We propose a new teletraffic model, named Wireless Engset Multi-rate Loss Model (W-EnMLM) for the call-level analysis of W-CDMA networks, supporting heterogeneous service-classes of finite population of traffic sources. Quasirandom arriving calls (generated by mobile users), compete for their admission to a W-CDMA cell under the Complete bandwidth Sharing (CS) policy. We present an approximate but recurrent formula for the efficient calculation of the system state probabilities and the call blocking probabilities in the uplink direction. The accuracy of the proposed model is verified by simulation results and found to be completely satisfactory. Moreover, the proposed model performs much better than the corresponding model of infinite number of traffic sources.

I. INTRODUCTION

The well-known Erlang Multi-rate Loss Model (EMLM) is used for the analysis of traditional networks with random (Poisson) arriving calls – i.e. assuming infinite population of traffic sources that generate calls. A recurrent formula developed by Kaufman and Roberts (K-R recursion) achieves the efficient and accurate calculation of call blocking probabilities (CBP) in the EMLM [9], [10]. An important extension of the EMLM is the Engset Multirate Loss Model (EnMLM), where the call arrival process is considered quasi-random - i.e. finite population of traffic sources is assumed [13]. The CBP calculation in the EnMLM is tough because enumeration and processing of the system state space are required. To avoid it and calculate the CBP efficiently in the EnMLM, Glabowski and Stasiak proposed in [6] an approximate but quite accurate method (G&S method). Note that in the EnMLM, there is a distinction between time and call congestion probabilities. They coincide in the case of Poisson arrivals (PASTA property [2]). In this paper by the term CBP we refer to the time congestion probabilities. These are determined by the portion of time the system is congested and can be measured by an outside observer.

The EMLM and EnMLM have been widely used in wired networks. In wireless networks the applicability of the aforementioned models is not possible and, in any case, not straightforward. Regarding cellular networks, infinite number of users roaming in a cell or in its vicinity is assumed in most publications, and therefore only the EMLM has been applied [8], [11]. More precisely, in cellular networks the radio resources are often being managed in a centralized manner (e.g. by RNC in UMTS networks). A single base station controlling a cell is modeled as a system of certain bandwidth capacity in the uplink direction, while accommodating an infinite number of mobile users. However, due to the limited coverage area of a cell, it is certainly more realistic to consider that this number is finite. This consideration is especially true in the case of microcells (small size cells). We avoid the case of picocells (used in indoor cellular systems), because the amount of handoff traffic that comes from users’ mobility is significant, compared to the traffic originated by the new calls of the users; in that case handoff traffic should be treated separately from the offered new traffic, independently from the considered number of users in picocells. In this paper we consider the handoff traffic of a cell or microcell, as one component of the offered quasi-random traffic ([2]), without distinguishing it from the traffic originated by new calls. This is in accordance with the widely used assumption that handoff traffic is Poisson and constitutes one component of the total random traffic [11], [15]. In any case, the explicit incorporation of the handoff traffic to teletraffic models needs further study. Concentrating on the uplink direction of W-CDMA networks, a few teletraffic models have been proposed for CBP calculation and eventually determination of the uplink capacity. In [11], the CBP calculation in the uplink of a W-CDMA cell is based on an extension of the K-R recursion, and an Interference-based Call Admission Control (CAC) algorithm [7]. According to it, a call is accepted, as long as there is enough bandwidth available in the cell. After acceptance, the signal-to-noise ratio of all in-service calls deteriorates. A call should not be accepted if it increases the noise of all in-service calls above a tolerable level, given that a call is noise for all other calls. This CAC corresponds to the CS policy in wired networks, since no restriction per service-class is set. In this paper, we refer to the model of [11], as Wireless-EMLM (W-EMLM). In [5], although finite source population is assumed, CBP calculation is also based on the K-R recursion, while considering a reduced load approximation method. In [3], [4] and [14], similar teletraffic models have been proposed for CDMA networks, accommodating a single service-class only.

In this paper we extend the EnMLM to become suitable for the analysis of W-CDMA networks. The new model is named Wireless Engset Multirate Loss Model (W-EnMLM). We propose an approximate but recurrent algorithm for the efficient CBP calculation in the uplink direction while incorporating the G&S method. Intuitively the proposed model is better than the other models which are based on the EMLM, because a realistic number of traffic sources in the cell is assumed. This is proved through comparison and relying to simulation results.

