BROADBAND MIMO SONAR SYSTEM: A THEORETICAL AND EXPERIMENTAL APPROACH

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Abstract: MIMO systems have raised a lot of interest in the recent years. The radar community pointed out the multiple advantages of MIMO systems such as diversity gain for target detection, angle of arrival and Doppler estimation. Coherent processing also allows super-resolution for target localisation. We explore in this paper broadband MIMO sonar systems. In the current literature, the channel matrix has been computed using point scatterer models. The limitation of such models for sonar (and broadband sonar in particular) is explained and a model based on the target form function is proposed. We will show that this deterministic MIMO model can naturally be extended to a statistical model. It has been shown that broadband sonars offer in situ great capability for target classification. Using widely separated transducers, it has been shown that channel matrices are decorrelated from one another, which means that the views of the potential target are independent. MIMO systems improve the process of classification thanks to these multiviews. We present experiments done in our tank (Width x Length x Depth: 3 x 4 x 2 m) with a broadband MIMO system (2 transmitters, and 4 receivers). The transmitters cover the frequency band of 30 kHz to 150 kHz. We demonstrate experimentally the advantage of Time Reversal for MIMO system by focusing the energy on the target independently of the medium. We propose a pseudo Time Reversal technique which focuses the energy directly back to the receivers increasing the SNR by a factor of \( N \) where \( N \) is the number of transmitters.

Keywords: Broadband sonar, MIMO system
VIRTUAL POINT SCATTERERS MODEL FOR CYLINDRICAL SHELL

The first formulation for MIMO systems has been done by the radar community. The MIMO system model can usually be expressed by: \( \mathbf{r} = \mathbf{H} \mathbf{s} + \mathbf{n} \), where \( \mathbf{r} \) represents the receivers, \( \mathbf{s} \) the transmitters, \( \mathbf{n} \) the noise, and \( \mathbf{H} \) the channel matrix. The channel matrix includes the wave propagation in the medium from any transmitters to any receivers and the target reflection. At first, targets were represented using the "point target" assumption [1]. Since then, several target models have been proposed such as rectangular-shape target in [2] composed of an infinite number of scatterers. The most popular model for a radar target model is the finite scatterer model [3,4].

**Fig. 1:** Sound interaction between a plane wave and a plastic cylindrical shell.

In this section we present an accurate multi-static model for a low impedance shell cylinder. In [5], we demonstrated that the sound scattering of a low impedance shell cylinder is analogous to the reflection by two spherical mirrors. Fig. 1 shows the echo formation of an acoustic wave reflected by a plastic cylindrical shell. The location of the two echo centres A1 and A2 (in Fig. 1) can be computed thanks to the well-known formula of reflection by a spherical mirror. The notation of Eq. 1 is explained in Fig. 2. A and A' represent respectively the source and the source image.

\[
\frac{1}{SA'} + \frac{1}{SA} = \frac{2}{SC}
\]  

**Fig. 2:** Reflection by a concave spherical mirror, construction of the source image \( A' \) from the source \( A \) in the axis \( CS \).

Assuming an incoming plane wave, the two echo centres, A1 and A2, are exactly in between the centre of the cylinder and respectively the front and the back of the cylinder. In our model A1 and A2 will represent the virtual scatterers. They act like point sources, but contrary to scattering points, they emit the received pulse with a delay. The transmitter Tx transmits a pulse \( s(t) \). The acoustic wave is reflected by the cylinder modelled by the virtual scatterers A1 and A2 to the receiver Rx. Eq.2 expresses the acoustic field \( r(t) \) received at the
receiver Rx. SC represent the radius of the cylinder, c the speed of sound in water, C the centre of the cylinder and \( \tau_{AB} \) the propagation time between A and B.

