DIGITAL SIGNAL PROCESSING **8,** 114–125 (1998)
ARTICLE NO. SP970308

A Deterministic Direct Data Domain Approach to Signal Estimation Utilizing Nonuniform and Uniform 2-D Arrays

Tapan K. Sarkar, Srikanth Nagaraja and Micheal C. Wicks*

Department of Electrical and Computer Engineering, Syracuse University, Syracuse, New York; and *Airforce Research Laboratory, Sensors Directorate, Z6 Electronic Parkway, Rome, New York 13441

Sarkar, T. K., Nagaraja, S., and Wicks, M. C., A Deterministic Direct Data Domain Approach to Signal Estimation Utilizing Nonuniform and Uniform 2-D Arrays, *Digital Signal Processing* **8** (1998), 114–125.

A direct data domain deterministic approach utilizing a nonuniform array to adaptively estimate the signal strength of an incoming signal in the presence of strong jammers, clutter, and thermal noise is presented. This method is based on choosing a weighted difference of neighboring antenna outputs, based on the direction of arrival of the signal of interest, in such a way that only the unwanted components remain. These unwanted signals are then nulled by adaptive weights. These weights are then used to estimate the signal strength for a set of sampling time instants. This approach is suited for a highly changing environment, particularly in the presence of blinking jammers and where clutter characteristics change rapidly, provided the signal of interest does not change significantly during the observation interval. Two main deterministic approaches are presented—one based on the solution of a generalized eigenvalue problem, the other based on the solution of a set of linear equations utilizing the conjugate gradient method. These methods are applied to nonuniform as well as uniform 2D arrays, and the results are compared.

I. INTRODUCTION

Consider an array of antenna elements placed in a plane and not necessarily uniformly spaced. Further, narrowband signals consisting of a signal of interest, jammers, and multipath, with a center frequency f_s, are assumed to impinge on this array. The signal we are interested in is assumed to arrive from a known direction with respect to the axes. This paper deals with the estimation of signal amplitude in the presence of jammer, noise, and multipath (clutter) signals. The signal is to be extracted from the measured voltages at the antenna elements of a nonuniform array of antennas placed in various configurations.

The approach followed is that of the deterministic least squares as in [1,2]. This work then is a continuation of the ideas presented in [2]. Whereas in [2] uniform linear arrays were discussed, the work presented here is mainly concerned with nonuniform arrays in a plane.

Section II will deal with the models used for the signal, jammer, clutter, and noise. Also the configuration of the antenna elements shall be made clear. Section III deals with the actual approach followed in order to estimate the signal parameter of interest. Section IV explains the eigenvalue approach to signal parameter estimation. In Section V we present a few examples for which these approaches have been used and the results are discussed therein. Finally we conclude with Section VI.

II. SIGNAL MODEL

The signal of interest, SOI, has a phasor representation of the form

$$S = |S| e^{j\theta_s}. \tag{1}$$

Here, S is the complex amplitude of the SOI, $|S|$ is the magnitude of the same, and θ_s is the phase. It is assumed that the signal is constant during the observation interval, as a function of time of interest.

There are several jammers assumed to be impinging on the array, and each is of the form

$$\text{Jammer} = J[1 + m \sin(2\pi f_m t + \phi_m)] \times \sin(2\pi f_c t + \phi_c). \tag{2}$$

1051-2004/98 $25.00

The above form of the jammer signals is that of an amplitude modulated signal, with the carrier frequency equal to that of the SOI, a modulating frequency of f_m, and a modulating index m. Furthermore each of the jammer signals, of amplitude J, is assumed to arrive at the array from a direction different from that of the SOI, with t being the time variable.

In addition to the above, there are clutter signals coming in at various angles to the array. We have modeled the clutter contribution as a number of amplitude modulated waves, identical to the jammer model elucidated above, and whose amplitudes and phases are sampled from a uniform distribution. Thermal noise contribution exists at each antenna element and is modeled as a real quantity, sampled from a uniform distribution between 0 and 1, and a phase, again sampled from a uniform distribution between 0 and 2π.

