A Structured Approach to Optimization of Energy Harvesting Wireless Sensor Networks

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Abstract—We analyze the data throughput maximization problem over fading channels for an energy harvesting wireless sensor network. The effective use of energy harvesting wireless sensor networks requires an apt understanding of the underlying environmental processes. Energy as a constrained resource must be carefully utilized so as to enable good performance through a horizon of epochs, trading off sensing performance and energy usage. We develop an algorithm for the allocation of power across sensors and time which is based on observations of the underlying structure of the optimization problem. We provide an analysis of particular features of the problem and present a maximum sum rate algorithm for generalized energy flow constraints, which proves to be optimal given known energy availability and fading coefficients. We present simulation results to validate the theory and algorithm.

Index Terms—energy harvesting, wireless sensor networks, water-filling, convex optimization

I. INTRODUCTION

In wireless sensor networks (WSN) which utilize energy harvesters, energy usage is constrained due to limited energy storage capabilities, and is in constant tension with sensing and communication performance. Distributed estimation, control, and other applications via WSNs presents a variety of interesting challenges to the limited energy available to sensors node. Rate maximization in such a framework is one such challenge. A common solution is to devise energy harvesting and management techniques to allocate power to sensor nodes at run-time [1], [2]. Since harvestable energy in the environmental energy is often limited and stochastic, the goal of rate maximization is to communicate efficiently while relying on unpredictable energy resources. Our work addresses this problem by designing and analyzing a WSN-specific, energy-harvesting aware rate maximization algorithm across sensors and epochs.

Work on individual energy-harvesting sensor-transmitters has been done in [3]–[5]. Buffer control is considered in [3] using a dynamic programming framework, and energy queues are utilized optimally with a back-pressure algorithm in [4]. The authors of [5] consider the maximum rate energy-neutral problem for a multiple epoch fading channel of a single transmitter and develop “directional water-filling” heuristics.

Previous work on rate allocation for wireless sensor networks has been done in [6]–[9]. The approach of utility maximization for the wireless sensor network is taken for linear utility functions in [6], [8]. More general assumptions of concave differential utility functions are used in [7], and non-differentiable concave utility functions in [9]. The authors in [8] propose an energy budgeting algorithm that specifies the energy levels for each node and epoch. Research presented in [10], [11] presents the Network Utility Maximization (NUM) problem for Internet congestion control which is adapted in [9] for a WSN application. Work in [12], [13] address utility maximization for energy-constrained systems that execute periodic real time assignments for solar-powered systems in which a highly dynamic energy harvesting model is assumed. However, [12] assumes a limited set of epochs (only two).

This paper highlights the structure of the constrained optimization framework that results in an intuitive algorithm for selecting active sensor nodes and allocating power usage in spatially uncorrelated fading channels for multiple epochs. Although rate maximization is a tractable problem, numerical convex programming techniques do not provide a clear understanding of the problem behavior, which we develop. Other similar works do not consider a horizon or non-uniform concave rate functions [7], [9]. A similar method in [13] involves extending the horizon as energy is utilized, while our method considers the entire sensor-horizon space simultaneously. Work in [8] simultaneously considers epochs and sensors, but the objective is assumed to be linear and not appropriate for rate maximization [9]. While our method uses a specific utility function ($\sum \log(\cdot)$), it considers a general constraint framework in which various routing structures or scheduling constraints can be included as opposed to [5], [9] which are closest to our work, but consider only either the sensor or epoch dimensions.

II. SYSTEM MODEL

We consider a cooperative network of wireless sensors communicating via some orthogonal signaling scheme over fading channels with additive Gaussian random noise. Each of these nodes utilizes an energy harvester, processing unit, and sensing devices. The $N$ nodes communicate data either directly to a centralized base station [14], or along a predefined routing path
to the base station [9]. An illustration of an example WSN setup is shown in Figure 1. The received signal at one node from another is given by $y_{mn}(k) = x_n(k) \cdot \sqrt{h_n(k) + n_n(k)}$ where $x_n(k)$ is the transmitted signal during epoch $k$, with $h_n(k)$ and $n_n(k)$ respectively the (squared) uncorrelated fading and additive Gaussian noise of the channel between nodes $n$ and $m$. Whenever a signal is transmitted for duration $L$, $\frac{L}{2} \log \left( 1 + \frac{h_n(k)p_n(k)}{\sigma^2_n(k)} \right)$ bits of data are sent to the receiving node from the queue of data at node $n$, where $\sigma^2_n(k)$ is the variance of the zero mean Gaussian random noise. We assume throughout the rest of our work that $\sigma^2_n(k) = 1 \ \forall \ n, k$, although the methods herein are easily adapted to case of non-uniform noise variance. This transmission costs $Lp_n$ units of energy from the battery. The indices $n, m, i, j, k \in \mathbb{N}$ unless otherwise noted.

