A Strategy for the Development of Coupled Ocean-Atmosphere Discontinuous Galerkin Models

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ONR Shallow Water Workshop

Background

Currently, in the U.S. there is a movement to construct one NWP model (NWS, Navy, Air Force – other partners include NASA and DOE). This National Board (NUOPC=National Unified Operational Prediction Capability) aims to develop a new model that is:

1. Highly scalable on current and future computer architectures
2. Global model that is valid at the meso-scale (i.e., non-hydrostatic)
3. Applicable to medium-range NWP and decadal time-scales

It would be ideal to couple such a model with a coastal ocean model.

Motivation

Our goal is to construct numerical methods for non-hydrostatic mesoscale and global atmospheric models (for NWP applications) as well as coastal ocean models for storm-surge modeling.

Our aim is to build a modeling framework with the following capabilities:

1. Highly scalable on current and future computer architectures (exascale computing and beyond and GPUs)
2. Flexibility to use a wide range of grids (e.g., statically and dynamically adaptive)
3. Model that is accurate, robust, and fully conservative

Our coupling strategy is as follows:

1. Develop a high-order discontinuous Galerkin global/regional atmospheric model.
2. Develop a high-order discontinuous Galerkin coastal ocean model.
3. Combine them using high-order adaptive/unstructured triangular prisms.
4. Resolving the disparate time-scales via extrapolation (multi-rate) methods.

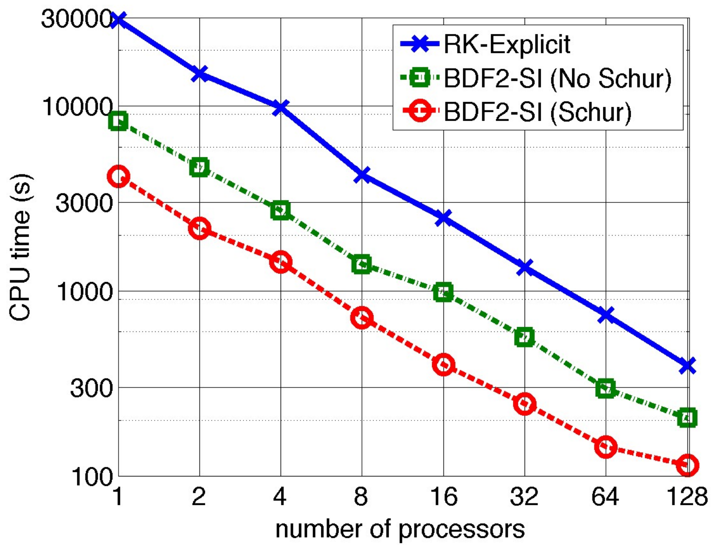
Talk Summary

1. New models need to exploit available computers
2. Numerical methods in new GFD models
3. What should we aim for in our new models
4. Where we plan to head with our GFD models

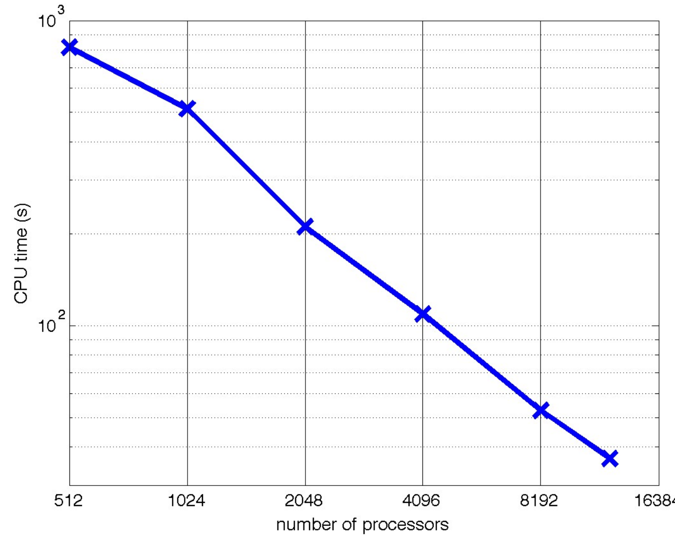
Talk Summary

1. New models need to exploit available computers
    - From Terascale to Petascale/Exascale Computing
    - 10 of Top 500 are already in the Petascale range
    - 3 of top 10 list are GPU-based machines
2. Numerical methods in new GFD models
3. What should we aim for in our new models
4. Where we plan to head with our GFD models

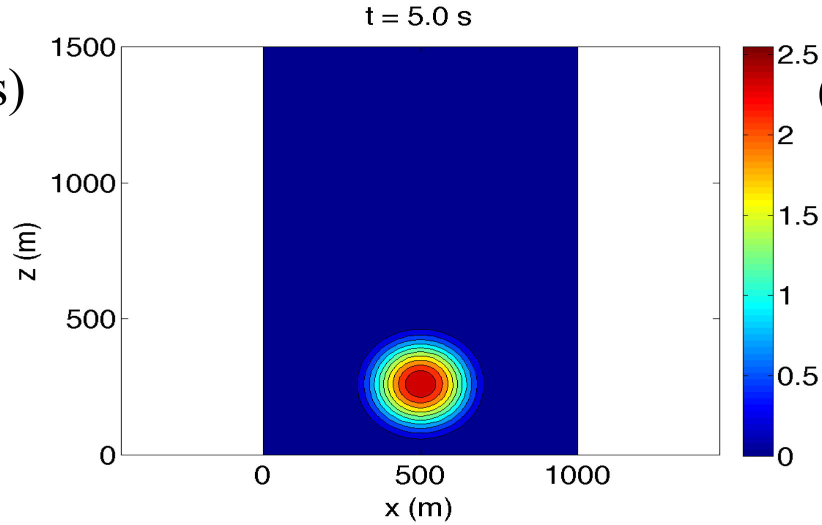
Performance of a Global/Mesoscale Non-Hydrostatic Model



(2 Million Grid Points)



(16 Million Grid Points)

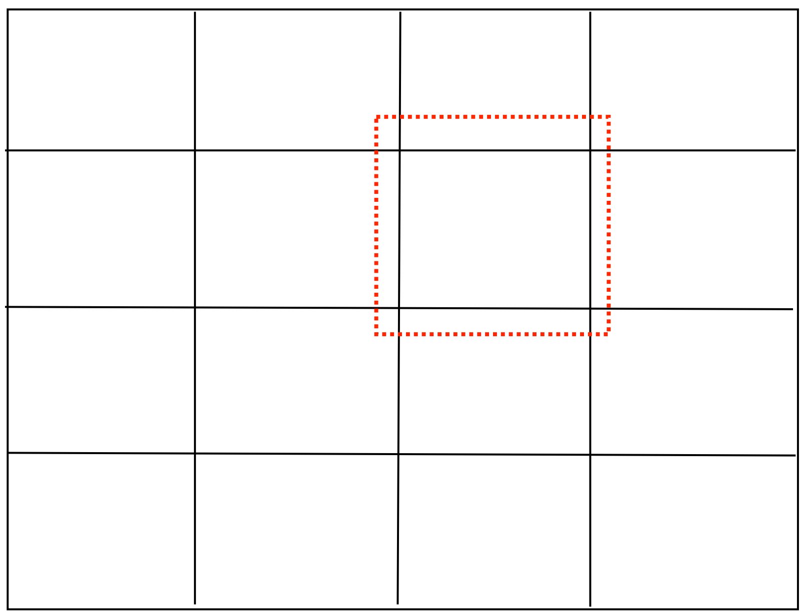


Talk Summary

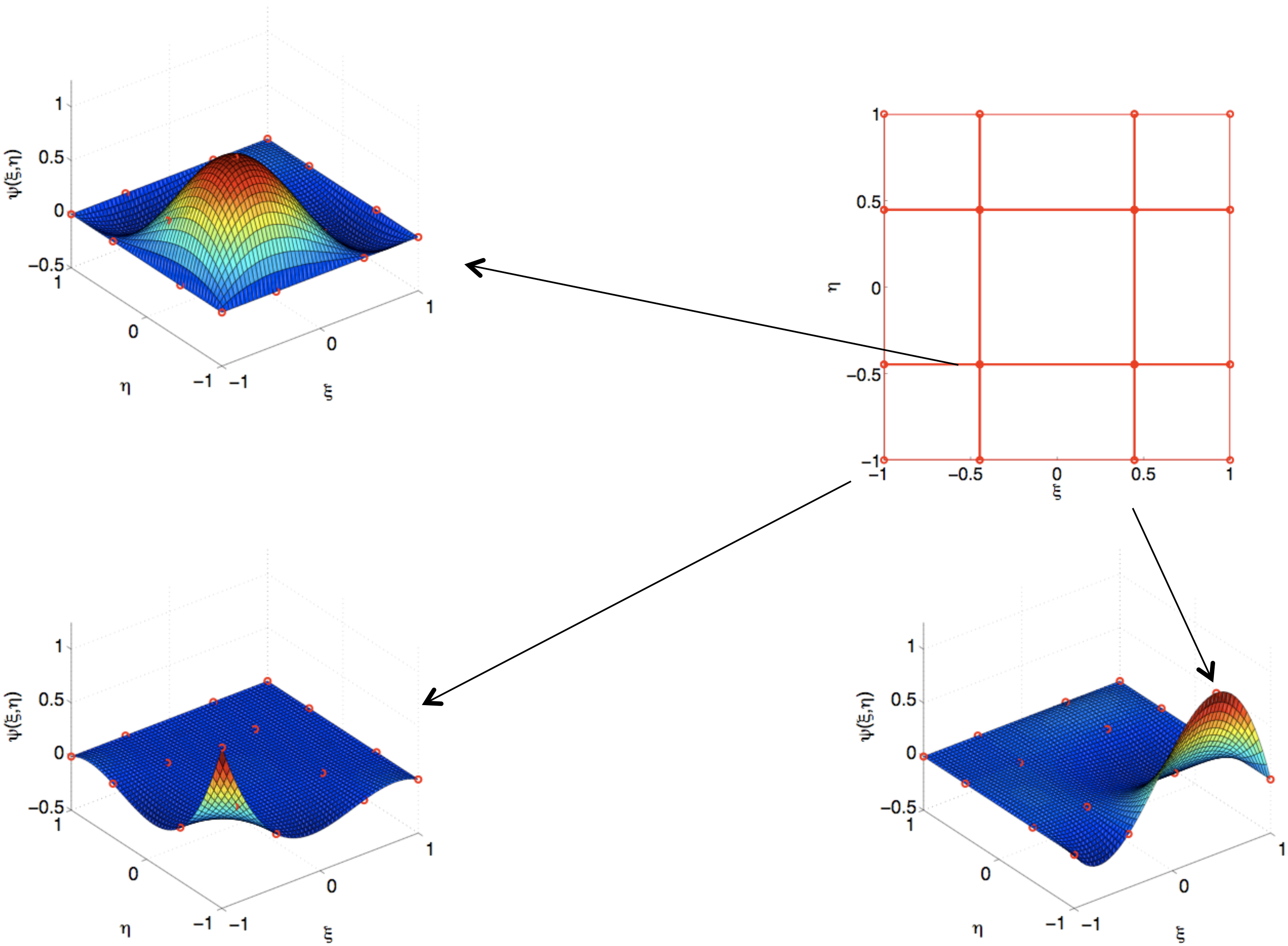
1. New models need to exploit available computers
2. Numerical methods in new GFD models
   - Time-Integration is important (e.g., explict, semi-implicit, fully-implicit)
   - Spatial Discretization methods is how we are able to take advantage of Parallel computers (i.e., domain decomposition of the physical grid)
3. What should we aim for in our new models
4. Where we plan to head with our GFD models

