Convective instability in proto–neutron stars

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ABSTRACT

The linear hydrodynamic stability of proto–neutron stars (PNSs) is considered taking into account dissipative processes such as neutrino transport and viscosity. We obtain the general instability criteria which differ essentially from the well-known Ledoux criterion used in previous studies. We apply the criteria to evolutive models of PNSs that, in general, can be subject to the various known regimes such as neutron fingers and convective instabilities. Our results indicate that the fingers instability arises in a more extended region of the stellar volume and lasts a longer time than expected.

Subject headings: convection – instabilities – stars: neutron

1. Introduction

According to theoretical models, neutron stars are formed in the aftermath of type II and Ib supernova explosions, associated to the gravitational collapse of massive stars (8 to $30M_{\odot}$) at the end of their lives. A number of authors have recognized that convection in the newly born hot neutron star can play an important role in both enhancing neutrino luminosities and increasing the energy deposition efficiency, which might lead to the explosion of a supernova (Epstein 1979; Livio, Buchler & Colgate 1980; Colgate & Petschek 1980; Smarr *et al.*, 1981; Lattimer & Mazurek, 1981; Burrows& Fryxell 1993; Janka & Muller 1994). Epstein (1979) pointed out that the negative lepton gradient that naturally arises in the outer layers of the PNS after the shock breaks out of the neutrinosphere can be convectively unstable. Later on, Colgate & Petschek (1980) suggested that this instability could lead to a complete overturn of the core and a strong enhancement of the neutrino transport needed for a powerful supernova explosion on a dynamical time scale. However, convection in PNSs can be driven not only by the lepton gradient but the entropy gradient as well. The suggestion that dissipation of the shock must result in a negative entropy gradient due to both neutronization and dissociation (Arnett, 1987) has been confirmed in a variety of models by Burrows & Lattimer (1988). As a matter of fact, despite different equations of state and differences in the consideration of neutrino transport, the development of negative entropy and lepton gradients seems to be common in many simulations of supernova models (Hillebrandt 1987, Bruenn & Mezzacappa 1994, Bruenn, Mezzacappa & Dineva 1995) and evolutionary models of PNSs (Burrows & Lattimer 1986, Keil & Janka 1995, Sumiyoshi, Suzuki & Toki 1995, Pons et al. 1999).

Concerning numerical simulations, the situation is controversial. Recent hydrodynamic simulations by Keil, Janka & Muller (1996) demonstrate that convection can arise in a rather extended region of PNSs and may generally last a relatively long time, > 1s. Bruenn and Mezzacappa (1994), using the mixing length approximation and Mezzacappa et. al (1998) in two dimensional hydrodynamic simulations found only mild convective activity in the region near the neutrinosphere. Note that the two-dimensional hydro code used by Keil, Janka & Muller is coupled to a radial gray equilibrium diffusion code, that suppresses the neutrino transport in the angular direction, essentially underestimating the stabilizing effect of neutrino transport. On the contrary, Mezzacappa et. al (1998) code works at the limit in which the neutrino transport in the angular direction is fast enough to render the neutrino distributions spherically symmetric, therefore overestimating the stabilizing effect. The answer is obviously at some point in between, that only a future self–consistent multidimensional calculation can determine.

Helpful insights into the nature and growth rates of fluid instabilities have been achieved thanks to semi-analytical investigations (Grossman, Narayan & Arnett, 1993; Bruenn & Dineva 1996), and this work is in that line. The presence of convection in PNSs is usually argued by the fact that the necessary condition of instability (the Ledoux criterion) is fulfilled in a some fraction of the stellar volume. (Epstein 1979, Livio; Buchler & Colgate 1980; Keil, Janka & Müller, 1996). However, the Ledoux criterion may have no bearing at all on the physics of convection. If dissipative effects caused by viscosity or energy and lepton transport are taken into account, the Ledoux condition does not apply and has to be substituted by the appropriate criteria. In the present paper, we derived the criteria of instability in PNSs employing the diffusion approximation for neutrino transport (Imshennik & Nadezhin 1972, Pons et al. 1999) and show that dissipative processes can substantially change the picture.