For the sake of simplicity in presentation, we will consider fixed epoch lengths while the authors in [5] model epochs as being of random size determined by some arrival distribution. The assumption of fixed epoch lengths caters to the implementation of many wireless sensor network systems, which utilize duty cycles of some practically fixed period. This reduces power consumption but can increase delay [1], [15]. It is a simple task to adapt the subsequent algorithm in the case of the non-uniform time step. An additional foregone assumption is that the fading can change at moments interspersed between energy arrivals. Instead, we consider a single fade and transmit power level per epoch. These assumptions fit for scenarios with slow and/or correlated fading across time. We also assume causal knowledge of this fading is available at the nodes.

In our system model, we consider the energy costs associated with sensing and processing to be negligible relative to the wireless transceiver energy consumption [9], [14]. Signal reception energy usage is controlled by the MAC layer, using various bandwidth division schemes [15]. Therefore, we model the per epoch energy cost for signal reception as a constant, $E_{rcv}$. We proceed further to eliminate this complication by noting that the arriving energy, denoted $E_n'(k)$, in each epoch and at each sensor can have its receive energy extracted before optimization of transmit energies. That is, energy which can be allocated for transmission is

$$E_n(k) = [E_n'(k) - E_{rcv}]^+,$$

where $\lfloor . \rfloor^+ = \max\{., 0\}$. In this way, our problem will only consider the assignment of transmission power levels.

### A. Energy Harvesting Model

The energy harvested at each node could be from a variety of sources, e.g., generated by wind, solar, or seismic activity. The collected energy is stored in a battery, super-capacitor, or other storage device which has a maximum transmit energy capacity denoted, $E_{\text{max}} = E_{\text{max}}' - E_{\text{rcv}}$, with $E_{\text{max}}'$ the actual energy storage capacity. We adopt the assumption made in [5] that all of the gathered energy quanta are less than the maximum storage capacity, i.e., $E_n(k) \leq E_{\text{max}}$. This assumption is intuitive since even if an energy harvesting device could harvest more than the capacity of its battery, the energy would be supplied to the transmitter through the battery, thus limiting the usable energy. We define epochs so that energy collected in epoch $k$ is denoted $E_n(k)$, with allocated power $p_n(k)$ which can use energy $E_n(j), j = 0, \ldots, k$. We also follow the common assumption that the harvested energy is uncertain but predictable [1], [7]–[9]. Constraints imposed on the optimization by the causality of the harvested energy and by the limited storage capacity will be discussed in Section III.

### B. Prediction Horizon

The prediction horizon, $K$, is the number of epochs wherein the harvested energy and correlated fading can be predicted reasonably well [9]. Our approach considers the optimal selection of transmission power levels over such a horizon of epochs with the assumption that the prediction of energy arrivals and fading coefficients has already been made. We will qualify the use of ‘optimal’ in the next section. The use of our algorithm as an online method can easily be reached by such simple extensions as stochastic Model Predictive Control, in which the action for the first epoch is taken, after which predictions and allocations are recalculated again for the remaining horizon.

### III. The Optimization Problem

In this section, we analyze the multi-sensor multi-epoch sum rate optimization problem. We claim optimality of our method if the harvested energy and fading levels are perfectly known (non-causally). The problem is to select the power levels $p_n(k) \forall n \in [1, N], k \in [1, K]$ achieving the optimal sum rate: $\max_{p} U$, with

$$U = \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{L}{2} \log \left( 1 + h_n(k)p_n(k) \right),$$

Figure 1. An example of the energy harvesting wireless sensor network setup. Nodes transmit data through additive Gaussian fading channel with (causal) channel state feedback.
subject to the constraints induced by the wireless network communications and the energy harvesting systems; \( p \) is the non-negative (an implicit constraint) vectorization of all power allocations, across sensors and epochs. We next develop some intuition from a more general problem and relax our indices to represent a set of channels for a single time index (which is neglected). When a communications system has the ability to distribute power to various channels with disparate losses under a total energy constraint, the optimal solution obeys the following Lemma.