Element-based Galerkin (EBG) Methods
(Definition and Examples)

- An element is chosen to be the basic building-block of the discretization and then a polynomial expansion is used to represent the solution inside the element



Element-based Galerkin Methods in a Nutshell



Spatial Discretization

- Primitive Equations: $$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{F} = S(q)$$
- Approximate the solution as: $$q_N = \sum_{i=1}^{M_N} \psi_i q_i \qquad \mathbf{F}_N = \mathbf{F}(q_N) \qquad S_N = S(q_N)$$
  - **Interpolation O(N)**
- Write Primitive Equations as: $$R(q_N) \equiv \frac{\partial q_N}{\partial t} + \nabla \cdot \mathbf{F}_N - S_N = \varepsilon$$
- Weak Problem Statement: Find $q_N \in \Sigma(\Omega)\ \forall\ \psi \in \Sigma$
$$\begin{cases} \Sigma = \left\{\psi \in H^1(\Omega) : \psi \in P_N(\Omega_e) \forall \Omega_e\right\} & \text{(CG)} \\ \Sigma = \left\{\psi \in L^2(\Omega) : \psi \in P_N(\Omega_e) \forall \Omega_e\right\} & \text{(DG)} \end{cases}$$
  - such that $$\int_{\Omega_e} \psi R(q_N)\, d\Omega_e = 0$$
    - **Integration O(2N)**

Spatial Discretization

- Integral Form: $$\int_{\Omega_e} \psi R(q_N)\, d\Omega_e = 0$$

- Matrix Form: $$M_{ij}^{(e)} \frac{dq_j^{(e)}}{dt} - \left(\mathbf{D}_{ij}^{(e)}\right)^T \mathbf{F}_j^{(e)} + \boxed{\mathbf{C}_i} = S_i^{(e)}$$

- Where each matrix is:

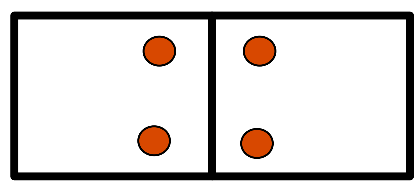
$$M_{ij}^{(e)} = \int_{\Omega_e} \psi_i \psi_j \, d\Omega_e$$

Integration O(2N)

$$\mathbf{D}_{ij}^{(e)} = \int_{\Omega_e} \nabla \psi_i \psi_j \, d\Omega_e$$

For DG: $$C_i = \left(\mathbf{M}_{ij}^{\Gamma}\right)^T \mathbf{F}_j^{(*)} \longrightarrow \mathbf{M}_{ij}^{\Gamma} = \int_{\Gamma} \mathbf{n} \psi_i \psi_j \, d\Gamma$$

For CG: $$q_i^{(e)}$$



Spatial Discretization

- Integral Form: $$\int_{\Omega_e} \psi R\left(q_N\right) d\Omega_e = 0$$

- Matrix Form: $$M_{ij}^{(e)} \frac{dq_j^{(e)}}{dt} - \left(\mathbf{D}_{ij}^{(e)}\right)^T \mathbf{F}_j^{(e)} + \boxed{\mathbf{C}_i} = S_i^{(e)}$$

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Integration O(2N)

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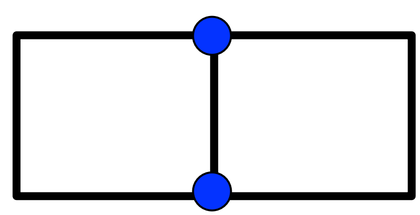
For CG: $$q_I = G\left(q_i^{(e)}\right) \quad (i,e) \longrightarrow I$$



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Integration O(2N)

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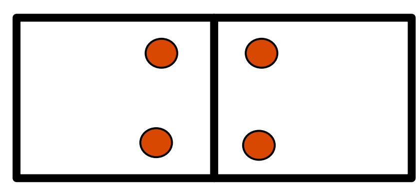
For DG: $$C_i = \left(\mathbf{M}_{ij}^{\Gamma}\right)^T \mathbf{F}_j^{(*)} \longrightarrow \mathbf{M}_{ij}^{\Gamma} = \int_{\Gamma} \mathbf{n} \psi_i \psi_j\, d\Gamma$$

For CG: $$q_i^{(e)} = S\left[q_I\right] \quad I \longrightarrow (i,e)$$



Talk Summary

1. New models need to exploit available computers
2. Numerical methods in new GFD models
3. What should we aim for in our new models
   - E.g., Conservation, Scalability, High-order Accuracy, Adaptivity
4. Where we plan to head with our GFD models

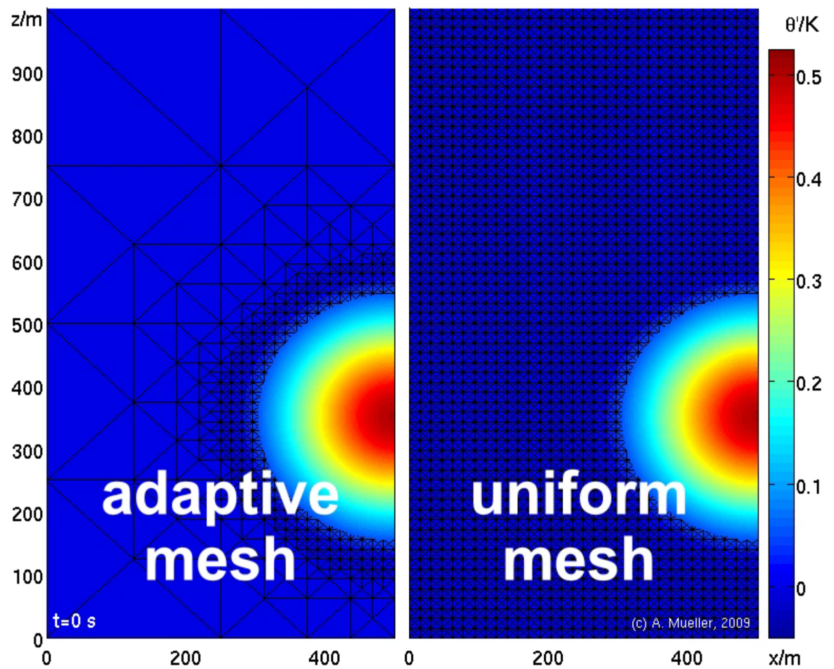
What Should We Aim For?

1. Conservation – Conservation of Mass and Energy are absolute musts; what else should we conserve?
2. Scalability – New models must be highly scalable because we will continue to get more processors
3. High-Order Accuracy – Accuracy is important, of course, but how do we measure this and what order accuracy is sufficient? This question is coupled to the accuracy of the physics, data assimilation, etc. From the standpoint of scalability, high-order is good (hp methods = on-processor work is large but the communication footprint is small). This is also a good strategy for exploiting MPI/Open MP Hybrid.
4. Adaptivity – Adaptive methods have improved tremendously in the past decade and it may offer an opportunity to solve problems not feasible a decade ago but we need to identify these applications (e.g., hurricanes, storm-surge modeling).
5. Coastal Ocean Model – Is a single-layer SW model sufficient? If not, how many layers?

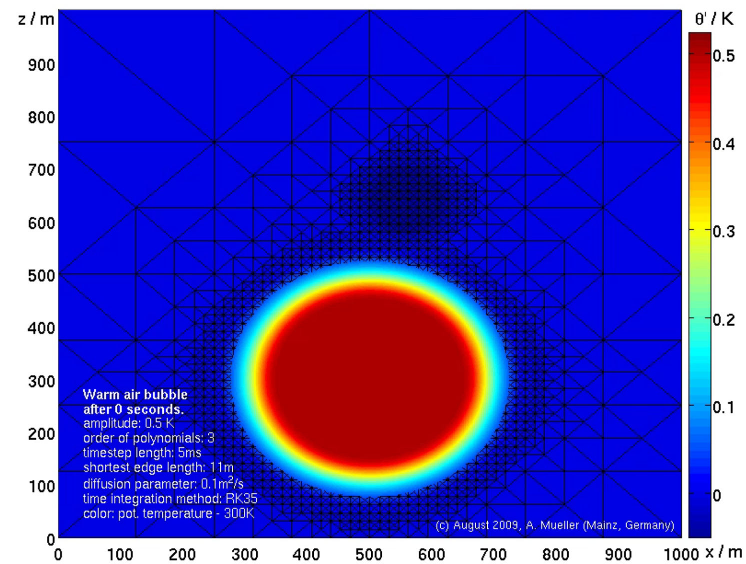
Some Standing Issues for Adaptive Methods

- **Parallelization/Domain Decomposition**: Modifying the data structures dynamically slows the computations. E.g., the domain decomposition needs to be a direct by-product of the adaptive mesh generator. A good first candidate for AM is statically adaptive grids where the grid is modified and held fixed for the entire simulation. This must work well before moving onto dynamically adaptive grids.