This paper is organized as follows: In section II we briefly review the EnMLM. In section III we propose the W-EnMLM; in subsection III.A the model description is given, while in subsections III.B to III.D we calculate the local blocking probabilities, state probabilities and CBP, respectively. Section IV is the evaluation section. We conclude in section V.
II. REVIEW OF THE ENMLM

A. Model Description

Consider a system of capacity $C$ bandwidth units (b.u.) that accommodates calls of $K$ independent service-classes. Calls of a service-class $k$ ($k=1, \ldots, K$) are generated by $N_k$ sources with arrival rate per idle source equal to $\gamma_k$. Calls compete for the available bandwidth under the complete sharing policy [1]. Each service-class $k$ call requests $b_k$ b.u. upon arrival. If they are available, the call is accepted in the system and the $b_k$ b.u. are occupied by the call for a time, exponentially distributed with mean $\mu_k^{-1}$. Otherwise the call is blocked.

B. Local Balance Equation and Bandwidth Share

The system state $j$ ($j=0, \ldots, C$) is defined as the total number of b.u. occupied by the calls. The probability that the system is in state $j$ is denoted by $q(j)$.

In the ENMLM the following local balance equation exists between adjacent system states [13]:

$$q(j) = \sum_{k=1}^{K} (N_k - n_k(j) + 1) a_k b_k q(j - b_k) = Y_k(j)b_k,$$

where $a_k = \gamma_k b_k^{-1}$ is the offered traffic-load per idle source of service-class $k$, $n_k(j)$ is the number of service-class $k$ calls in state $j$ at a given time and $Y_k(j)$ is the average number of service-class $k$ calls in state $j$. The offered traffic-load of service-class $k$ is denoted by $a_k$.

We calculate the bandwidth share (proportion of the system bandwidth capacity occupied by calls of a specific service-class) of service-class $k$ calls in state $j$ ($j>0$), $P_k(j)$ as follows:

$$P_k(j) = \frac{(N_k - n_k(j) + 1) a_k b_k q(j - b_k)}{Y_k(j)b_k}$$

C. State Probabilities

To calculate the normalized state probabilities, $q(j)$, the following recursion is used [13]:

$$q(j) = \sum_{k=1}^{K} (N_k - n_k(j) + 1) a_k b_k q(j - b_k), \text{for } j = 1, \ldots, C$$

and $q(j) = 0$ for $j < 0$, where $\sum_{j=0}^{C} q(j) = 1$

The calculation of $q(j)$’s through (3) is not direct; we need to know the exact number of each service-class’ calls, $n_k(j)$, in different system states $j$. To overcome this problem, we apply the tractable method of [6] whereby $n_k(j)$ is approximated by the average number of service-class $k$ calls with requirement $b_k$, in state $j$, assuming infinite population for each service-class:

$$n_k(j) = a_k q(j - b_k) / q(j)$$

where $a_k$ and $q(j)$ are the parameters of the corresponding infinite model (EMLM).

D. Call Blocking Probability

A new arriving call with requirement $b_k$ is accepted in the system only if $j \leq C - b_k$. Thus for the service-class $k$ the states $j=C-b_k+1, \ldots, C$ are blocking states. The CBP of service-class $k$ is determined by the formula:

$$B_k = \sum_{j=C-b_k+1}^{C} q(j)$$

III. THE WIRELESS ENMLM (W-ENMLM)

A. Model Description

Assume a W-CDMA system that supports $K$ independent service-classes. This system consists of a reference cell surrounded by neighboring cells. We consider only the uplink – i.e. calls from the mobile users to the base station (BS).

Four parameters characterize a service-class $k$ call:

- $R_k$: Transmission bit rate
- $(E_b/N_0)_k$: Bit error rate (BER) parameter
- $v_k$: Activity factor
- $N_k$: Number of sources that generate calls

The arrival rate per idle source of service-class $k$ calls is $\gamma_k$. The service-class $k$ calls’ holding time is exponentially distributed with mean $\mu_k^{-1}$. The offered traffic-load per idle source of service-class $k$ is defined as: $a_k = \gamma_k b_k^{-1}$. The offered traffic-load of service-class $k$ is denoted by $a_k$.

1) Interference and Call Admission Control

We distinguish the intra-cell interference, $I_{\text{intra}}$, caused by users of the reference cell and the inter-cell interference, $I_{\text{inter}}$, caused by users of the neighboring cells. We also consider the existence of the thermal noise, $P_N$, which corresponds to the interference of an empty system. We assume perfect power control – i.e. at the BS, the received power from each service-class $k$ call is the same [7].