Thus the net signal received at each antenna element at a particular instant of time is of the form

$$\text{NetSignal} = \text{SOI} + \text{Jammer} + \text{Clutter} + \text{ThermalNoise}. \quad (3)$$

Antenna Configuration

We are basically interested in four kinds of configurations. Each of these is an arrangement of antenna elements in a plane. The four different arrangements we have used are as follows:

- A nonuniform array placed in the x-y plane (Fig. 1).
- A circular array (Fig. 2).
- A sinusoidally spatially modulated array (Fig. 3).
- A hexagonal array (Fig. 4).

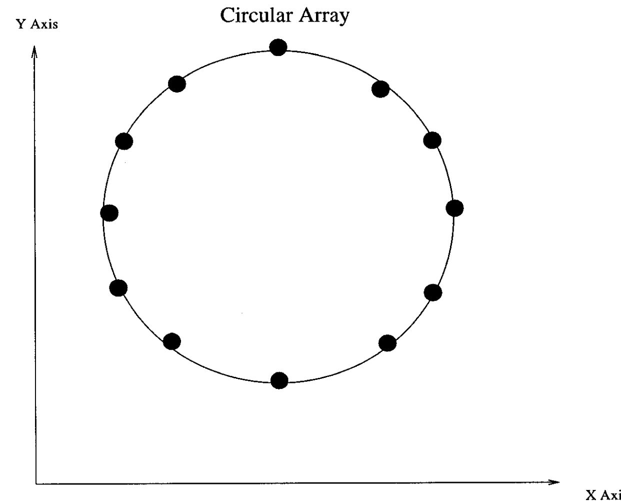

FIG. 2. Circular array.

In each of the above cases the distance between neighbouring antenna elements is chosen to be less than half a wavelength corresponding to the frequency of the SOI.

III. DETERMINISTIC DIRECT DATA DOMAIN APPROACH

1. Forward Method

The basic idea behind this approach has been explained in [1]. What follows is an extension of the same to our case of nonuniform as well as uniform 2-D arrays in a plane.

Consider N antenna elements placed in the x-y plane at locations (x_i, y_i), $i = 1, 2, \ldots, N$. The SOI is

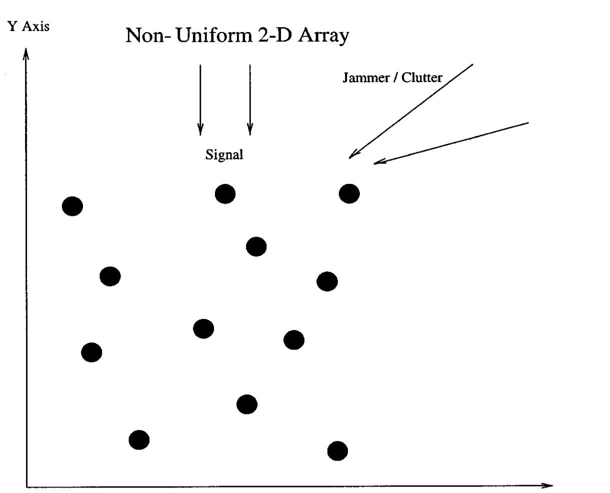

FIG. 1. Nonuniform array in the plane.

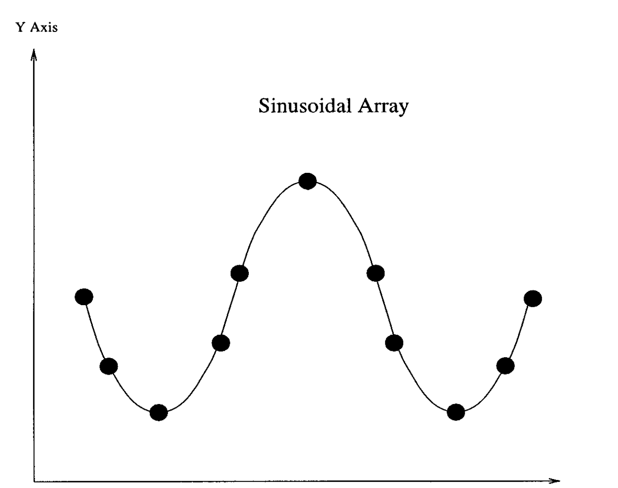

FIG. 3. Sinusoidal array.