Non-hydrostatic Adaptivity Examples
(Müller, Behrens, Giraldo, Wirth 2010)



Rising Thermal Bubbles



Two (Warm/Cold) Thermal Bubbles

Talk Summary

1. New models need to exploit available computers
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4. Where we plan to head with our GFD models
   - Weather and Climate Models
   - Coastal Ocean Models
   - Coupled System

Atmospheric Models: Compressible Navier-Stokes with Stratification

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \mathbf{U} = 0$$ (Mass)

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left(\frac{\mathbf{U} \otimes \mathbf{U}}{\rho} + P\mathbf{I}_3 \right) = -f\left(\mathbf{k} \times \mathbf{U}\right) - \rho g \mathbf{k}$$ (Momentum)

$$\frac{\partial E}{\partial t} + \nabla \bullet \left(\frac{(E+P)\mathbf{U}}{\rho} \right) = 0$$ (Energy)

$$P = (\gamma - 1)\left(E - \frac{\mathbf{U} \bullet \mathbf{U}}{2\rho} - \rho\varphi \right)$$ (Equation of State)

$$e = c_v T + \frac{\mathbf{u} \bullet \mathbf{u}}{2} + \varphi$$

$$\varphi = gz$$

$$\mathbf{U} = \rho \mathbf{u},$$

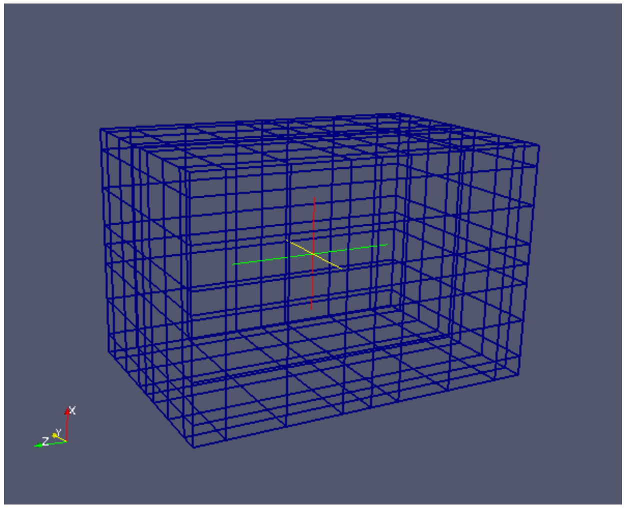
$$E = \rho e,$$

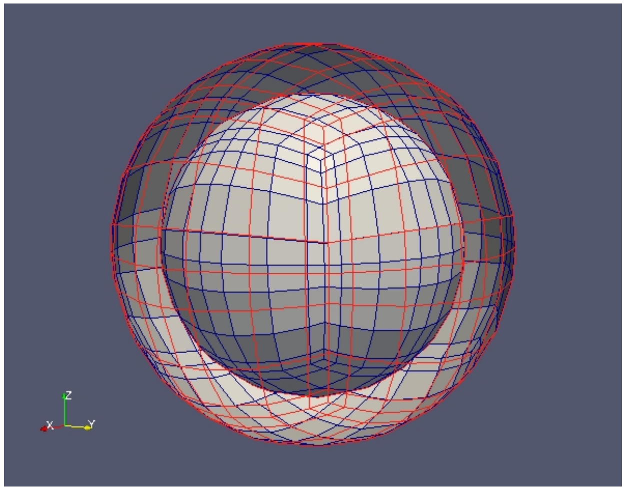
$$\mathbf{u} = (u, v, w)^T,$$

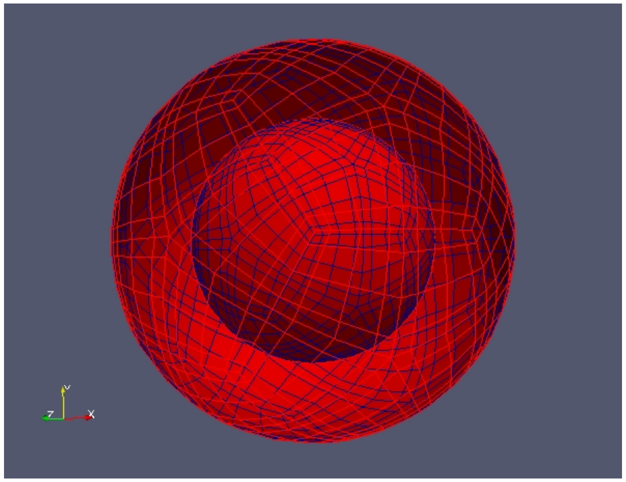
$$\mathbf{x} = (x, y, z)^T,$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)^T$$

Example of 3D Grids







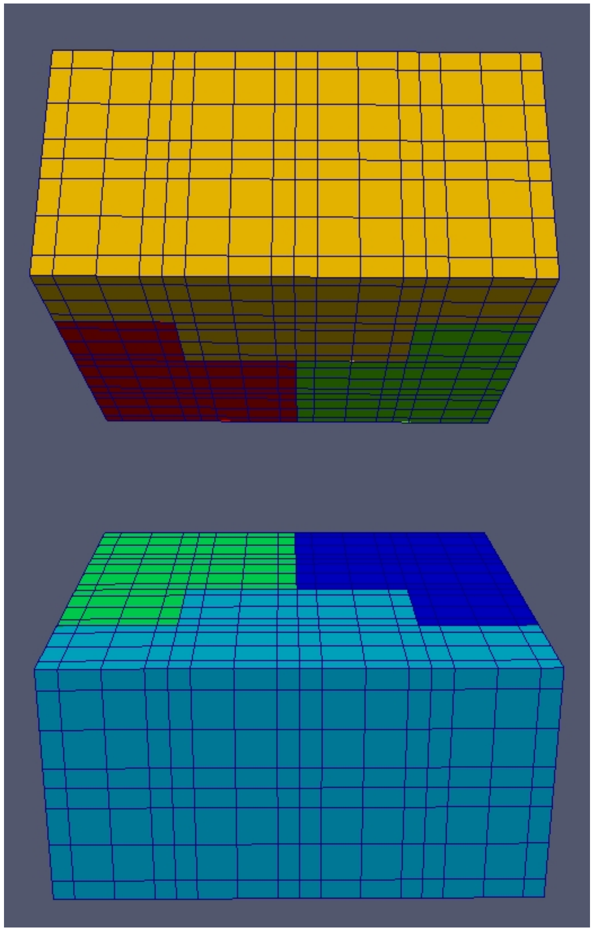
Mesoscale Modeling Mode

Global Modeling Mode (Cubed-Sphere)

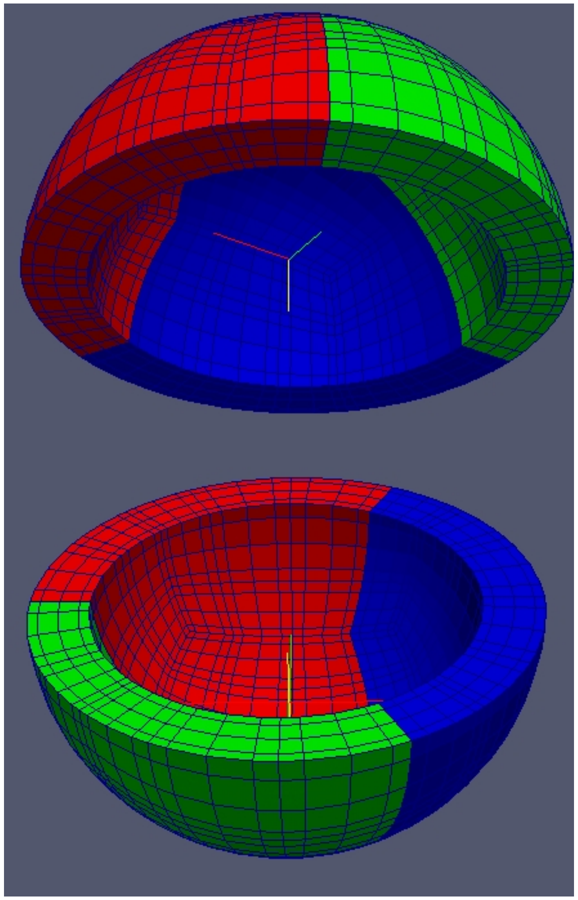
Global Modeling Mode (Icosahedral)

- NUMA runs in either Limited-Area or Global Mode.
- Currently, any grid can be used including completely unstructured grids.
- Parallel Domain Decomposition handled by METIS.