**Lemma 1** (Water-filling). Given \( h \in \mathbb{R}^{N \times 1} \), \( h_n \geq 0 \) \( \forall \ n \), the optimization problem

\[
\max_{p} \sum_{n=1}^{N} \frac{L}{2} \log(1 + h_n p_n) \quad \text{subject to} \quad \sum_{n=1}^{N} L p_n \leq E_{\text{tot}}
\]

with respect to non-negative power allocations \( p \in \mathbb{R}^{N \times 1} \), has the well known water-filling solution (when the total constraint \( E_{\text{tot}} \) is met with equality). The optimal power allocation is

\[
p^* = \left[ \frac{1}{L \lambda} - \frac{1}{h_n} \right]^+ = \left[ \frac{1}{|A|} \left( E_{\text{tot}} + \sum_{\sigma(1) \in A} \frac{L}{h_{\sigma(i)}} \right) - \frac{1}{h_n} \right]^+,
\]

\( \forall n \), where

\[
A = \left\{ j \mid \sum_{i=m}^{N} \frac{1}{L \lambda(i)} \left( E_{\text{tot}} + \sum_{\sigma(1) \in A} \frac{L}{h_{\sigma(i)}} \right) - \frac{1}{h_{\sigma(i)}} \leq E_{\text{tot}} \right\}
\]

with \( \lambda(m) = 1/(E_{\text{tot}} + \sum_{i=m}^{N} \frac{1}{h_{\sigma(i)}}) \) the constraint Lagrange multiplier, \( A \) the set of indices of the non-zero power allocations, \( |\cdot| \) the number of elements in a set, \( ^+ \) indicating optimal, \( E_{\text{tot}} \) the total energy usage constraint, and \( \sigma(\cdot) \) some permutation of the natural numbers \( 1, \ldots, N \) such that the elements of \( h \) are ordered as in the definition of \( A \) above.

The proof of this is well known and is found easily from Lagrange analysis [16]. The loss levels \( h_n \) are arranged in decreasing order by the permutation \( \sigma(\cdot) \). The solution can be found by iterating: adding the channel with the smallest loss, and then the next smallest, and up to largest. The set \( A \) is found by excluding the loss level (and all greater than it) that produce a violation of the total energy constraint. Reconsidering the time dimension, the constraint in the water-filling solution could be implemented as a per epoch total transmit energy constraint, (see Section V).

### A. Energy Harvesting Induced Constraints

As specified in the introduction to the model, the energy harvesting problem induces causality and storage limitation constraints of the form

\[
\sum_{k=1}^{j} L p_n(k) \leq \sum_{k=0}^{j} E_n(k)
\]

\[
\sum_{k=0}^{j} E_n(k) - \sum_{k=1}^{j} L p_n(k) \leq E_{\text{max}},
\]

\( \forall n \in [1, N], j \in [1, K] \), with \( E_n(0) \) the initial battery level and \( E_n(1) \) the initial collected energy available at node \( n \).

### B. Generalized Energy Flow Constraints

A structure of the energy flow constraint which generalizes for the total epoch, link budget, causality, and battery constraints, given below:

**Proposition 2.** The constraints in (3), (5), and (6) are generalized by the linear constraint

\[
f_{\text{nk}}^e(p, E) = a_{nk} L \sum_{m:(m,j)\in F_{nk}} \sum_{j:(j,m)\in F_{nk}} p_m(j) - b_{nk} \sum_{m:(m,j)\in G_{nk}} E_m(j) - C_{nk}
\]

where \( a_{nk}, b_{nk} \in \{-1, 1\} \) and \( C_{nk} \geq 0 \) are constants, \( F_{nk}, G_{nk} \) represent fixed sets of pairs of indices (e.g., \( (m, j) \)), and \( E \) is the vectorization of available energy across sensors and epochs. We will use the superscript \( :^z \) to denote the type of constraint (causality \((c)\), energy storage \((s)\), total energy \((t)\), or link budget \((b)\)), which will aid in describing the algorithm.