The CAC in the W-CDMA system under consideration is performed by measuring the noise rise, $NR$, which is defined as the ratio of the total received power at the BS, $P_{\text{total}}$ to the thermal noise power, $P_N$:

$$NR = \frac{I_{\text{total}}}{P_N}$$

When a new call arrives, the admission control estimates the noise rise and if it exceeds a maximum value, $NR_{\text{max}}$, the new call is blocked and lost.

2) User Activity

A user, during his call’s duration, alternates between transmitting and silent periods. This behavior is characterized by the activity factor $v_k$, which represents the fraction of the call’s holding time during which the user is occupying system resources. Users that at a time instant occupy system resources are referred to as active users. The rest of the users (passive users) are in silent period and do not occupy any
system resources. The service-class $k$ user activity at a new call arrival can be modelled by a Bernoulli random variable with probability of success equal to the activity factor $v_k$ [14].

3) Load Factor and Cell Load

The cell load, $n$, is defined as the ratio of the received power from all active users (at the reference or neighbouring cells) to the total received power:

$$n = \frac{I_{\text{intra}} + I_{\text{inter}}}{I_{\text{intra}} + I_{\text{inter}} + P_N}$$  \hspace{1cm} (7)

Hence from (6) and (7) we can derive the relation between the noise rise and the cell load:

$$NR = \frac{1}{1 - n} \quad \text{and} \quad n = \frac{NR - 1}{NR}$$  \hspace{1cm} (8)

We define the maximum value of the cell load, $n_{\text{max}}$, as the cell load that corresponds to the maximum noise rise, $NR_{\text{max}}$. Typical value is $n_{\text{max}} = 0.8$ and it can be considered as the shared system resource.

The load factor, $L_k$ of (9) can be considered as the bandwidth requirement of a service-class $k$ call [11]:

$$L_k = \frac{(E_b / N_0)_k * R_k}{W + (E_b / N_0)_k * R_k}$$  \hspace{1cm} (9)

where $W$ is the chip rate of the W-CDMA carrier (3.84 Mcps).

The cell load can be written as the sum of the intra-cell load, $n_{\text{intra}}$ (cell load that derives from the users of the reference cell) and the inter-cell load, $n_{\text{inter}}$ (cell load that derives from the users of the neighboring cells). They are defined in (10) and (11), respectively:

$$n_{\text{intra}} = \sum_{k=1}^{K} m_k L_k$$  \hspace{1cm} (10)

$$n_{\text{inter}} = (1 - n_{\text{max}}) \frac{I_{\text{inter}}}{P_N}$$  \hspace{1cm} (11)

According to the adopted CAC policy, the following condition is used at the BS in order to decide whether to accept a new call or not:

$$n + L_k \leq n_{\text{max}}$$  \hspace{1cm} (12)

B. Local Blocking Probability

Due to (12), the probability that a new service-class $k$ call is blocked when arriving at an instant with intra-cell load $n_{\text{intra}}$ is called local blocking probability (LBP) and is calculated by:

$$\beta_k(n_{\text{intra}}) = P(n_{\text{intra}} + n_{\text{inter}} + L_k > n_{\text{max}})$$  \hspace{1cm} (13)

To calculate the LBP of (13) we use (10)-(12) where the only unknown parameter is the inter-cell interference, $I_{\text{inter}}$.

Similarly to [12], we model $I_{\text{inter}}$ as a lognormal random variable (with parameters $\mu_i$ and $\sigma_i$), that is independent of the intra-cell interference. Hence, the mean, $E[I_{\text{inter}}]$ and the variance, $Var[I_{\text{inter}}]$ are given by (14) and (15):

$$E[I_{\text{inter}}] = e^{\mu_i + \frac{\sigma_i^2}{2}}$$  \hspace{1cm} (14)

$$Var[I_{\text{inter}}] = (e^{\sigma_i^2} - 1)e^{2\mu_i + \sigma_i^2}$$  \hspace{1cm} (15)