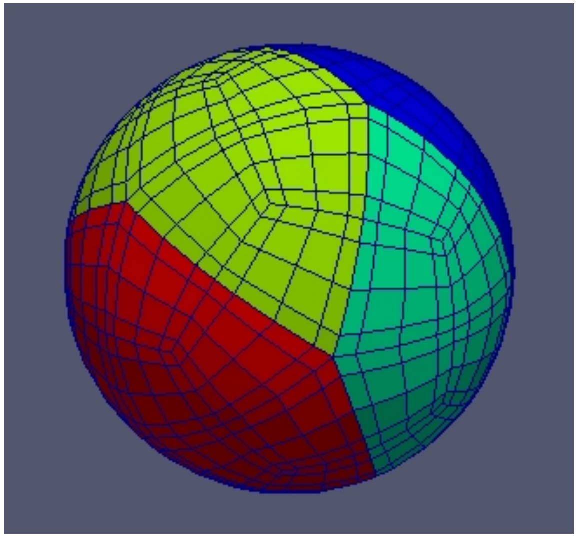
Domain Decomposition via METIS



1: Decomposition of a 3D Cartesian domain using 64 spectral elements (fourth order).



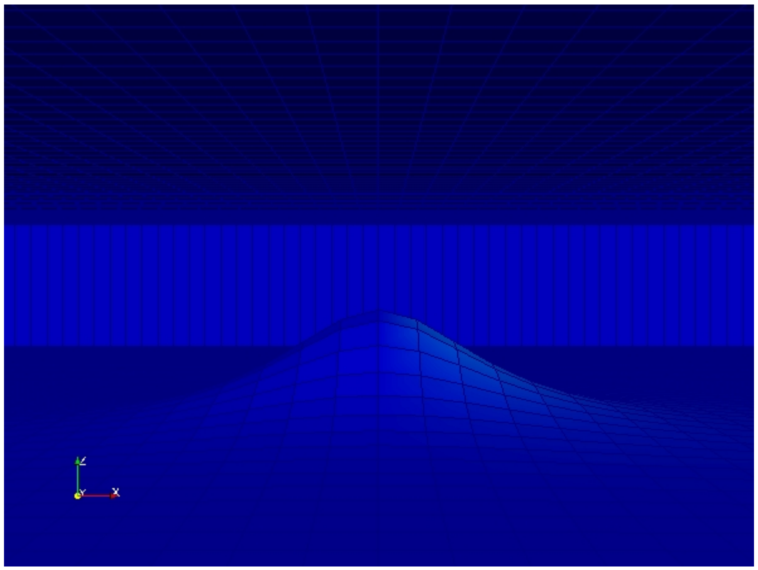
2: Decomposition of a “cubed sphere” using 96 spectral elements (fourth order).



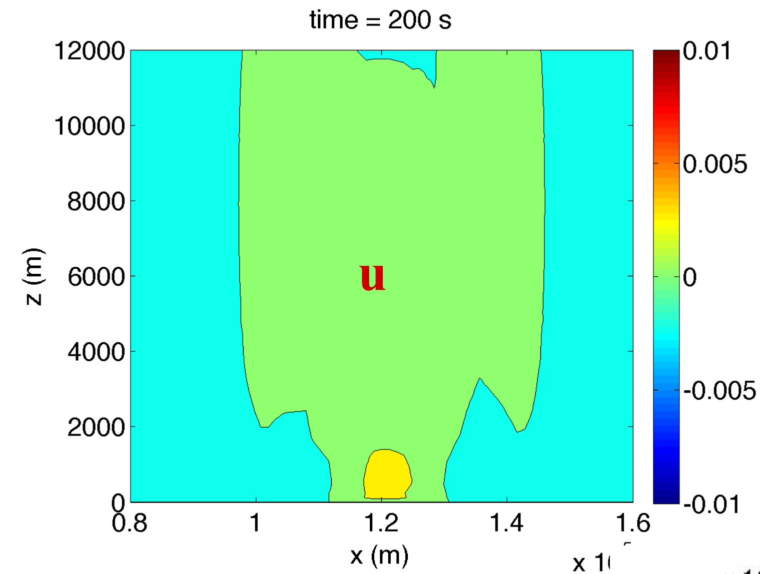
3: Decomposition of an “icosahedral sphere” using 96 elements (fourth order).

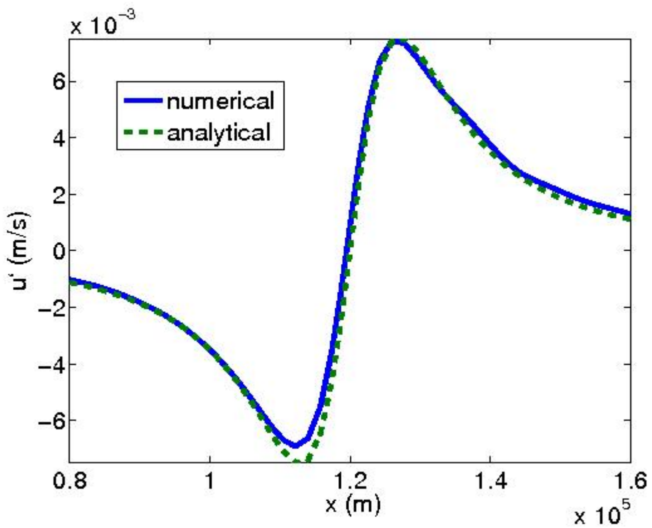
Limited-Area Mode:
Linear Hydrostatic Isolated Mountain

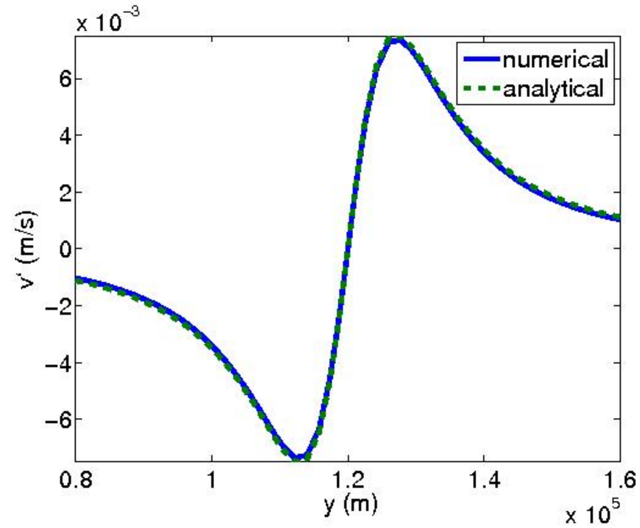
- Flow of U=20 m/s in an isothermal atmosphere.
- LH Mountain: Solid of revolution of Witch of Agnesi: Mountain height = 1 m with radius 10 km.
- Absorbing (sponge) boundary condition implemented on lateral and top boundaries.



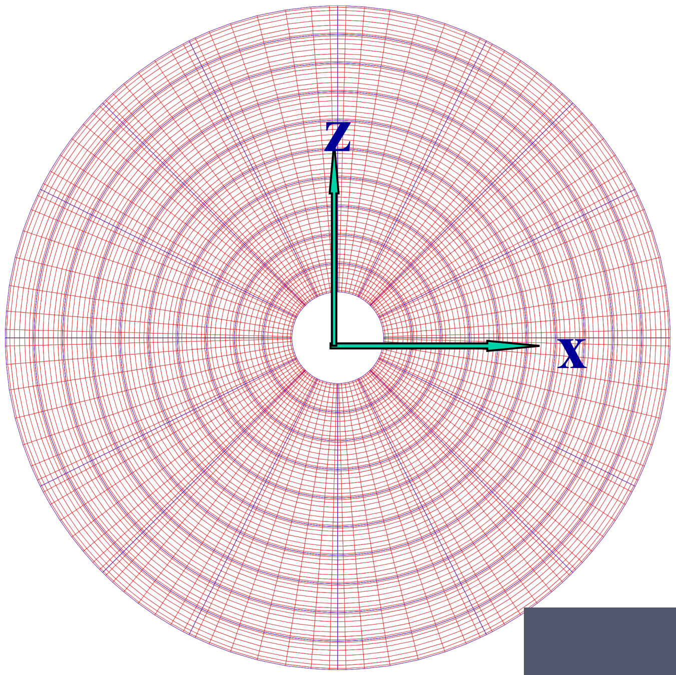
Limited-Area Mode:
Linear Hydrostatic Isolated Mountain

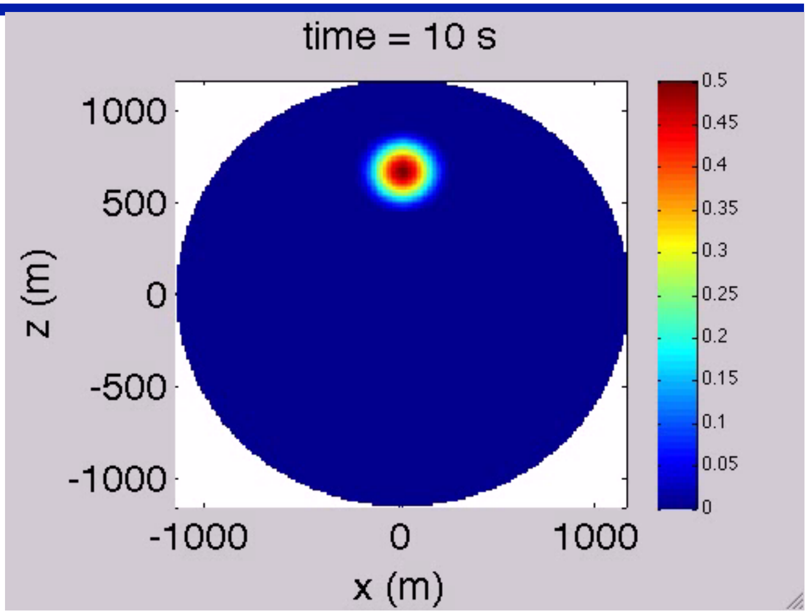


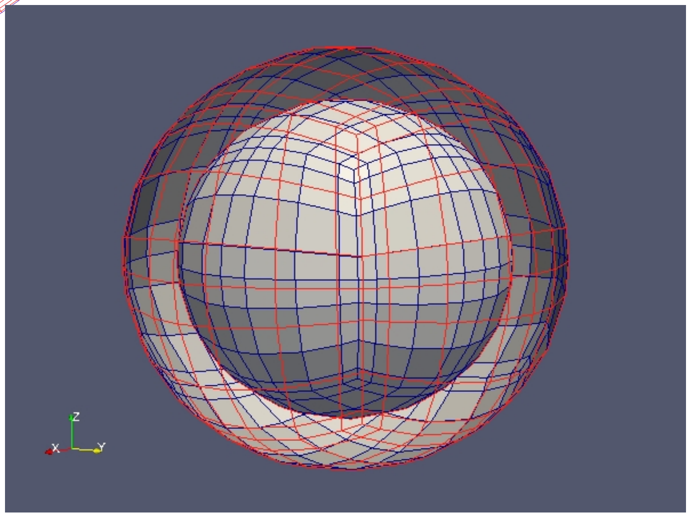




Global Mode:
Rising Thermal Bubble







Coastal Ocean Model: Shallow Water Equations

$$\frac{\partial \varphi}{\partial t} + \nabla \bullet \mathbf{U} = 0$$

(Mass)