These types of constraints may not be indexed at all or may only be referenced by a single dimension. For example, consider the total energy constraint indexed only by epoch,

\[
f_{k}^e(p, E) = a_k L \sum_{m:(m,j)\in F_k} \sum_{j:(j,m)\in F_k} p_m(j) - b_k \sum_{m:(m,j)\in G_k} E_m(j) - C_k.
\]

We now examine the link budget constraint which fits this framework.

**Example** In the model we specified that the nodes communicate either directly to the central base station, or through a predefined route. The general water-filling solution already illustrates the former. The link budget energy constraint associated with a predefined route can be expressed, at a particular node \( n \) and epoch \( k \), as

\[
L p_n(k) + \sum_{m:S_n\in \rho_k(S_m)} L p_m \leq B_n(k)
\]

where \( \{S_1, \ldots, S_N\} \) represents the nodes in a graph-like sense, and \( \rho_k(S_m) \) in epoch \( k \) is the sequence of nodes from \( S_m \) to the base station, \( S_0 \). It is typically assumed these paths (links) form an acyclic graph [9]. Alternatively, the budget could be constrained over epochs.

### C. Lagrange Analysis

We explore the Karush-Kuhn-Tucker conditions [17], attempting to uncover the structural behavior of the problem. We already have the result of the KKT analysis for a total constraint in the form of Lemma 1. If we focus on the energy causality and limited storage constraints (i.e., for a single node, \( n \)) the Lagrangian becomes
\[ \mathcal{L}_\lambda = \sum_{k=1}^{K} \log(1 + h_n(k)p_n(k)) - \sum_{j=1}^{\lambda_{nj}} \left( \sum_{k=1}^{j} Lp_n(k) - \sum_{k=1}^{j} E_n(k) \right) - \sum_{j=1}^{\mu_{nj}} \left( \sum_{k=1}^{j} E_n(k) - \sum_{k=1}^{j} Lp_n(k) - E_{max} \right), \]

where \( \lambda_{nj} \) and \( \mu_{nj} \) are the Lagrange multipliers of the causality and energy storage constraints, respectively. The first order condition produces the optimal power allocation (across epochs),

\[ p_n^*(k) = \left[ \frac{1}{\sum_{j=1}^{\lambda_{nj}} L(\lambda_{nj} - \mu_{nj})} - \frac{1}{h_n(k)} \right]^+. \]

Motivated by this solution and the complementary slackness conditions, a result detailed in \([5], [13]\) is the following.

**Lemma 3.** For \( E_{max} = \infty \) (which implies \( \mu_{nj} = 0 \quad \forall \quad n, j \)), regardless of whether all the energy in each epoch is used in that epoch (whether \( \lambda_{nj} = 0 \) for any \( k < K \)), the water levels \( \nu_{nk} = \frac{1}{\sum_{j=k}^{\lambda_{nj}} L(\lambda_{nj} - \mu_{nj})} \) will monotonically increase, since \( \lambda_{nj} \geq 0 \quad \forall \quad n, j \).

The proof of this is found in \([5]\) and does not necessarily hold for finite \( E_{max} \). Continuing our quest to expose familiar and novel elements of the water filling problem in Lemma 1, we provide the following motivating example.

**Example** If we are given a scenario satisfying \( E_n(1) \leq E_n(2) \leq \cdots \leq E_n(K) \) and \( h_n(1) \leq h_n(2) \leq \cdots \leq h_n(K) \), with \( \sum_{k=1}^{K} E_n(k) \leq E_{max} \). Such an ordering provides us with a solution where energy only flows to future epochs. Since by construction, the battery constraint is never violated, epochs with the smallest loss (largest \( h_n(k) \)) will contain the largest power levels. In fact, the solution mimics that of the solution in Lemma 1. This is because the fading levels and energy availability are ‘pre-ordered’ such that the there is no re-ordering necessary to find the optimal common ‘water level’ over the epochs.

Similarly, if the available energy is ordered the same but with the fading coefficients in reverse order, we have the following.