2. Basic equations

Consider the condition of instability in a plane-parallel layer between $z = 0$ and $z = d$ with the gravity $\mathbf{g}$ directed in the negative z-direction. We neglect a non-uniformity of $\mathbf{g}$ as well as general relativistic corrections to hydrodynamic equations. Calculations show that initially the instability arises in a surface layer of PNS thus the plane-parallel approximation seems to be accurate, at least, during the initial evolutionary stage and, should give qualitatively correct results during the late stage. We assume that the characteristic cooling time scale of a PNS is much longer than the growth time of instability thus it can be treated in a quasi-stationary approximation. Since convective velocities are typically smaller than the speed of sound, one can describe instability by making use of the standard Boussinesq approximation (see, e.g., Landau & Lifshitz 1959). We consider the linear instability when the equations governing small perturbations can be obtained by the linearization of hydrodynamic equations. In what follows, small perturbations of hydrodynamic quantities will be marked by a subscript "1". The linearized momentum and continuity equations read

$$\rho \dot{\mathbf{v}}_1 = -\nabla p_1 + \mathbf{g} \rho_1 + \rho \nu \Delta \mathbf{v}_1, \tag{1}$$

$$\nabla \cdot \mathbf{v}_1 = 0, \tag{2}$$

where p and ρ are the pressure and density, respectively, ν is the kinematic viscosity. We assume that the matter inside a PNS is in chemical equilibrium thus the density is generally a function of the pressure p, temperature T and lepton fraction $Y = (n_e + n_\nu)/n$, with n_e and n_ν being the net (particles minus antiparticles) number densities of electrons and neutrinos, respectively, and $n = n_p + n_n$ is the number density of baryons. Since in the Boussinesq approximation, the perturbations of pressure are negligible because the fluid motions are assumed to be slow and the moving fluid elements are nearly in pressure equilibrium with surroundings, the perturbations of density (ρ_1) and entropy per baryon (s_1) can be expressed, in terms of the perturbations of temperature (T_1) and lepton fraction

Y_1,

$$\rho_1 \approx -\rho(\beta \frac{T_1}{T} + \delta Y_1), \tag{3}$$

$$s_1 \approx m_B c_p \frac{T_1}{T} + \sigma Y_1 \tag{4}$$

where m_B is the mass of the baryon (we neglect the mass difference between protons and neutrons), β and δ are the coefficients of thermal and chemical expansion; $\beta = -(\partial \ln \rho / \partial \ln T)_{pY}$, $\delta = -(\partial \ln \rho / \partial Y)_{pT}$, $c_p = (T/m_B)(\partial s / \partial T)_{pY}$ is the specific heat at constant pressure and $\sigma = (\partial s / \partial Y)_{pT}$.

The above equations should be complemented by the equation driving the evolution of chemical composition and heat balance. We employ the equilibrium diffusion approximation (EDA) which can be reliable during the early stage of evolution of the PNS, when the mean free path of neutrino is short compared to the density and temperature length scales. In this approximation, the diffusion and thermal balance equations read

$$n \frac{dY}{dt} = -\nabla \cdot \mathbf{F}, \tag{5}$$

$$nT \frac{ds}{dt} + n\mu \frac{dY}{dt} = -\nabla \cdot \mathbf{H}, \tag{6}$$

where μ is the neutrino chemical potential and $\mathbf{F}$ and $\mathbf{H}$ are the lepton and heat fluxes, respectively. Note that in the equation for the heat balance we have neglected viscous heat production since this term is second order in velocity. In the EDA, these fluxes are given by

$$\mathbf{F} = -a_T \nabla T - a_\eta \nabla \eta, \tag{7}$$

$$\mathbf{H} = -b_T \nabla T - b_\eta \nabla \eta, \tag{8}$$

where $\eta = \mu / k_B T$ is the degeneracy parameter of the neutrino gas, k_B being the Boltzmann constant; a_T, a_η and b_T, b_η are the coefficients of diffusion and the thermal conductivities, respectively. These kinetic coefficients can easily be related to the coefficients D_2, D_3 and

D_4 introduced by Pons et al. (1999), $a_T = \Gamma D_3$, $a_\eta = T\Gamma D_2$, $b_T = k_B T\Gamma D_4$, $b_\eta = k_B T^2\Gamma D_3$, where $\Gamma = (k_B T)^2 k_B/6\pi^2 c^2\hbar^3$.

For hydrodynamic considerations, it is more convenient to express **F** and **H** in terms of ∇T and ∇Y rather than ∇T and $\nabla\eta$. Then, neglecting the contribution of the pressure gradient (barodiffusion), we have

$$\mathbf{F} = -\left(\chi_T \nabla T + \chi_Y \nabla Y\right), \tag{9}$$

$$\mathbf{H} = -(\xi_T \nabla T + \xi_Y \nabla Y), \tag{10}$$

where

$$\begin{aligned} \chi_T &= a_T + a_\eta \left(\frac{\partial\eta}{\partial T}\right)_{p,Y}, \quad \chi_Y = a_\eta \left(\frac{\partial\eta}{\partial Y}\right)_{p,T}, \\ \xi_T &= b_T + b_\eta \left(\frac{\partial\eta}{\partial T}\right)_{p,Y}, \quad \xi_Y = b_\eta \left(\frac{\partial\eta}{\partial Y}\right)_{p,T}. \end{aligned} \tag{11}$$