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left(\frac{\mathbf{U} \otimes \mathbf{U}}{\varphi} + \frac{1}{2}\left(\varphi^2 - \varphi_B{}^2\right)\mathbf{I}_2 - \nu \nabla \mathbf{U} \right) = -\varphi_S \nabla \varphi_B - f(\mathbf{k} \times \mathbf{U}) + g\frac{\mathbf{t}}{\rho} - \gamma \mathbf{U}$$

(Momentum)

$$\mathbf{u} = (u, v)^T,$$

$$\mathbf{U} = \varphi \mathbf{u},$$

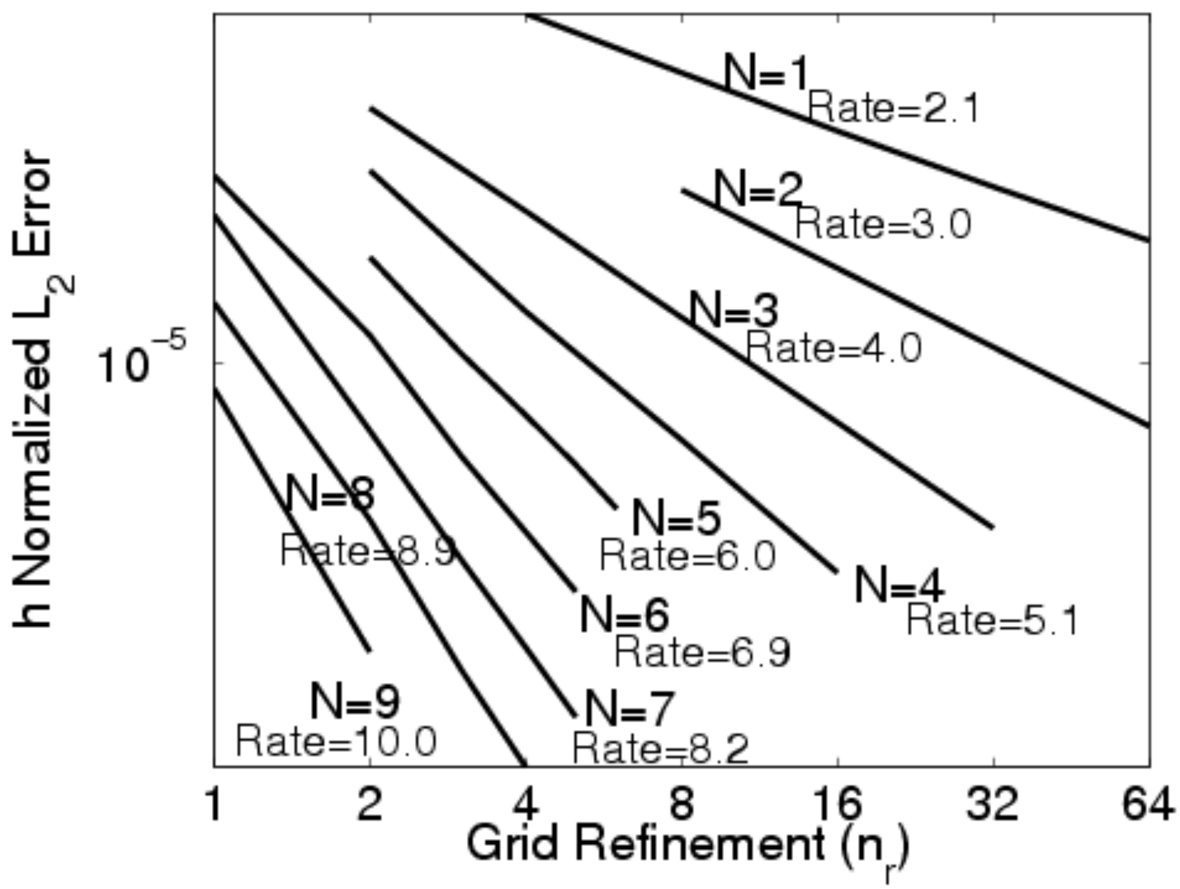
$$\mathbf{x} = (x, y)^T,$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)^T$$

$$\varphi = g\left(h_S + h_B\right)$$

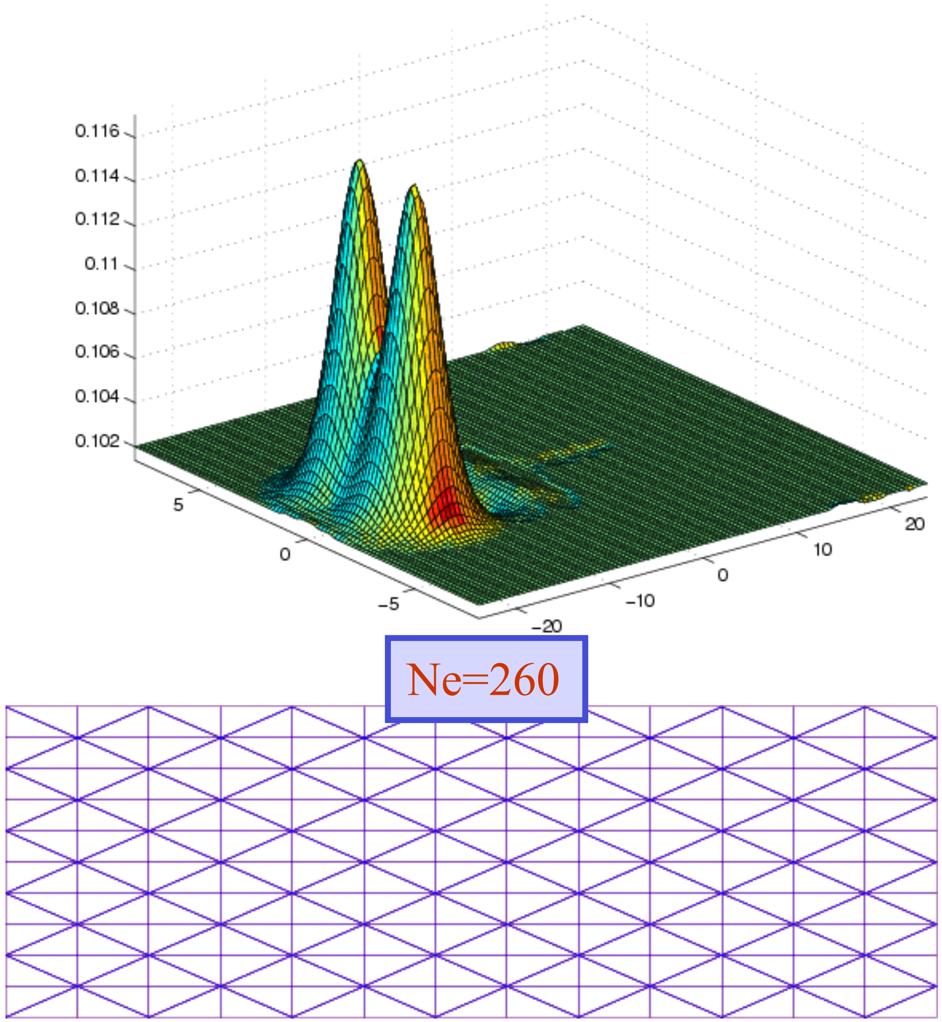
(Geopotential)

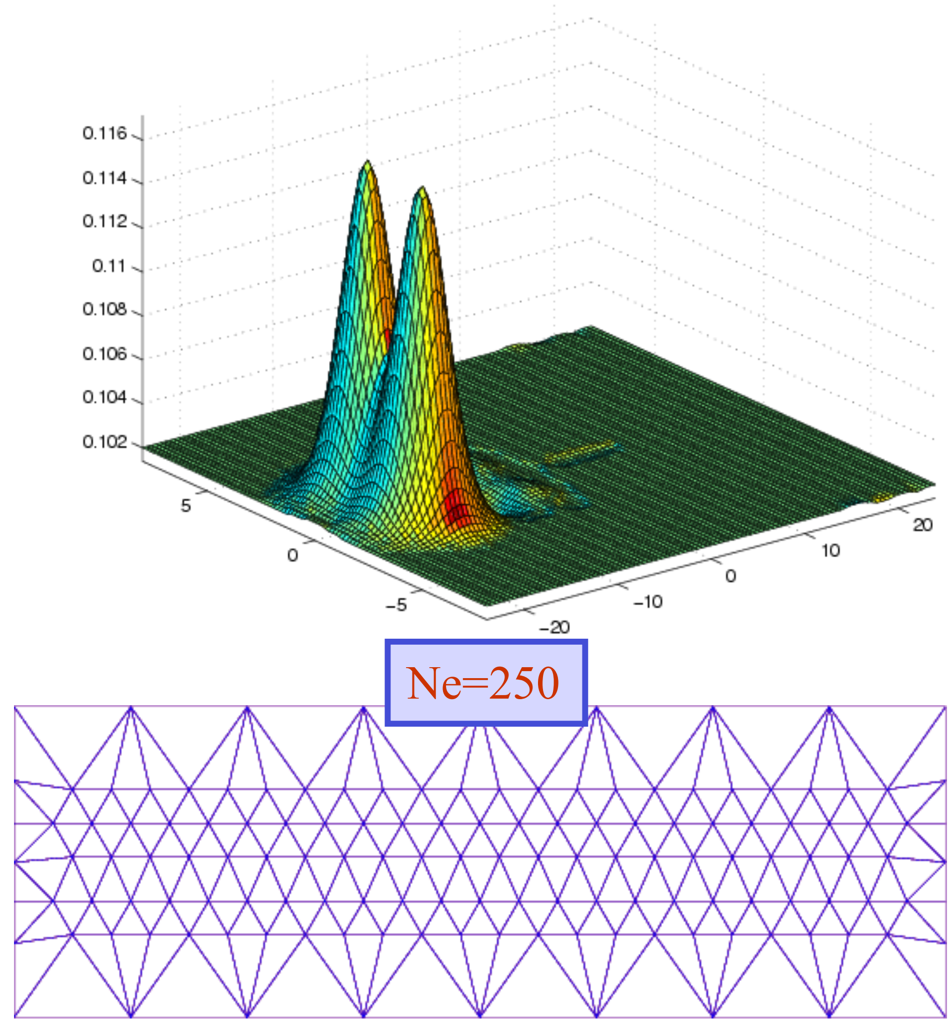
Convergence Rate for DG Triangles (Linear Standing Wave)



