-vertex is proportional to  $\bar{\theta}^\alpha$  whereas the remaining vertices are all proportional to  $\bar{\theta}^2$ .

## References

- [1] N. Seiberg and E. Witten, JHEP **9909** (1999) 032 [arXiv:hep-th/9908142].
- [2] H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **7** (2003) 53 [arXiv:hep-th/0302109];  
H. Ooguri and C. Vafa, Adv. Theor. Math. Phys. **7** (2004) 405 [arXiv:hep-th/0303063].
- [3] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, Phys. Lett. B **574** (2003) 98, hep-th/0302078.
- [4] N. Seiberg, JHEP **0306** (2003) 010 [arXiv:hep-th/0305248].
- [5] S. Ferrara and M. A. Lledo, JHEP **0005**, 008 (2000) [arXiv:hep-th/0002084].
- [6] D. Klemm, S. Penati and L. Tamassia, Class. Quant. Grav. **20**, 2905 (2003) [arXiv:hep-th/0104190].
- [7] S. Ferrara, M. A. Lledo and O. Macia, JHEP **0309** (2003) 068 [arXiv:hep-th/0307039].
- [8] M. Billo, M. Frau, I. Pesando and A. Lerda, JHEP **0405** (2004) 023 [arXiv:hep-th/0402160].
- [9] E. Ivanov, O. Lechtenfeld and B. Zupnik, JHEP **0402** (2004) 012 [arXiv:hep-th/0308012];  
S. Ferrara and E. Sokatchev, Phys. Lett. B **579** (2004) 226 [arXiv:hep-th/0308021];  
S. Ferrara, E. Ivanov, O. Lechtenfeld, E. Sokatchev and B. Zupnik, Nucl. Phys. B **704** (2005) 154 [arXiv:hep-th/0405049];  
I. L. Buchbinder, E. A. Ivanov, O. Lechtenfeld, I. B. Samsonov and B. M. Zupnik, Nucl. Phys. B **740** (2006) 358 [arXiv:hep-th/0511234].
- [10] S. Terashima and J. T. Yee, JHEP **0312** (2003) 053 [arXiv:hep-th/0306237].
- [11] R. Britto, B. Feng and S. J. Rey, JHEP **0307** (2003) 067 [arXiv:hep-th/0306215];  
R. Britto, B. Feng and S. J. Rey, JHEP **0308** (2003) 001 [arXiv:hep-th/0307091].
- [12] M. T. Grisaru, S. Penati and A. Romagnoni, JHEP **0308**, 003 (2003) [arXiv:hep-th/0307099];  
M. T. Grisaru, S. Penati and A. Romagnoni, Class. Quant. Grav. **21**, S1391 (2004) [arXiv:hep-th/0401174].
- [13] I. Jack, D. R. T. Jones and R. Purdy, arXiv:0808.0400 [hep-th].
- [14] R. Britto and B. Feng, Phys. Rev. Lett. **91**, 201601 (2003) [arXiv:hep-th/0307165];  
A. Romagnoni, JHEP **0310**, 016 (2003) [arXiv:hep-th/0307209].

- [15] S. Penati and A. Romagnoni, JHEP **0502** (2005) 064 [arXiv:hep-th/0412041];  
M. T. Grisaru, S. Penati and A. Romagnoni, JHEP **0602**, 043 (2006) [arXiv:hep-th/0510175].
- [16] I. Jack, D. R. T. Jones and L. A. Worthy, Phys. Lett. B **611**, 199 (2005) [arXiv:hep-th/0412009];  
I. Jack, D. R. T. Jones and L. A. Worthy, Phys. Rev. D **72**, 065002 (2005) [arXiv:hep-th/0505248].
- [17] O. Lunin and S. J. Rey, JHEP **0309** (2003) 045 [arXiv:hep-th/0307275];  
D. Berenstein and S. J. Rey, Phys. Rev. D **68** (2003) 121701 [arXiv:hep-th/0308049].
- [18] I. Jack, D. R. T. Jones and L. A. Worthy, Phys. Rev. D **75**, 045014 (2007) [arXiv:hep-th/0701096].
- [19] S.J. Gates, M.T. Grisaru, M. Rocek and W. Siegel, “Superspace”, Benjamin-Cummings, Reading, MA, 1983. *Second printing*: Front. Phys. **58** (1983) 1 [arXiv:hep-th/0108200].
- [20] M. T. Grisaru and W. Siegel, “Supergraphity. 2. Manifestly Covariant Rules And Higher Loop Finiteness,” Nucl. Phys. B **201** (1982) 292 [Erratum-ibid. B **206** (1982) 496];  
M. T. Grisaru and D. Zanon, “Covariant Supergraphs. 1. Yang-Mills Theory,” Nucl. Phys. B **252** (1985) 578.
- [21] M. T. Grisaru and D. Zanon, *In \*West, P.c. ( Ed.): Supersymmetry: A Decade Of Development\**, 32-68.
- [22] I. Jack, D. R. T. Jones and R. Purdy, arXiv:0901.2876 [hep-th].