Assuming that unperturbed quantities are homogeneous, equations (5) and (6) can be linearized and written in the form

$$\dot{Y}_1 + \mathbf{v}_1 \cdot \nabla Y = \lambda_T \frac{\Delta T_1}{T} + \lambda_Y \Delta Y_1 \,, \tag{12}$$

$$\frac{\dot{T}_1}{T} - \mathbf{v}_1 \cdot \frac{\Delta\nabla T}{T} = \kappa_T \frac{\Delta T_1}{T} + \kappa_Y \Delta Y_1 \tag{13}$$

where

$$\Delta\nabla T = -\frac{T}{m_B c_p}\left(\nabla s - \sigma\nabla Y\right) = \left(\frac{\partial T}{\partial p}\right)_{s,Y} \nabla p - \nabla T \tag{14}$$

is the superadiabatic temperature gradient that gives the difference between the temperature gradient of a fluid with constant entropy and composition and the actual temperature gradient, and we have introduced the following characteristic conductivities

$$\begin{aligned} \kappa_T &= \frac{1}{m_B c_p}\left[\frac{\xi_T}{n} - \lambda_T(\eta+\sigma)\right], \qquad \lambda_T = \frac{T\chi_T}{n}, \\ \kappa_Y &= \frac{1}{m_B c_p}\left[\frac{\xi_Y}{nT} - \lambda_Y(\eta+\sigma)\right], \qquad \lambda_Y = \frac{\chi_Y}{n}. \end{aligned} \tag{15}$$

The system formed by equations (1), (2), (12) and (13), together with the corresponding boundary conditions, completely determines the behaviour of small perturbations. For the sake of simplicity, we consider the case when perturbations are vanishing at the boundaries $z = 0$ and $z = d$. Note that other boundary conditions cannot change the main conclusion of our paper qualitatively.

3. The dispersion equation

The dependence of all perturbations on time and horizontal coordinate can be chosen in the form $\exp(\gamma t - ikx)$, where γ is the inverse growth (or decay) timescale of perturbations and k is the horizontal wavevector. For such perturbations, we have

$$\Delta = \frac{d^2}{dz^2} - k^2. \tag{16}$$

The dependence on the vertical coordinate should be obtained from equations (1), (2), (12) and (13), which can be reduced to one equation of a higher order, say for v_{1z}. The coefficients of this equation are constant in our simplified model, therefore the solution for the fundamental mode with "zero boundary conditions" can be taken in the form $v_{1z} = \sin(\pi z/d)$. Then, the dispersion equation for the fundamental mode is

$$\gamma^3 + a_2\gamma^2 + a_1\gamma + a_0 = 0, \tag{17}$$

where

$$\begin{aligned} a_2 &= \omega_\nu + \omega_T + \omega_Y, \\ a_1 &= \omega_T\omega_Y - \omega_{TY}\omega_{YT} + \omega_\nu(\omega_T + \omega_Y) - (\omega_g^2 + \omega_L^2), \\ a_0 &= \omega_\nu(\omega_T\omega_Y - \omega_{TY}\omega_{YT}) - \omega_g^2(\omega_Y - \frac{\delta}{\beta}\omega_{YT}) \\ &\quad -\omega_L^2\left(\omega_T - \frac{\beta}{\delta}\omega_{TY}\right). \end{aligned} \tag{18}$$

In these expressions, we introduced the characteristic frequencies

$$\omega_T = \kappa_T Q^2, \qquad \omega_Y = \lambda_Y Q^2,$$
$$\omega_{YT} = \lambda_T Q^2, \qquad \omega_{TY} = \kappa_Y Q^2, \tag{19}$$
$$\omega_\nu = \nu Q^2, \quad \omega_g^2 = \frac{\beta g k^2}{Q^2} \cdot \frac{(\Delta \nabla T)_z}{T}, \qquad \omega_L^2 = -\frac{\delta g k^2}{Q^2} \cdot \frac{dY}{dz},$$

where $(\Delta \nabla T)_z$ is the z-component of $\Delta \nabla T$ and $Q^2 = (\pi/d)^2 + k^2$. The quantities ω_ν, ω_T, and ω_Y are the inverse time scales of dissipation of perturbations due to viscosity, thermal conductivity and diffusivity, respectively; ω_{YT} characterizes the rate of diffusion caused by the temperature inhomogeneity (thermodiffusion), and ω_{TY} describes the influence of the chemical inhomogeneity on the rate of heat conduction; ω_g is the frequency (or, in the case of instability, the inverse growth time) of the buoyant wave; ω_L characterizes the dynamical time scale of the processes associated with the lepton gradient.