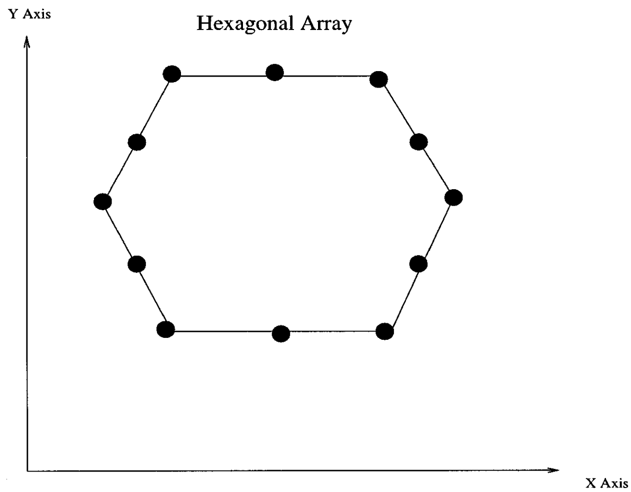

FIG. 4. Hexagonal array.

assumed to arrive at the array from a known direction ψ_s with respect to the x-axis. The signal strength is unknown and is the parameter to be estimated, in the presence of the jammer, clutter, and noise. The directions of arrival of the jammer and clutter are unknown but they are assumed to be different from that of the SOI. The amplitudes of the jammer and clutter are assumed unknown.

Let v_i^p denote the net output voltage of the ith antenna element at the pth sampling instant and let $|s|e^{j\theta_s}$ denote the incoming signal, where $|s|$ denotes the magnitude, and θ_s, the phase. n_i^p is the term accounting for the jammer, clutter and thermal noise contribution at the ith antenna element, at the pth instant of time. Thus we have

$$v_i^p = z_i|s|e^{j\theta_s} + n_i^p \tag{4}$$

$$z_i = e^{j\{2\pi[x_i \sin\psi_s + y_i \cos\psi_s]\}/\lambda}. \tag{5}$$

The z's in the above equation indicate the contribution due to the progression of the signal phase front across the array elements. λ is the wavelength corresponding to the frequency of the SOI. ψ_s is the angle of arrival of the SOI with respect to the x-axis. (x_i, y_i) indicate the coordinates of the ith antenna element in the plane.

Now, consider the following weighted difference between the voltages induced in two particular antenna elements, j and $(j+1)$, at the same sampling instant, p:

$$\frac{v_j^p}{z_j} - \frac{v_{j+1}^p}{z_{j+1}}. \tag{6}$$

The above weighted difference does not contain any contribution due to the SOI and is only a weighted difference of the unwanted components in the voltages at the two elements. In light of the above fact consider the following system of equations:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_1^1}{Z_1} - \frac{V_2^1}{Z_2} & \frac{V_1^2}{Z_1} - \frac{V_2^2}{Z_2} & \cdots & \frac{V_1^M}{Z_1} - \frac{V_2^M}{Z_2} \\ \frac{V_2^1}{Z_2} - \frac{V_3^1}{Z_3} & \frac{V_2^2}{Z_2} - \frac{V_3^2}{Z_3} & \cdots & \frac{V_2^M}{Z_2} - \frac{V_3^M}{Z_3} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_{M-1}^1}{Z_{M-1}} - \frac{V_1^M}{Z_M} & \frac{V_{M-1}^2}{Z_{M-1}} - \frac{V_M^2}{Z_M} & \cdots & \frac{V_{M-1}^M}{Z_{M-1}} - \frac{V_M^M}{Z_M} \end{bmatrix}_{M\times M} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}_{M\times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{M\times 1}. \tag{7}$$

In the above system of equations, the first row is a constraint equation on the set of weights, w_i. This fixes the gain of the array along the direction ψ_s. It is seen in the above that for a set of M weights, we need M time snapshots of at least M antenna voltages. This is so that we have at least M equations to solve for in M variables.

Once the weights are solved for by using the above set of equations, the signal strength can be estimated from

$$\tilde{s} = \frac{1}{N}\sum_{i=1}^{N}\sum_{p=1}^{P}\frac{v_i^p}{z_i}w_p, \tag{8}$$

where N is the total number of antenna elements, P is the total number of time samples utilized in forming the system matrix above, and w_p are the set of adaptive weights.