Consequently, because of (11), the inter-cell load, $n_{\text{inter}}$ will also be a lognormal random variable. Its mean, $E[n_{\text{inter}}]$ and the variance, $Var[n_{\text{inter}}]$ are calculated by:

$$E[n_{\text{inter}}] = e^{\mu_n + \frac{\sigma_n^2}{2}} = \frac{1 - n_{\text{max}}}{P_N} E[I_{\text{inter}}]$$  \hspace{1cm} (16)

$$Var[n_{\text{inter}}] = (e^{\sigma_n^2} - 1)e^{2\mu_n + \sigma_n^2} = \frac{1 - n_{\text{max}}}{P_N} Var[I_{\text{inter}}]$$  \hspace{1cm} (17)

where $\mu_n$ and $\sigma_n$ are the parameters of $n_{\text{inter}}$, which can be found by solving (16) and (17):

$$\mu_n = \ln(E[I_{\text{inter}}]) - \frac{\ln(1 + CV[I_{\text{inter}}]^2)}{2} + \ln(1 - n_{\text{max}}) - \ln(P_N)$$  \hspace{1cm} (18)

$$\sigma_n = \sqrt{\ln(1 + CV[I_{\text{inter}}]^2)}$$  \hspace{1cm} (19)

The coefficient of variation, $CV[I_{\text{inter}}]$ is defined as:

$$CV[I_{\text{inter}}] = \sqrt{\frac{Var[I_{\text{inter}}]}{E[I_{\text{inter}}]}}$$  \hspace{1cm} (20)

Note that (13) can be rewritten as:

$$1 - \beta_k(n_{\text{intra}}) = P(n_{\text{inter}} \leq n_{\text{max}} - n_{\text{intra}} - L_k)$$  \hspace{1cm} (21)

The right hand side of (21), is the cumulative distribution function (CDF) of $n_{\text{inter}}$. It is denoted by $P(n_{\text{inter}} \leq x) = F_n(x)$ and can be calculated from:

$$F_n(x) = \frac{1}{2}(1 + \text{erf}\left(\frac{\ln x - \mu_n}{\sigma_n \sqrt{2}}\right))$$  \hspace{1cm} (22)

where $\text{erf}(\cdot)$ is the well-known error function.

Hence, if we substitute $x = n_{\text{max}} - n_{\text{intra}} - L_k$ into (22), from (21) we have:

$$\beta_k(n_{\text{intra}}) = \begin{cases} 1 - F_n(x), & x \geq 0 \\ 1, & x < 0 \end{cases}$$  \hspace{1cm} (23)

C. State Probabilities

In W-CDMA networks the cell load, $n$ can be considered as shared bandwidth capacity and the load factor, $L_k$ as the
bandwidth requirement of a service-class \( k \) call. In what follows we calculate the state probabilities in the W-EnMLM. The EnMLM considers discrete state space. In the W-EnMLM, the discretization of the cell load \( n \) and the load factor, \( L_k \) is performed by the use of basic cell load unit, \( g \):

\[
C = \frac{n_{\text{max}}}{g} \quad \text{and} \quad b_k = \text{round}(\frac{L_k}{g}) \quad (24)
\]

The resultant discrete values \( C \) and \( b_k \) of (24) can be considered as the system capacity and the service-class \( k \) bandwidth requirement, respectively.

In W-CDMA networks we can exploit the fact that passive users do not consume system bandwidth, that is, in the W-EnMLM a state \( j \) does not represent the total number of occupied b.u. as it happens in the EnMLM. It represents, however, the total number of occupied b.u. when all users are active. We denote by \( c \) the total number of occupied b.u. at an instant. Note that in the EnMLM, \( c \) is always equal to \( j \), while in the W-EnMLM, we have \( 0 \leq c \leq j \). When \( c=0 \), all users are passive, whereas when \( c=j \), all users are active.

We denote by \( q(j) \) the probability that the system is in state \( j \). The bandwidth occupancy \( A(c|j) \) is defined as the conditional probability that \( c \) b.u. are occupied, when the state is \( j \) and, similarly to [11], can be calculated from (25):

\[
A(c|j)=\sum_{k=1}^{K} P_k(j)\left[ v_k A(c-b_k|j-b_k) +(1-v_k)A(c|j-b_k) \right]
\]

(25)

for \( j=1,...,j_{\text{max}} \) and \( c \leq j \)

where \( A(0|0)=1 \), \( A(c|j)=0 \) for \( c>j \) and \( j_{\text{max}} \) represents the highest reachable system state.

In W-CDMA systems, due to the inter-cell interference, blocking of a service-class \( k \) call may occur at any state \( j \) with a probability \( LB_k(j) \). This is called local blocking factor (LBF) and, similarly to [11], can be calculated from (26):

\[
LB_k(j) = \sum_{c=0}^{j} \beta_k(c)A(c|j) \quad (26)
\]

Note that for \( j=0 \) we have \( LB_k(0)=\beta_k(0) \).