Equation (17) describes three essentially different modes which generally exist in a chemically inhomogeneous fluid. The condition that at least one of the roots has a positive real part (unstable) is equivalent to fulfilling one of the following inequalities (see, e.g., Aleksandrov, Kolmogorov & Laurentiev 1963)

$$a_2 < 0, \quad a_0 < 0, \quad a_1 a_2 < a_0. \tag{20}$$

Since ν, κ_T and λ_Y are positive defined quantities, the first condition $a_2 < 0$ will never apply, and the discussion will be reduced to the other two conditions.

In the particular case of chemically homogeneous plasma with a "standard" transport ($\omega_{TY} = \omega_{YT} = 0$), we have

$$(\gamma + \omega_Y)[(\gamma + \omega_\nu)(\gamma + \omega_T) - \omega_g^2] = 0. \tag{21}$$

The first root, $\gamma_1 = -\omega_Y$, describes a stable diffusive mode. Two other roots correspond to the ordinary buoyancy modes one of which can be unstable if $\omega_g^2 - \omega_\nu \omega_T > 0$. Then, the necessary condition of instability reduces to the Schwarzschild condition $(\Delta\nabla T)_z > 0$, or in terms of the entropy gradient, $ds/dz < 0$.

If $dY/dz \neq 0$ but dissipative effects are negligible ($a_2 = 0$, $a_0 = 0$), we have

$$\gamma^3 + a_1\gamma = 0 \tag{22}$$

with $a_1 = -(\omega_g^2 + \omega_L^2)$. The first root is degenerate in this case and the two other roots are given by $\pm\sqrt{-a_1}$. The condition of instability is $(-a_1) = \omega_g^2 + \omega_L^2 > 0$, or

$$\left(\frac{\Delta\nabla T}{T}\right)_z - \frac{\delta}{\beta}\frac{dY}{dz} > 0, \tag{23}$$

that represents the familiar Ledoux criterion.

4. Instability criteria in the general case.

Generally, when $dY/dz \neq 0$ and dissipative effects are important, the conditions of instability are more complex and depend on a horizontal wavevector of perturbations, k. The temperature and lepton gradients required for instability can be quite different for perturbations with different k and to obtain the criteria we have to find the minimal values of these gradients. From the properties of kinetic coefficients we have $\kappa_T\lambda_Y - \kappa_Y\lambda_T > 0$, therefore we can deduce from conditions (20) that the gradients are minimal for perturbations with the wavevector k minimizing the quantity Q^6/k^2, which is reached at $k^2 = (\pi/d)^2/2$. Then, we have

$$\left(\frac{Q^6}{k^2}\right)_{min} = \frac{27}{4}\left(\frac{\pi}{d}\right)^4. \tag{24}$$

The instability criteria can now be obtained from the last two conditions in (20), which read, respectively,

$$\frac{dY}{dz}(\delta\kappa_T - \beta\kappa_Y) \quad - \quad \left(\frac{\Delta\nabla T}{T}\right)_z (\beta\lambda_Y - \delta\lambda_T)$$

$$+ \quad \frac{27\pi^4\nu}{4gd^4}(\kappa_T\lambda_Y - \kappa_Y\lambda_T) < 0\,, \tag{25}$$

$$\frac{dY}{dz}(\delta(\nu+\lambda_Y)+\beta\kappa_Y) \quad - \quad \left(\frac{\Delta\nabla T}{T}\right)_z(\beta(\nu+\kappa_T)+\delta\lambda_T) +$$
$$+ \quad \frac{27\pi^4}{4gd^4}(\kappa_T+\lambda_Y)\left[(\nu+\kappa_T)(\nu+\lambda_Y)-\lambda_T\kappa_Y\right] < 0. \tag{26}$$

In the case of a negligible diffusivity, the first criterion yields the Rayleigh-Taylor condition of instability, $\delta dY/dz < 0$. This instability can be responsible, for instance, for the salt fingers phenomena in the terrestrial oceans. Therefore, following Bruenn & Dineva (1996), we can conventionally call the instability associated to condition (25) neutron fingers. We refer to a *neutron finger unstable* region as a region where condition (25) is fulfilled but not condition (26). In the case of a "standard" transport with small viscosity and diffusivity, the second criterion yields the Schwarzschild criterion of convection, although convection in PNS is, in general, quite different from the Schwarzschild convection and may arise in oscillatory or nonoscillatory regimes. A region is said to be *convectively unstable* when both conditions (25) and (26) are satisfied. If condition (26) is satisfied but condition (25) is not, the system is said to be *semiconvectively unstable*.

Despite the fact that convective instabilities in PNS were originally argued by applying the Ledoux criterion (see, e.g., Epstein 1979, Livio, Buchler & Colgate 1980), it appears that the Ledoux criterion is not the valid criterion for instability if we allow for conduction of heat, diffusion of particles and/or viscosity. In this case, the true criteria of instability are (25) and (26), and only in two limiting cases our criteria reduces to the Ledoux form.