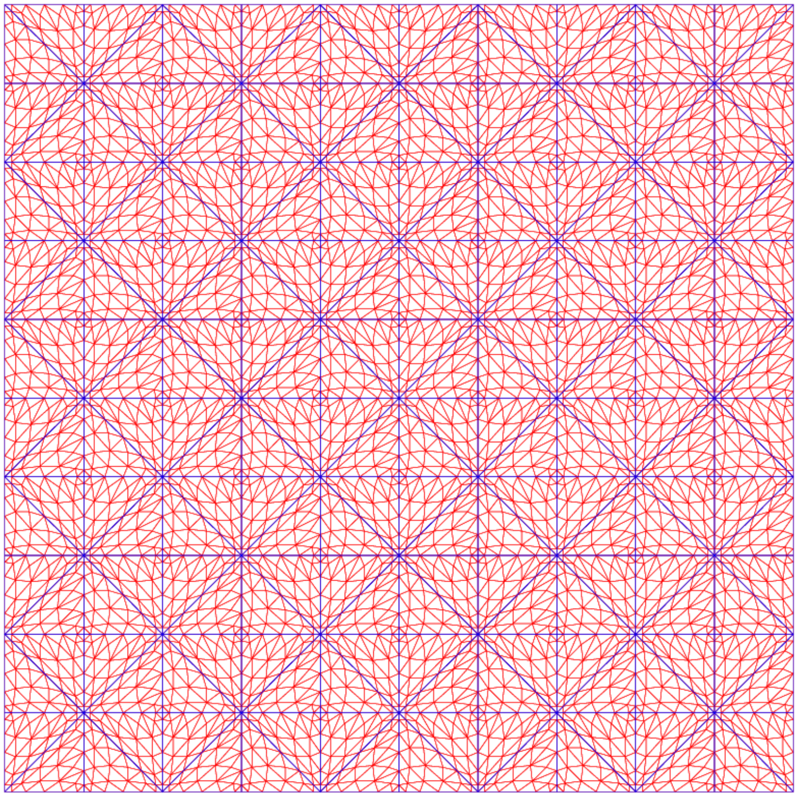
Note that the method achieves the expected convergence rate; i.e., error $=O(h^{N+1})$

Coastal Ocean Model
(Rossby Soliton Wave in a Channel)
(8^{th} Order Polynomials)

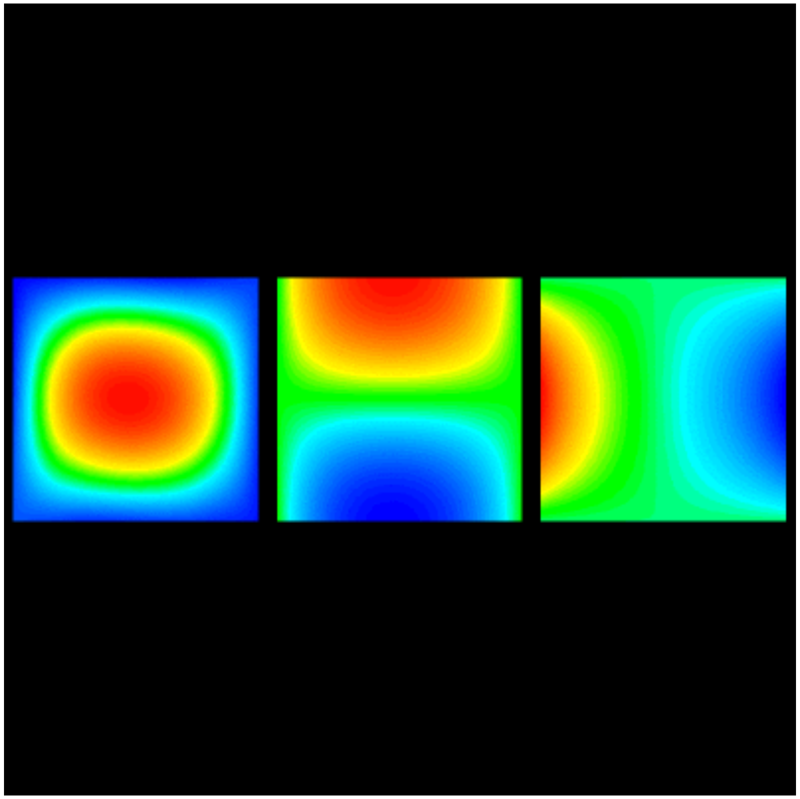




Linear Stommel Problem
(Grid Dimensions: Np=3721, Ne=200, N=6)
(Operators Tested: Wind Stress, Coriolis, and Bottom Friction)