\[ r(t) = s \left( t - \tau_{TxC} - \frac{3}{2} \frac{SC}{c} - \tau_{A,Rx} \right) e^{i\phi_1} + s \left( t - \tau_{TxC} + \frac{3}{2} \frac{SC}{c} - \tau_{A,Rx} \right) e^{i\phi_2} \]  \hspace{1cm} (2)

The two terms \(-3SC/2c\) and \(+3SC/2c\) represent the negative and positive delays of the virtual scatterers. Fig.3 compares the echo spectra of our model with the analytic solution given by Doolittle in [6]. In this example, the cylindrical shell is made of PVC, its diameter is 32cm and its thickness is 3mm. The receiver is placed at 4m from the shell at an angle of 30°. An excellent match is found between the theoretical prediction and our model.

### PROBLEM REFORMULATION FOR BROADBAND MIMO SONAR

Haimovich et al. in [3] formulates narrowband MIMO radar using a finite point target model with Q scattering points \{X_q\}. The transmitter \( k \) send a pulse \( s_k(t) \), the receiver \( l \) receives from the transmitter \( k \) \( z_{lk}(t) \):

\[ z_{lk}(t) = \frac{E}{M} \sum_{q=1}^{Q} h_{lk}^{(q)}(t - \tau_{k,X_q} - \tau_{l,R}) \]

\[ = \frac{E}{M} \sum_{q=1}^{Q} \zeta_q \exp(-2j\pi f_r [\tau_{k}(X_q) + \tau_{l}]) \]

(3)

The notations can be found in [3]. Assuming the Q scattering points are close, we can write that

\[ s_k(t - \tau_{k,X_q}) = s_k(t - \tau_{k,X_0}) = s_k(t,X_0) \]

where \( X_0 \) is the centre of gravity of \{X_q\}. So the previous equation becomes:

\[ z_{lk}(t) = \frac{E}{M} \sum_{q=1}^{Q} \zeta_q \exp(-j2\pi f_r [\tau_{k}(X_q) + \tau_{l}]) \]

\[ = \frac{E}{M} \sum_{q=1}^{Q} \zeta_q \exp(-j2\pi f_r [\tau_{k}(X_q) + \tau_{l}]) \]

(4)
The term \( \sum \zeta \exp(-j2\pi f_c [\tau_{ik}(X_q) + \tau_{rl}(X_q)]) \) corresponds to a random walk in the complex plane. It explains the phenomena of fading observed in radar. Indeed this value can be statistically lower than the noise level.

We saw in [5,7] that for broadband sonar a formulation in the Fourier domain is more appropriate. Eq.3 becomes:

\[
Z_{lk}(\omega) = \sqrt{\frac{E}{M}} \sum_{q=1}^{Q} h_{lk}^{(q)} \exp[-j\omega(\tau_{ik}(X_q) + \tau_{rl}(X_q))] \]

Using the following notations:

\[
\tau_{ik}(X_q) = \tau_{ik}(X_0) + \tilde{\tau}_{ik}(X_q)
\]

\[
\tau_{rl}(X_q) = \tau_{rl}(X_0) + \tilde{\tau}_{rl}(X_q)
\]

and

\[
H_{lk}(X_0,\omega) = \sqrt{\frac{E}{M}} \exp[-j(2\pi f_c + \omega)[\tau_{ik}(X_q) + \tau_{rl}(X_q)]]
\]

we arrive to:

\[
Z_{lk}(\omega) = H_{lk}(X_0,\omega) \left( \sum_{q=1}^{Q} \tilde{h}_{lk}^{(q)} \exp[-j\omega(\tau_{ik}(X_q) + \tau_{rl}(X_q))] \right) S_k(\omega) = H_{lk}(X_0,\omega) F_{\omega}(\omega, \theta_l, \phi_k) S_k(\omega)
\]

\( \theta_l \) is the angle of view of the target from the transmitter, and \( \phi_k \) is the angle of view of the target from the receiver. Eq.6 can be interpreted as follows: the first term corresponds to the propagation of the wave to and from the target, the second term is the form function of the target, the third term is the transmitted signal.