Generally, the region where any of the conditions (25,26) are fulfilled can be very different to that given by the Ledoux criterion.

5. Results and discussion.

To obtain the different thermodynamical derivatives, diffusion coefficients and conductivities appearing in the coefficients of the dispersion equation, we used the results from numerical simulations of PNS evolution performed by Pons *et al.* (1999). The results discussed in this paper correspond to the model labelled GM3np, with a baryonic mass of 1.6 $M_{\odot}$. The simulation was carried out using a Henyey–like spherically symmetric evolution code, coupled to a 1D neutrino transport scheme in the flux–limited diffusion approximation. Details about the code and the calculation of the diffusion coefficients using opacities consistent with the underlying EOS (Reddy, Prakash & Lattimer, 1998) can be found in Pons *et al.* (1999).

Since calculations of viscosity at high densities are unreliable (compare, for instance, van den Horn & van Weert 1981, Goodwin & Pethick 1982, Thompson & Duncan 1993), we calculate the unstable region for different values of ν. In Figure 1 (upper panel) we plot the stability regions of a 1.6 $M_{\odot}$ PNS corresponding to case $\nu = 0$. In all the simulations we use a value for d given by the pressure scale, $d =\mid d \ln p/dr \mid^{-1}$. Three different situations can clearly be distinguished. Initially, there is a small convectively unstable zone (darkest) near the surface that lasts for only a few seconds, surrounded by a neutron finger unstable region (intermediate gray tone). The inner part of the star is stable (lighter). After a few seconds, the neutron finger instability moves inward, occupying a large portion of the PNS, whereas the stable region shrinks. Later on, at $t = 12$ s, the innermost core becomes convectively unstable for a few seconds, while the neutron finger unstable region begins to shrink. By about $t = 30$ s, the PNS is mostly stable, except a small region near the center that is still subject to the neutron fingers instability. Instability completely disappears after ≈ 40 s.

To study the effect of viscosity, Figure 1 (lower panel) shows the time dependence of the boundaries of unstable regions for $\nu = \eta^2 \kappa_T$ (van den Horn & van Weert 1981).

Qualitatively, the unstable regions evolve in a similar fashion to the inviscid case. The reason why viscosity hardly influences the neutron finger instability region is that the third term on the l.h.s. of (25), which describes dissipation of perturbation due to neutrino transport and depends on ν, is relatively small compared to the first two terms which are independent of ν. Therefore, the neutron finger unstable region is approximately the same as in Fig.1 at any evolutionary stage. On the contrary, convective instability is sensitive to the value of viscosity through condition (26). The third term on the l.h.s. of (26) again does not yield too much contribution, however, two other terms depend on viscosity. Due to this, the region unstable to convection turns out to be more extended at high viscosity. Notice that in the case $\eta \gg 1$ the viscosity is much greater than the diffusivity and thermal diffusivity but, nevertheless, is not sufficiently large to make the term in the r.h.s. important, thus recovering Ledoux criterion from (26). Note, however, that for both models the convectively unstable region always lies inside the region unstable to neutron fingers. In our simulations we have not found any region unstable to semiconvection. Although this result might seem different from Bruenn & Dineva (1996) we should keep in mind that the thermodynamic conditions used in this work to describe the interior of the PNS are quite different from the conditions used by Bruenn & Dineva (1996) to describe the matter just below the neutrinosphere of the collapsed core. Bruenn & Dineva (1996) studied the instability in a region, typically with a density of $3{\times}10^{13}$ g/cm^{3}and with a high entropy per baryon $s = 4$, since at such early times ($\approx 50 - 230$ ms after bounce) the excess of entropy of the shocked mantle has not been yet radiated away. In this work, however, we focus on the long term evolution of a PNS, and most of the region of our interest is at supranuclear densities ($\rho > 3 \times 10^{14}$ g/cm^3) and moderate entropies.

Since the superadiabatic temperature gradient and the gradient of lepton number play a fundamental role in the stability criterion we have shown in Figure 2 graphics of these quantities for different times in the evolution. We plot thicker lines for the unstable regions according to the model shown in the lower panel of Figure 1. As we can see from the figures, at early and middle times in the evolution, instability region corresponds approximately to that with a positive value of the superadiabatic temperature gradient. The effect of the negative lepton fraction gradient and/or dissipative processes increase only slightly the unstable region. At late time however, the unstable region almost disappears while the superadiabatic temperature gradient remains positive in a much wider region. This is chiefly caused by the change in the sign of the thermodynamic derivative δ, when the lepton fraction drops some critical value.