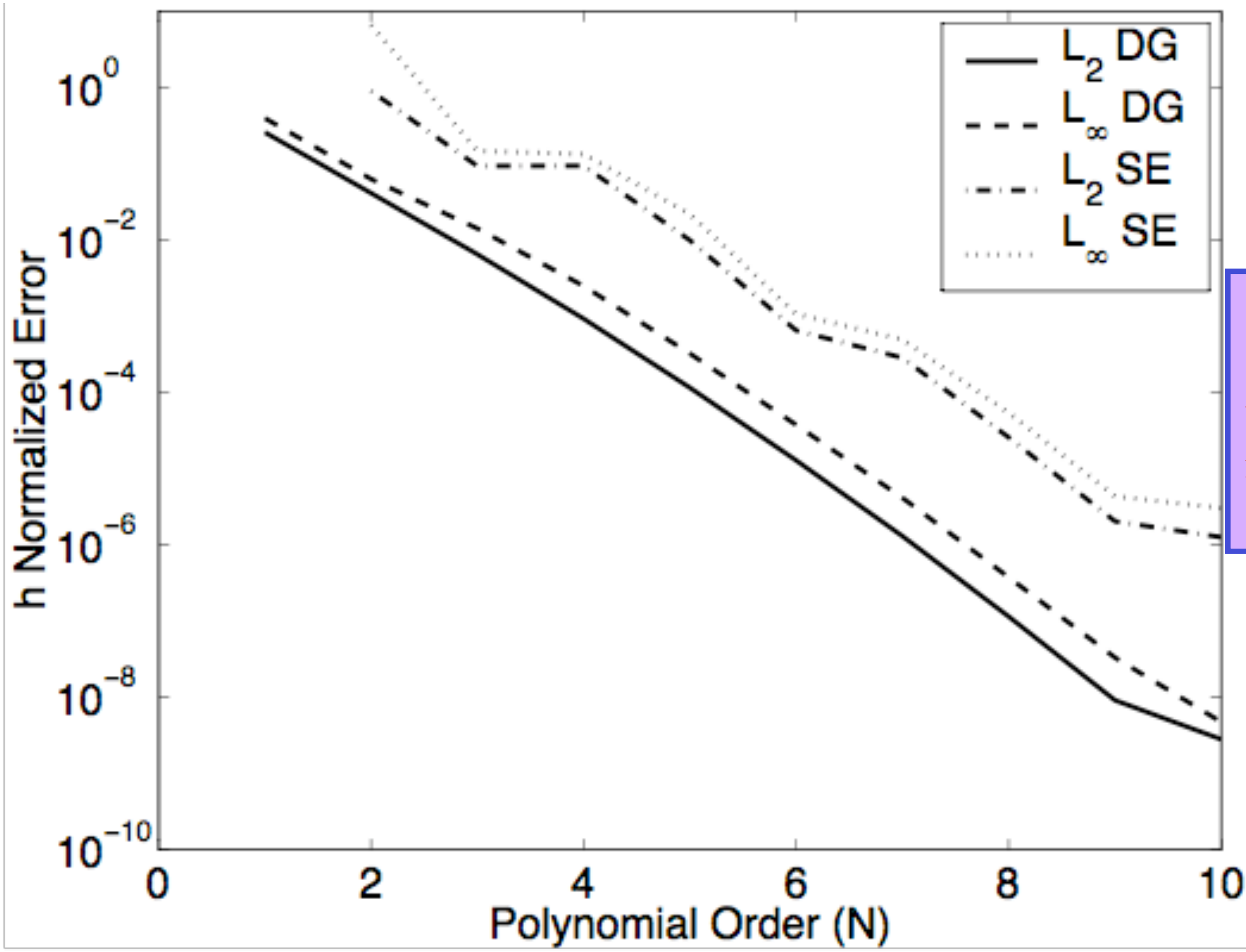


Grid



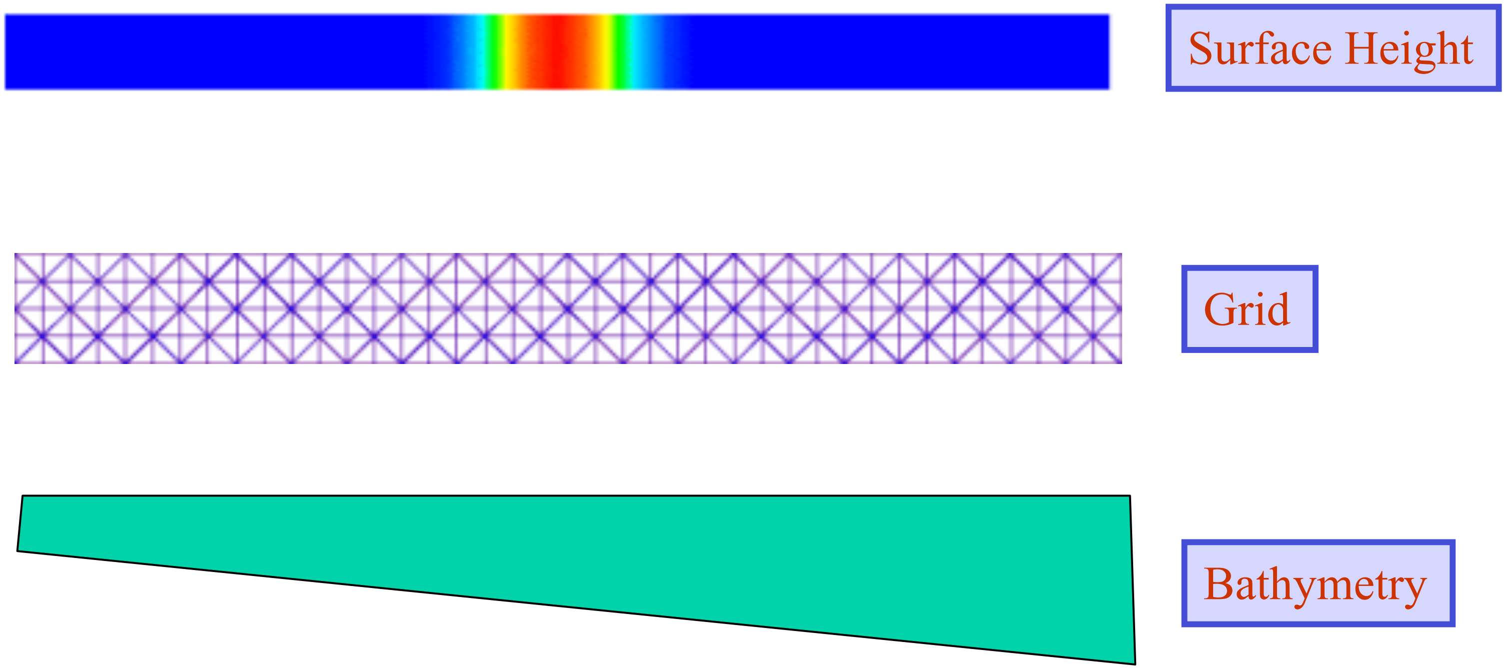
Free Surface Height, U and V

Linear Stommel Problem
(Ne=32)

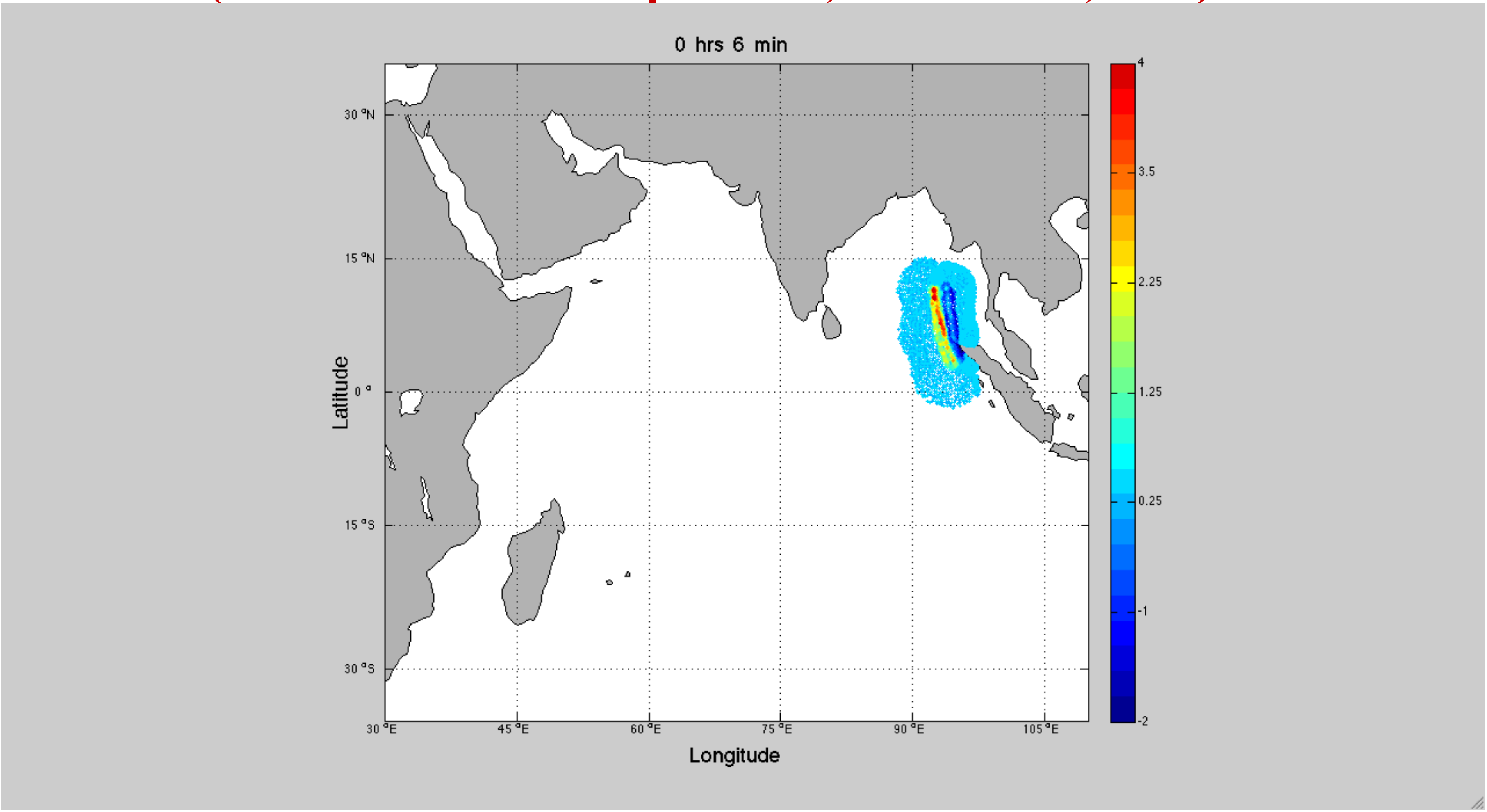


Semi-Implicit 3 times faster than explicit method

Cosine Wave on a Sloping Beach
(Grid Dimensions: Np=205, Ne=320, N=1)
(Operators Tested: Nonlinear Terms and Bathymetry)

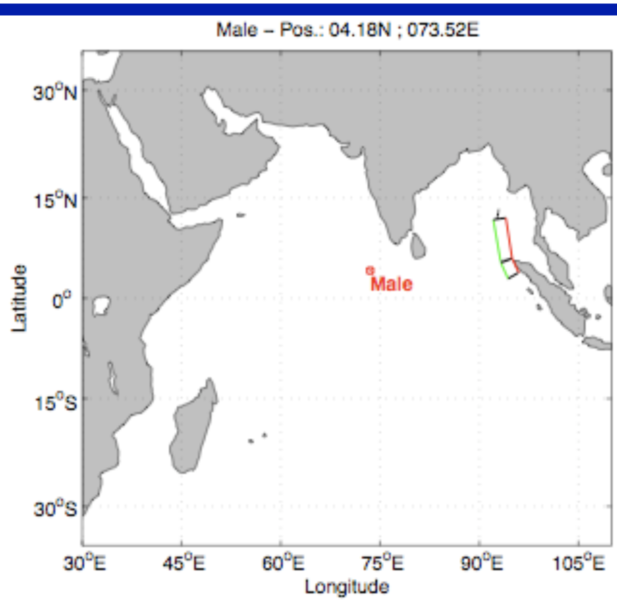


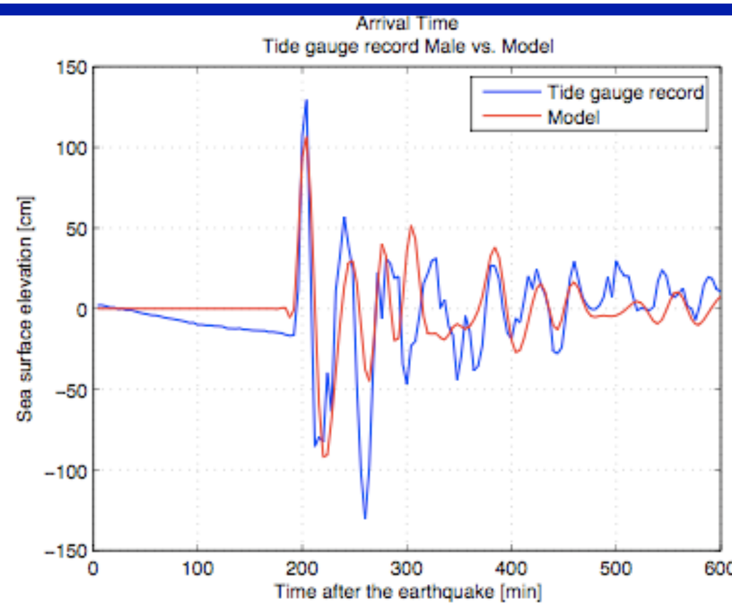
Propagation of the 2004 Indian Ocean Tsunami
(Grid Dimensions: Np=66715, Ne=130444, N=1)