**Corollary 4.** If \( \lambda_{nj} \neq 0 \quad \forall \quad n, j \) then the optimal power allocation is

\[ p_n^*(k) = \frac{E_n(k)}{L} \quad \forall \quad n, k \]

and this solution is only possible when the following hold:

\[ E_n(1) \leq E_n(2) \leq \cdots \leq E_n(K) \]

and

\[ h_n(1) \geq h_n(2) \geq \cdots \geq h_n(K). \]

The conceptual proof is as follows. Since the fading loss \((1/h_n(k))\) is least in the beginning epochs, more energy should be allocated to them. However, energy availability is also the least in the beginning epochs, and since causality prevents energy from reaching past epochs, only \( E_n(k) \) is available.

**IV. SPACE-TIME WATER-FILLING ALGORITHM**

We now present an algorithm based on observations in the previous section and the Lemma presented next. If we first return to the total energy constrained water-filling solution in (3) and rewrite the objective as

\[ U = \sum_{n=1}^{N} \frac{L}{2} \log \left( \frac{1}{h_n} + p_n \right) - \frac{L}{2} \log \left( \frac{1}{\lambda_n} \right). \]

We see that the first order condition dictates that

\[ \frac{1}{h_n} + p_n = \frac{1}{h_m} + p_m = \frac{L}{\lambda} \quad \forall \quad n, m \in \mathcal{A}, \]

where we use the previous notation for \( \lambda \) and \( \mathcal{A} \). We observed that the water-filling solution (in (3)) leverages the following principle \([9], [13]\).

**Lemma 5** (Optimal Concave Assignment). Given concave function \( U(\cdot) \) and variables \( c_1, \ldots, c_N \geq 0 \) such that \( \sum_{i=1}^{N} c_i \leq C_{tot} \), with \( C_{tot} \) a non-negative constant, then \( \sum_{i=1}^{N} U(c_i) \) is maximized when \( c_1 = \cdots = c_N = \frac{C_{tot}}{N} \).

The proof of this is offered in \([13]\) and follows from Jensen’s inequality for concave functions. This principle will guide the development of our algorithm since our objective is a sum of concave functions. We will now discuss the minimum-improvement space-time water-filling (MIST-WF) method in Algorithm 1.

A prerequisite to analyzing the optimality of our algorithm is verifying feasibility. As opposed to other types of concave function maximization schemes with flow constraints which must often project back into the feasible space \([7]\), we propose a solution which achieves constraints (or remains feasible) exactly in each update. It could be considered a type of minimum cost network flow problem \([18]\), but for the use of concave reward functions. Therefore, our method does not require primal-dual or other such iterations, nor must we address convergence, as the problem produces an exact solution in a fixed maximum number of steps.

**Lemma 6** (Feasibility). Given a set of energy flow constraints, \( f_{nk}(p, E) \leq 0 \quad \forall \quad n, k, z \quad \text{and} \quad \{c, e, t, b\} \), available energy \( E \), and associated power levels \( p \), which are selected according to Algorithm 1, then it holds that

\[ f_{nk}(p, E) \leq 0 \quad \forall \quad n, k, z \]

at every update of the algorithm.

Lemma 6 follows simply from the fact that our algorithm constructs allocations such that no infeasibility is ever incurred.

For the next discussion, we extend the definition of the set \( \mathcal{A} \) to denote the set of pairs of indices \((n, k)\) for which power
allocation is non-zero and uniform according to Lemma 5. In applying this Lemma, we have used \( \frac{1}{n_{\text{tot}}} + p_n(k) \)'s in the place of \( c_n \)'s. If constraints with slack do not interact across non-zero power allocations, then there exists for each current non-zero allocation a maximum increment in flow restricted by the associated constraints. Mathematically put, an iteration of the algorithm at such a juncture finds

\[
p_n(k) := p_n(k) + \min_{(m,j,z):(n,k) \in F_{mj}} \left\{ \frac{-f_{mj}(p,E)}{|A \cap F_{mj}|} \right\} \tag{14}
\]

for all \((n,k) \in A\). We are now ready to summarize Algorithm 1 for maximizing sum rate over fading channels with generalized linear energy flow constraints.