To solve the above system of equations efficiently, we use the *conjugate gradient* method, as noted in [1]. For the solution of $[A][W] = [Y]$, this method starts with an initial guess, $[W]_0$, and defines

$$R_0 = Y - AW_0 \tag{9}$$

$$P_0 = G_0 = A^*R_0. \tag{10}$$

A^* denotes the adjoint operator for A and is defined by

$$(Au, v) = (u, A^*v). \tag{11}$$

For $i = 0, 1, 2, \ldots,$ let

$$a_i = \frac{\|G_i\|^2}{\|AP_i\|^2} \tag{12}$$

$$W_{i+1} = W_i + a_i P_i \tag{13}$$

$$R_{i+1} = R_i + a_i A P_i \tag{14}$$

$$G_{i+1} = A^* R_{i+1} \tag{15}$$

$$P_{i+1} = G_{i+1} + b_i P_i \tag{16}$$

$$b_i = \frac{\|G_{i+1}\|^2}{\|G_i\|^2}. \tag{17}$$

$\|\cdot\|$ defines the norm of a vector as

$$\|v\| = [v]^H [v], \tag{18}$$

where the superscript H denotes the complex conjugate and transpose operations.

The above iterations are performed until a desired error criterion is met. In our case this criterion is defined as

$$\frac{\|[A][W]_i - [Y]\|}{\|Y\|} \leq 10^{-6}. \tag{19}$$

Further characteristics of the conjugate gradient method are discussed in [1] and [3].

2. Backward Method

The above deterministic direct data domain method can be applied to the antenna outputs in three different ways. One is the *forward* method, indicated in the previous section. Another is the *backward* method, where the sampled sequence is estimated by observing it in the reverse direction. For this, we have to conjugate the data and form the reverse sequence. We thus end up with the system of equations

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_M^{1*}}{z_M^*} - \frac{V_{M-1}^{1*}}{z_{M-1}^*} & \frac{V_M^{2*}}{z_M^*} - \frac{V_{M-1}^{2*}}{z_{M-1}^*} & \cdots & \frac{V_M^{M*}}{z_M^*} - \frac{V_{M-1}^{M*}}{z_{M-1}^*} \\ \frac{V_{M-1}^{1*}}{z_{M-1}^*} - \frac{V_{M-2}^{1*}}{z_{M-2}^*} & \frac{V_{M-1}^{2*}}{z_{M-1}^*} - \frac{V_{M-2}^{2*}}{z_{M-2}^*} & \cdots & \frac{V_{M-1}^{M*}}{z_{M-1}^*} - \frac{V_{M-2}^{M*}}{z_{M-2}^*} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_2^{1*}}{z_2^*} - \frac{V_1^{1*}}{z_1^*} & \frac{V_2^{2*}}{z_2^*} - \frac{V_1^{2*}}{z_1^*} & \cdots & \frac{V_2^{M*}}{z_2^*} - \frac{V_1^{M*}}{z_1^*} \end{bmatrix}_{M\times M} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}_{M\times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{M\times 1}, \tag{20}$$

where the superscript $*$ denotes the complex conjugate of the signal.

The estimate for the signal strength remains the same as before. Also the requirements in terms of number of time snapshots and number of weights remains the same as for the forward case.

3. Forward–Backward Method

In this method, the number of equations to work with is doubled by considering the data in both the forward and the reverse directions. This method works because we are dealing with a signal which is assumed to be of the form of an exponential function with a purely imaginary argument, which has some noise terms associated with it. In the absence of this noise term this method would not yield an extra set of independent equations.

Thus, as the number of independent equations increases, the number of adaptive weights also increases, in comparison to the forward (and the backward) method. The system of equations that needs to be solved is