In the upper panel of Figure 3 we show the maximum of the real parts of the roots of equation (17) at two different stages in the evolution. A positive value of this quantity implies instability and, in that case, the instability growth time is calculated by taking its inverse (lower panel). As we can contrast in the lower panel of Figure 3 the characteristic growth time of instabilities at the beginning of the evolution (t=1 s) is of the order of tenths of millisecond. Since this time is much shorter than the evolution time scale, convective heat transport might be important in this region. At the middle stages of the evolution (t=20 s), two different unstable regions can be distinguished, a first one with a growth time of the order of 1 ms (convection) and a second region with associated growth times of the order of few tens of millisecond (corresponding to the region unstable to neutron fingers). The growth time in both cases is, again, much shorter than the time evolution and, in principle, instabilities can grow fast enough to influence the heat transport.

In summary, we have to emphasize that neutrino transport plays a significant role in the stability properties of PNSs. Criteria (25) and (26) obtained in the present paper correspond properly to the conditions of PNS where heat transport and diffusion can be sufficiently fast. These criteria differ essentially from those used by other authors in this context. Our analysis shows that the unstable region inside the PNS can be more extended than predicted, for instance, by the Ledoux criterion alone. Generally, neutron fingers instability arises in a more extended region of a stellar volume and lasts a longer time than convection. Therefore, this instability seems to be as important as convection for the early evolution of a newly born neutron star. Our main conclusion is that, surprisingly, diffusive effects allow for the existence of new unstable modes and the role of convection in PNS should be carefully reexamined.

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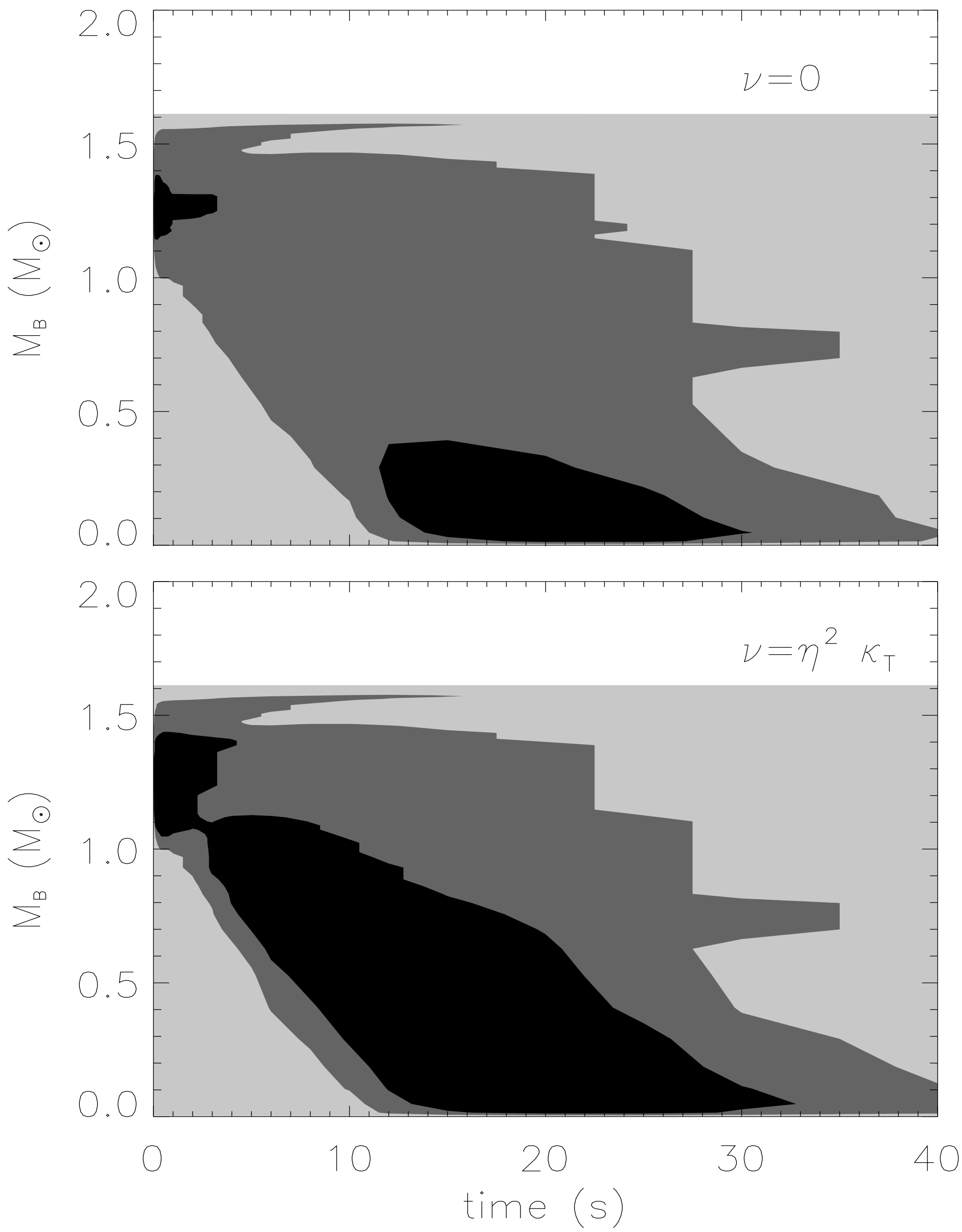
Figure captions