The main advantage of this formulation is the clear separation between propagation terms and target reflection terms. In our formulation the target form function \( F_{\omega} \) is independent of any particular model. The generalization of this equation including multipath and attenuation terms is straightforward:

\[
Z_{lk}(\omega) = \sum_{p=1}^{P} A^{(p)}(\omega) H_{ik}^{(p)}(X_0,\omega) F_{\omega}(\omega, \theta_l^{(p)}, \phi_k^{(p)}) S_k(\omega)
\]

where \( P \) is the number of multipath and \( A^{(p)}(\omega) \) the attenuation through the path \( p \).

**EXPERIMENTAL RESULTS AND PSEUDO TIME REVERSAL**

**Experiments**

Sonar MIMO experiments have been done in our test tank (L x W x D: 4m x 3m x 2m) using 2 transmitters (a low frequency transducer: 30kHz-90kHz and a high frequency transducer: 60kHz-150kHz) and 3 wideband receivers. A display of the configuration of the experiment can be found in Fig.4. Fig.4 displays as well the geometrical reconstruction of the
echo taking into account the arrangement of the transducer. The PVC cylindrical shell echoes from the front and the back of the cylinder are clearly visible.

Fig. 4: (left) configuration of the experiment. (centre) geometrical reconstruction of the MIMO echoes without the PVC cylinder. (right) geometrical reconstruction of the MIMO echoes with the PVC cylinder.

**Pseudo Time Reversal**

In MIMO systems, the total signal received at each receiver is the sum over the transmitters, i.e. $\sum z_{kl}(t)$. The classical assumption made in MIMO is the orthogonality of the transmitted pulses $s_k(t)$. So in the detection problem, the total received signal is projected into each transmitted pulse space, in order to recover the channel matrix elements $h_{kl}$. The optimal detector is then given by the likelihood ratio of the recovered channel matrix [2].

Fig. 5: Spatial sound focus using two transmitters.

We consider here a deepwater propagation type, which means no multi-path. By playing with the delay between the transmitted pulses, the combined sound is focused on certain parts of the space. Fig.5 displays an example of sound focusing using two transmitters. By knowing the geometry of the transmitters, time delays can be computed to focus on a particular point in space.

The idea of our Pseudo time reversal is to use this combined energy on the target to improve the detection. We want to maximize the total signal from the receiver point of view that means maximizing $\Sigma_k z_{kl}(t-\tau_k)$ where $\tau_k$ is the delay used to focus the beams. Usually the pulses used are coherent (their cover the same frequency band) so the sum term $\Sigma_k z_{kl}(t-\tau_k)$ is a coherent summation, which can result in destructive interferences.

Fig. 6 illustrates the interferences due to the coherence in the summation of the same chirp. If the two chirps do not overlap in frequency, the two pulses are incoherent, and no interferences are observed. By using different frequency bands for each transmitter, the sum $\Sigma_k z_{kl}(t-\tau_k)$ becomes incoherent, and as a result: $\max(\Sigma_k z_{kl}(t-\tau_k)) = \Sigma_k \max(z_{kl}(t))$. 

Assuming that the K transmitters can emit the same energy in all the frequency bands, the SNR increases by a factor of K.

![Fig.6: Maximum amplitude of the summation of two chirps. The chirps are windowed by a gaussian. The chirp duration is 200µs.](image)

**CONCLUSION**

In this paper we proposed a new formulation for broadband MIMO sonar systems by separating clearly the terms of propagation and the terms of target reflection. This formulation is more flexible for different target models integration. A new model for cylindrical shell has been proposed using virtual scatterers. A new method of pseudo time reversal has been proposed in order to increase the SNR by a factor of K (where K is the number of transmitters) and improve the detection performance of the system. Future works include a demonstration in tank of the Pseudo Time Reversal.

**REFERENCES**