The service-class \( k \) bandwidth share in state \( j \), \( P_k(j) \), is derived from (2) by incorporating the LBFs of (26):

\[
P_k(j) = \frac{\left( N_k - n_k(j)+1 \right)q_k(1-LB_k(j-b_k))b_k q(j-b_k)}{q(j)} \quad (27)
\]

The state probabilities, \( q(j) \), can be calculated by extending (3) due to the presence of local blockings:

\[
q(j)=\sum_{k=1}^{K} (N_k-n_k(j)+1)\alpha_k(1-LB_k(j-b_k))b_k q(j-b_k),
\]

(28)

for \( j=1,...,j_{\text{max}} \) and \( q(j)=0 \) for \( j<0 \), where \( \sum_{j=0}^{C} q(j)=1 \)

In the W-EnMLM (as it happens also in the EnMLM) the calculation of \( q(j) \)'s through (28) is not direct; we need the exact number of each service-class’ calls, \( n_k(j) \), in different system states \( j \). To overcome this problem, the tractable method of [6] can be applied whereby \( n_k(j) \) is approximated by the average number of service-class \( k \) calls with requirement \( b_k \) in state \( j \), assuming infinite population for each service-class:

\[
n_k(j) = a_k q(j-b_k)(1-LB_k(j-b_k))/q(j) \quad (29)
\]

where \( a_k \), \( q(j) \) and \( LB_k(j) \) are the parameters of the corresponding infinite model (W-EMLM).

D. Call Blocking Probabilities

The CBP of service-class \( k \) are given by adding all the state probabilities multiplied by the corresponding LBFs for all possible system states:

\[
B_k = \sum_{j=0}^{j_{\text{max}}} q(j)LB_k(j) \quad (30)
\]

IV. EVALUATION

We compare the analytical versus simulation CBP results for the W-EnMLM in order to show its accuracy. The simulation results are mean values from 6 runs with confidence interval of 95%. The resultant reliability ranges of the measurements are small and therefore we present only the mean CBP results. Furthermore, we compare the CBP results of the W-EnMLM to the CBP results of the W-EMLM ([11]).

A. Application Example

We consider a W-CDMA system with two service-classes: data and video. The traffic parameters used for each service-class are as follows (see also Table I): Data service-class requires a transmission rate of \( R_{t}=64 \) Kbps. The activity factor is chosen to be \( v_1=1.0 \) and the required BER parameter is \( (E_b/N_0)_1=4 \) dB. Video service-class requires a transmission rate of \( R_{t}=144 \) Kbps. The activity factor is chosen to be \( v_2=0.3 \) and the required BER parameter is \( (E_b/N_0)_2=3 \) dB. We take measurements for ten different traffic-load points (x-axis of Fig. 1) for W-EnMLM and W-EMLM. Each traffic-load point for the W-EnMLM corresponds to a number of sources of both services. The number of sources is the same for both service-classes. Each traffic-load point for both the W-EnMLM and the W-EMLM corresponds also to some values of the effective offered traffic-load of the two services as presented in Table II.

In this example the mean value and the coefficient of variation for the inter-cell interference are: \( E[I_{\text{inter}}]=2*18 \) mW and \( CV[I_{\text{inter}}]=1 \), respectively. In Fig.1, in order to validate the accuracy of the W-EnMLM, we present analytical and simulation CBP results of the W-EnMLM for both service-classes. The results show that the model’s accuracy is completely satisfactory. In Fig.1 we also present the CBP results of the corresponding W-EMLM (dashed lines), in order to compare it to the W-EnMLM. The W-EMLM results in much higher CBP compared to the W-EnMLM. This shows the necessity of the W-EnMLM in the call-level analysis of W-CDMA networks. The results of Fig. 1 are also presented in Table III, for better clarity.
We propose a new model, the Wireless Engset Multirate Loss Model (W-EnMLM), for the call-level analysis of W-CDMA systems with quasi-random call arrival processes. We provide a recurrent formula for the calculation of the call blocking probabilities. The accuracy of the proposed calculations is completely satisfactory as was verified by simulations. Besides, the W-EnMLM performs much better than the corresponding model of infinite number of traffic sources.

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<th>Table 1: Traffic parameters for the application example</th>
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<td><strong>Traffic-load point</strong></td>
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<th>Table 2: Offered traffic-load for the application example</th>
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<td><strong>Traffic-load point</strong></td>
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<th>Table 3: CBP (%) for the application example</th>
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<td><strong>Teletraffic Model</strong></td>
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