The MIST-WF method initiates all power allocations to zero. It then finds the minimum loss and next minimum loss \((1/n_{\text{tot}})\) over sensors and epochs. The method then computes the largest energy flow allowed before achieving equality with any of the constraints. The minimum between the minimal increment, and the maximum allowable flow (which is divided by the number of nodes/epochs in the active allocation set and associated constraint, \(|M_t \cap F_{mj}|\) is selected for the update. It is essential to note that the minimum losses found on line 5 of Algorithm 1 are almost never singletons, which is exactly the approach of the method. When common minimums are found, they are incremented evenly based on the available flow, or evenly up to the next minimum loss. An example of a step in this process is illustrated in Figure 2 for a simple setup with \(E_{\text{max}} = 12eu\) and \(E_{\text{tot}} = 10eu\), where \(eu\) is an arbitrary unit of energy. The figure shows the available energy flows induced by the various constraints for the set of non-zero allocated bins/epochs. The additional subfig in the lower left shows the flow slack for the total energy constraint in each epoch.

It is intuitive, that if any sensor and epoch, \((n,j)\), achieves its causal energy constraint, i.e., \(\sum_{k=1}^{j} E_n(k) = \sum_{k=1}^{j} E_n(k)\), then all of the previous epoch power levels cannot be increased without violating the constraint. When a such condition is met, then the epochs \(1, \ldots, j\) for that sensor are removed from future updating and the \(p_n(j), j = 1, \ldots, k\) values are final. There is a similar procedure followed for the total energy or link budget constraints, removing all of the power levels from continued processing associated with the achieved constraint.

Another important aspect of the algorithm is how to deal with a battery at maximum capacity. Since the method has found a partial solution where no more energy can flow to later epochs, the problem is decomposed in the epochs before, between, and after the achieved constraints. These new subproblems can be solved optimally since the across epoch and across sensor constraint flows are independent.

The algorithm complexity is \(O(N^3K^3)\) as the upper bound. This follows since the deepest for loop include \(K\) possible operations done possibly \(N\) times, subsumed by an possible \(NK\) iteration loop within the final possible \(NK\) iteration loop. Scenarios with close fading levels or not achieving battery limits much produce much quicker runtimes. Convex programing methods have complexity \(O(N^3K^3)\). The benefit gained is a model specific algorithm that proceeds from a greater understanding of the underlying behavior.
V. SIMULATION RESULTS

We adopt the energy harvesting model in [5] with Poisson harvested energy arrivals. However, we ‘collect’ the energy until the beginning of the next epoch to use for transmission. We use a correlated Rayleigh distribution to draw fading coefficients across time with a maximum Doppler spread of 1 Hz for a time step of 1 s (moderately correlated). We utilize a linear Wiener predictor to estimate the future coefficients. The mean of the harvested energy stochastic process is used to estimate future collected energy. In each of our test scenarios a fixed number of epochs is used.

We compare our method against the numerical convex programming solution with non-causal information. This serves as an upper bound (UPBD). The use of Model Predictive Control methods [19] is a computationally viable alternative to full, infinite-horizon stochastic dynamic programming. Thus, we execute the MIST-WF method with causal information with MPC (MIST-WF-MPC) using the predictors above. We also compare the solution obtained using our method with non-causal information (MIST-WF-NC). A Random Feasible Allocation (RFA) heuristic is included for comparison. The RFA method randomly selects a node/epoch to update with a maximum improvement such that all associated constraints remain feasible. We execute the above scenario over 20 Monte Carlo runs to obtain the averaged results in Figure 3. Figure 3(a) shows the sum rate performance of the MPC and non-causal MIST-WF solution versus the upper bound as a function of increasing epoch length for five sensors (N = 5). Figure 3(b) illustrates variation of the sum rate versus sensors for K = 5 epochs. Preserving the time step of 1 s, Figure 3(c) shows sum rate versus maximum Doppler spread for N = 5 and K = 5. Smaller Doppler-spread produces more highly correlated fading and better sum rate performance. We see that the MPC-like method does reasonable well using only the mean from the statistics of the harvested energy and fading coefficients. We also see that MIST-WF method achieves the upper bound when using non-causal information.

VI. CONCLUSIONS

We have investigated the maximization of rate for a energy harvesting wireless sensor network with spatially uncorrelated fading channels. A algorithm for power allocation is presented for space-time water-filling. Our algorithm is easily adapted to a decentralized framework and is forthcoming in future work.

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