Fig. 1.– Evolution of the different unstable zones in a $M = 1.6 M_\odot$ PNS. The darker zone corresponds to the convectively unstable regions, the lighter zone to convectively

stable regions and the intermediate zone presents the secular instability usually denoted by neutron fingers. The upper and lower panels displays the results of the inviscid and viscous ($\nu = \eta^2 \kappa_T$) models, respectively.

Fig. 2.– Superadiabatic temperature gradient (left panels) and lepton fraction gradient (right panels) for three different times in the evolution. Thicker lines correspond to the unstable region according to Figure 1.

Fig. 3.– Maximum value of the real parts of the roots of equation (17) at two different stages in the evolution (upper panel) and growth time for the unstable regions (lower panel).



2.0
1.5
1.0
0.5
0.0
$\nu=0$
M_B ($M_\odot$)
2.0
1.5
1.0
0.5
0.0
$\nu=\eta^2\ \kappa_T$
M_B ($M_\odot$)
0
10
20
30
40
time (s)

Fig. 1.—

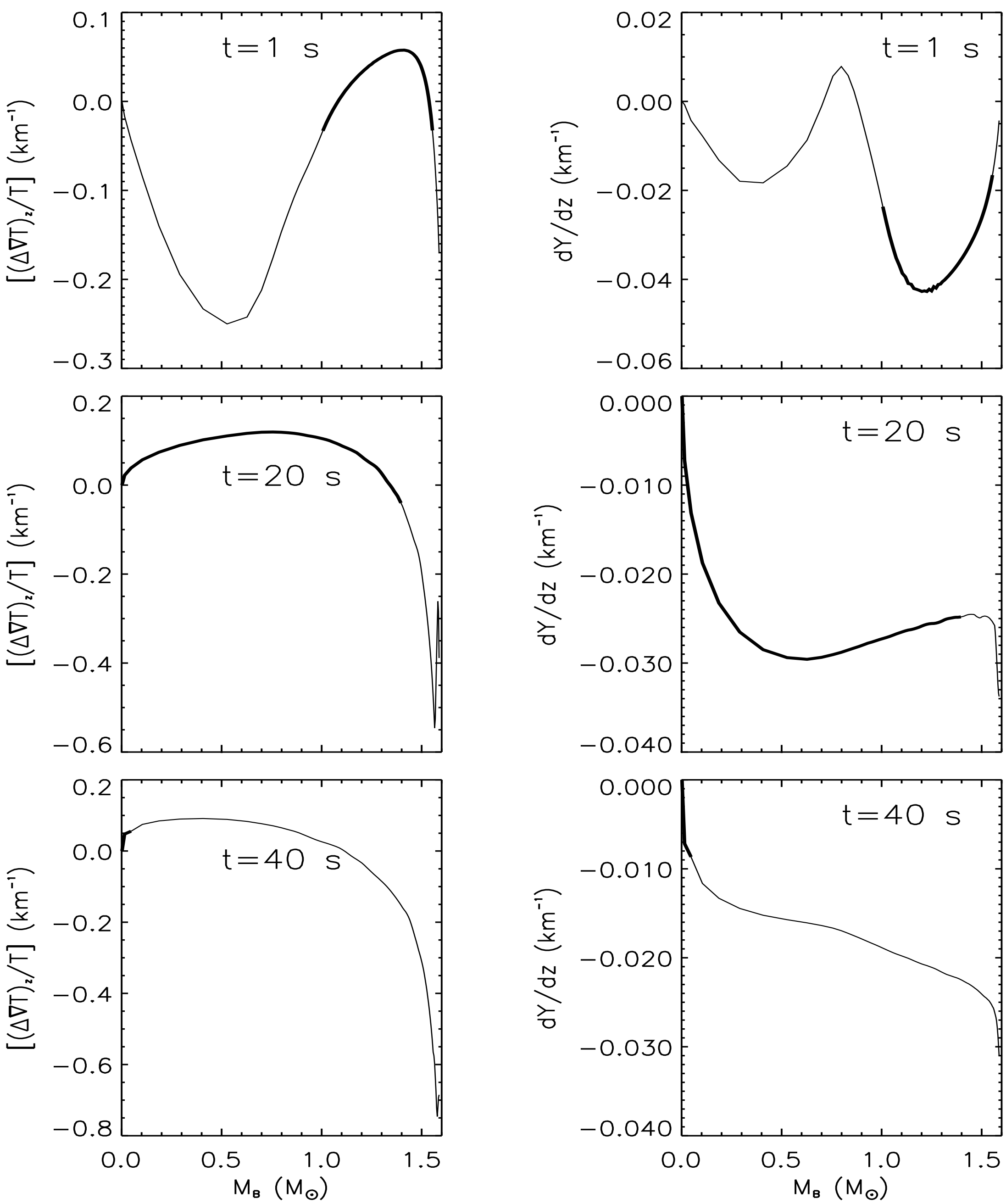


Fig. 2.—

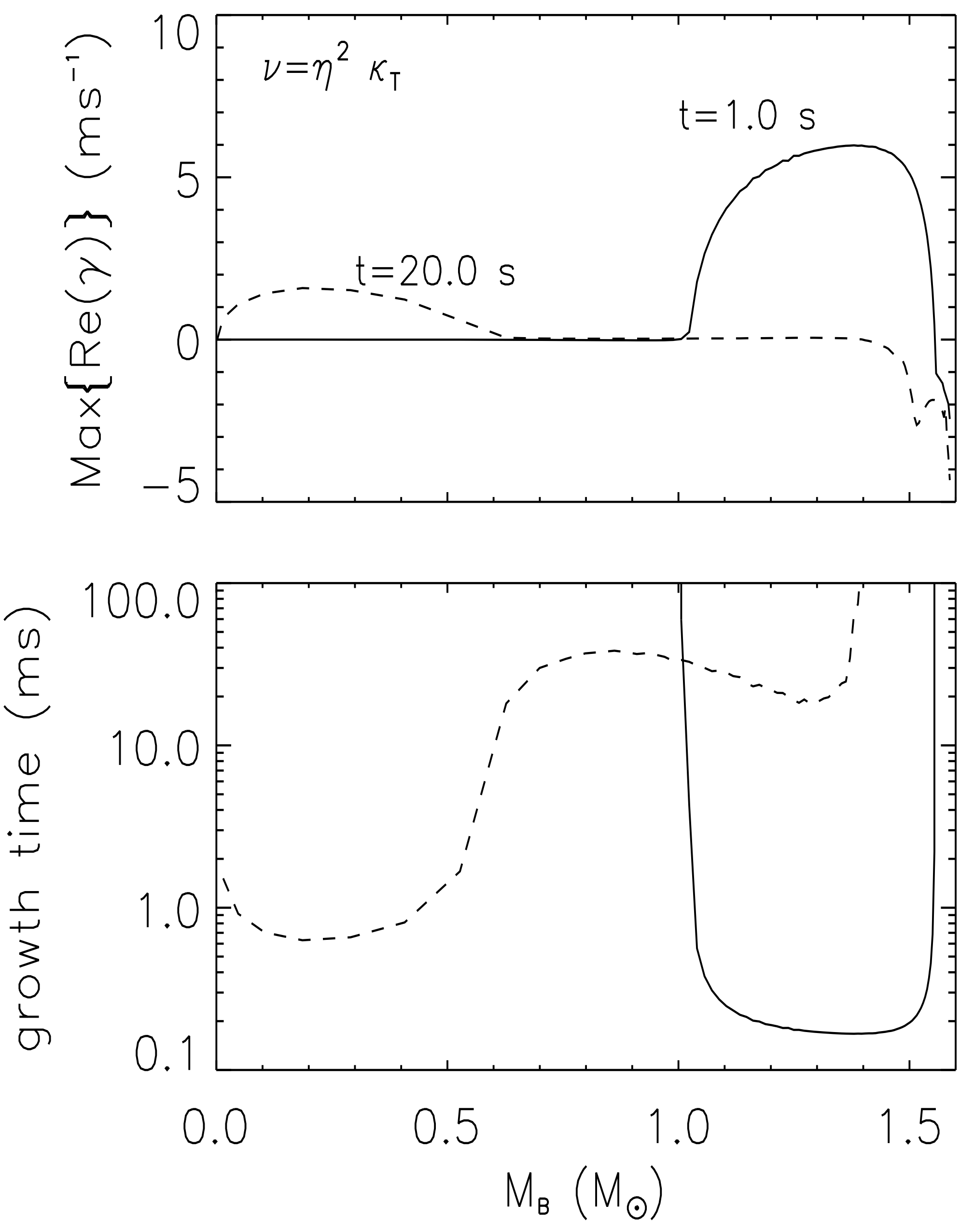

$\nu=\eta^2\ \kappa_T$
t=1.0 s
t=20.0 s
Max{Re(γ)} (ms⁻¹)
growth time (ms)
M_B ($M_\odot$)
10
5
0
−5
100.0
10.0
1.0
0.1
0.0
0.5
1.0
1.5

Fig. 3.—