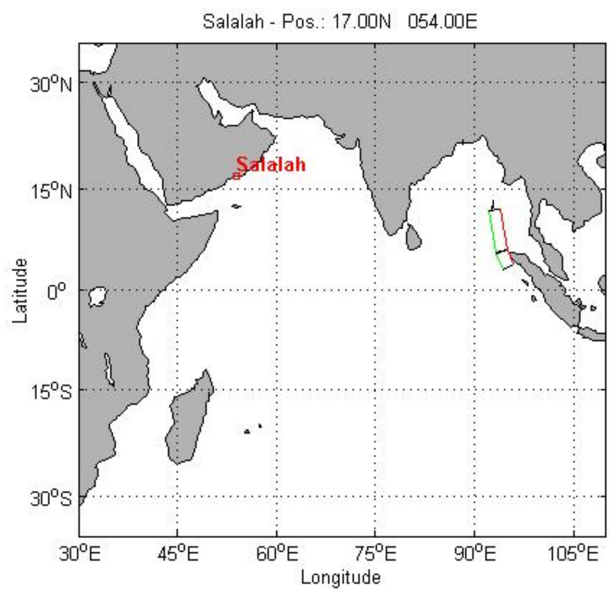


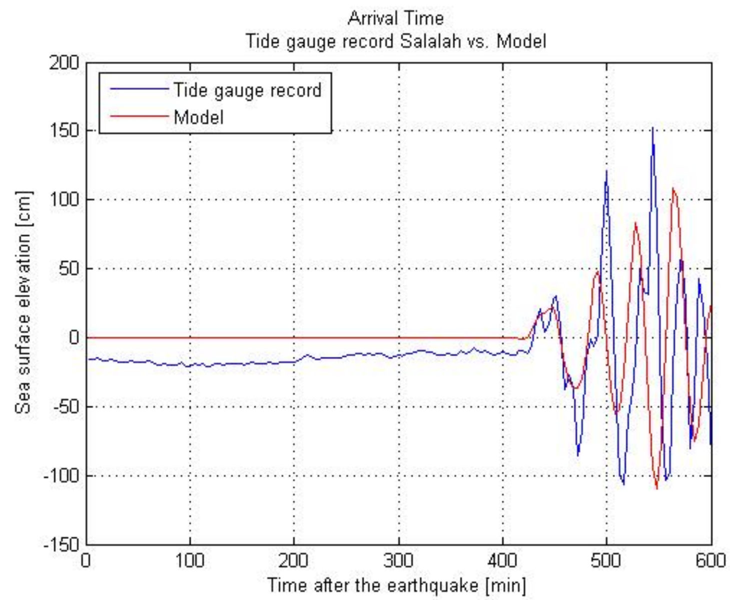
Time evolution of the water surface height (grid data provided by J. Behrens, AWI, and data formatted by D. Alevras, NPS)

Propagation of the 2004 Indian Ocean Tsunami
(Comparisons between Obs and Model)









Propagation of the 2004 Indian Ocean Tsunami
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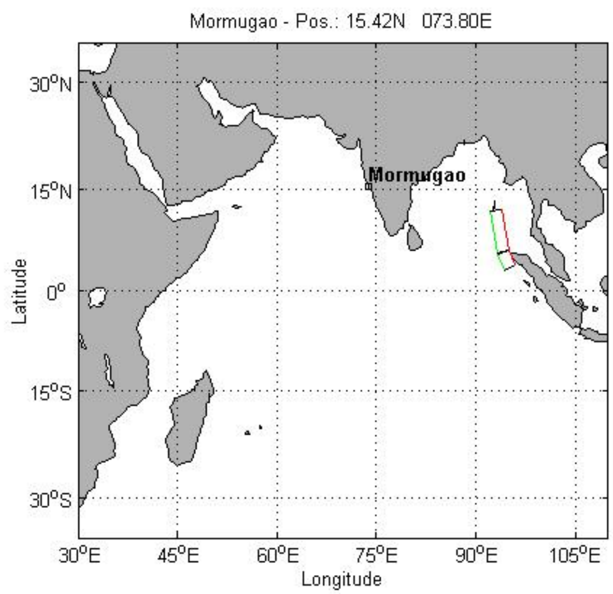

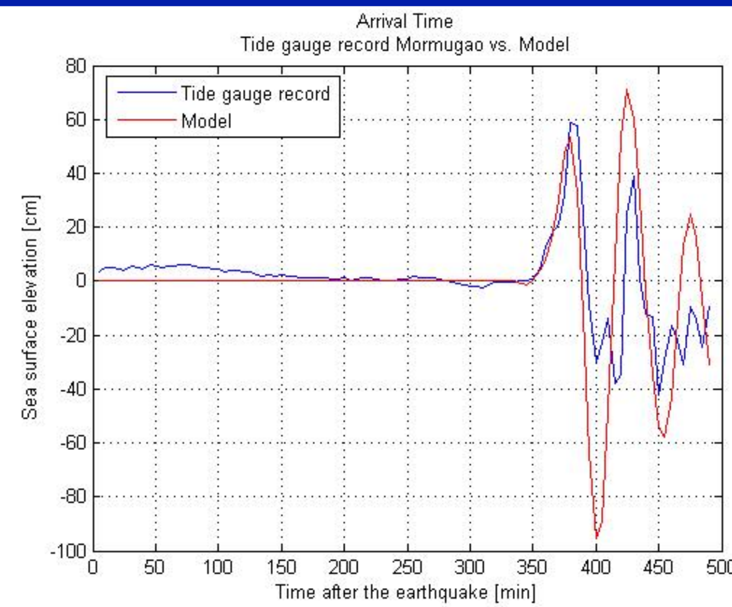

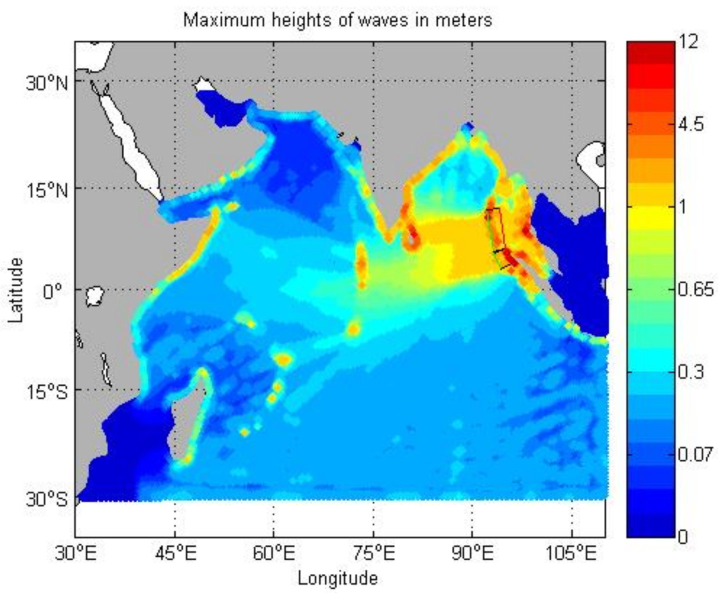

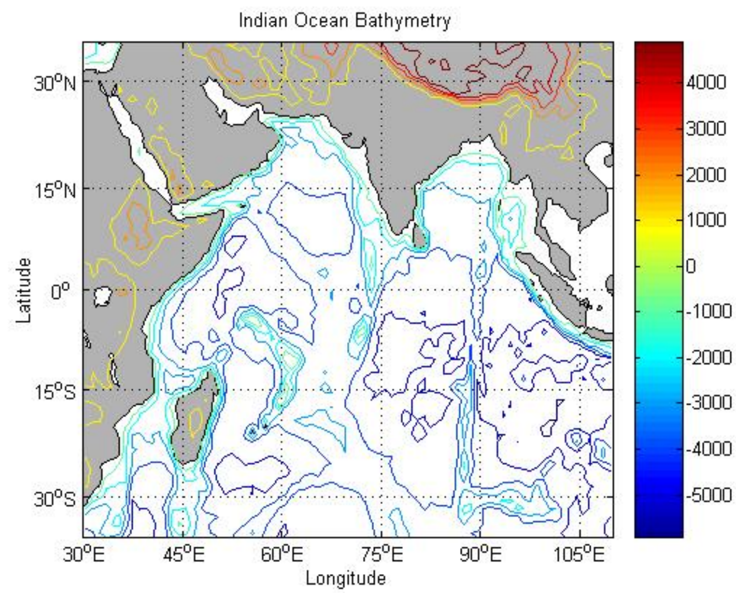

A Multitude of Challenges Remain

- Discontinuous Galerkin method is a great choice for both atmospheric and ocean models.
- Using (high-order) triangular elements allows for straightforward use of adaptive unstructured grids for complex geometries (e.g., coastlines, etc.).
- The atmospheric model is quite mature:
  - 3D and MPI
- The coastal ocean model needs additional work:
  - only 2D and serial – need to consider 3D and MPI.
  - Wetting and drying algorithms work for N=1 but are implementing new methods that preserve well-balanced and positivity (for wetting and drying).
- Coupling the models is possible especially if triangular prisms are used.
  - This is feasible in the context of a coupled atmosphere-ocean model especially for limited area only and with statically adaptive grids.
  - The disparate time-scales between the atmosphere-ocean can be handled using extrapolation (multi-rate) methods.
  - We have (unanswered) questions about whether we need to go to multi-layer shallow water equations or is the 2D SWE sufficient? What about going to full INSE? Is this overkill?
  - Future projects include adding wave models (perhaps SWAN is a good choice as in Joannes' talk)