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_1^1}{Z_1} - \frac{V_2^1}{Z_2} & \frac{V_1^2}{Z_1} - \frac{V_2^2}{Z_2} & \cdots & \frac{V_1^M}{Z_1} - \frac{V_2^M}{Z_2} \\ \frac{V_2^1}{Z_2} - \frac{V_3^1}{Z_3} & \frac{V_2^2}{Z_2} - \frac{V_3^2}{Z_3} & \cdots & \frac{V_2^M}{Z_2} - \frac{V_3^M}{Z_3} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_{k-1}^1}{Z_{k-1}} - \frac{V_k^1}{Z_k} & \frac{V_{k-1}^2}{Z_{k-1}} - \frac{V_k^2}{Z_k} & \cdots & \frac{V_{k-1}^M}{Z_{k-1}} - \frac{V_k^M}{Z_k} \\ \frac{V_l^{2*}}{z_l^*} - \frac{V_{l-1}^{1*}}{z_{l-1}^*} & \frac{V_l^{2*}}{z_l^*} - \frac{V_{l-1}^{2^*}}{z_{l-1}^*} & \cdots & \frac{V_l^{M*}}{z_l^*} - \frac{V_{l-1}^{M*}}{z_{l-1}^*} \\ \frac{V_{l-1}^{1*}}{z_{l-1}^*} - \frac{V_{l-2}^{1*}}{z_{l-2}^*} & \frac{V_{l-1}^{2*}}{z_{l-1}^*} - \frac{V_{l-2}^{2*}}{z_{l_2}^*} & \cdots & \frac{V_{l-1}^{M*}}{z_{l-1}^*} - \frac{V_{l-2}^{M*}}{z_{l-2}^*} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_2^{1*}}{z_2^*} - \frac{V_1^{1*}}{z_1^*} & \frac{V_2^{2*}}{z_2^*} - \frac{V_1^{2*}}{z_1^*} & \cdots & \frac{V_2^{M*}}{z_2^*} - \frac{V_1^{M*}}{z_1^*} \end{bmatrix}_{M\times M} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix}_{M\times 1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{M\times 1}, \tag{21}$$

where $k = [M - 1]/2$, $[X]$ indicates the largest integer less than or equal to X, and $k + l = M - 1$.

Upon solving for the weights in the above system, the estimate for the signal strength is computed as before. Also note that in the forward–backward (FB) method, for the same number of degrees of freedom, the number of data samples can be reduced by half. Alternatively, if the number of data samples remains the same, a set of least squares equations can be formed to improve the accuracy of the solution.

IV. METHOD BASED ON THE SOLUTION OF AN EIGENVALUE EQUATION

We consider the same problem, of estimating the signal strength of a signal coming in at a known angle of arrival to the array, in the presence of noise, clutter, and jamming signals. Using the same notation as earlier for these various components, we apply the matrix pencil

$$[V] - \alpha[S], \tag{22}$$

where

$$[V] = \begin{bmatrix} \frac{V_1^p}{z_1} & \frac{V_2^p}{z_2} & \cdots & \frac{V_M^p}{z_M} \\ \frac{V_2^p}{z_2} & \frac{V_3^p}{z_3} & \cdots & \frac{V_{M+1}^p}{z_{M+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_M^p}{z_M} & \frac{V_{M+1}^p}{z_{M+1}} & \cdots & \frac{V_N^p}{z_N} \end{bmatrix}_{M\times M} \tag{23}$$

and

$$[S] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{M\times M} . \tag{24}$$

The difference at each element, $V_i - \alpha S_i$, represents the contribution due to the unwanted components of the incoming signal, viz. jammer, clutter, and thermal noise. Every term of the form V_i/z_i reduces to

$$\frac{V_i}{z_i} = 1 + \frac{(J + C + N)_i^p}{z_i}, \tag{25}$$

where the z_i's are as defined before. $(J + C + N)_i^p$ represents the jammer, clutter, and thermal noise contribution at the ith antenna element at the pth instant of time.

Now, for our adaptive process of estimation, the weights $[W]$ are chosen in such a way that the contribution due to the unwanted components is nulled out [2]. Thus, if the following is defined as a generalized eigenvalue problem,

$$[R][W] = ([V] - \alpha[S]) \cdot [W] = 0, \tag{26}$$

then α is the estimate for the strength of the signal, and is given by the generalized eigenvalue of the above system. The adaptive weights, $[W]$, are given by the generalized eigenvector of the system. Since we have to deal with only one SOI, the matrix $[S]$ is of rank unity and therefore the eigenvalue problem has only one eigenvalue, which is the estimate we are looking for at the pth instant of time.

It can be observed in the above system that the number of weights, M, is related to the number of antenna elements, N, as

$$N = 2M - 1. \tag{27}$$

V. APPLICATIONS TO SOME SELECTED EXAMPLES

Example 1: Jammer Only Case

First, we consider a case where we have five jammer signals arriving at the array along with the SOI. Thermal noise and clutter contributions are neglected in this first example. The power levels of the jammer signals, with respect to the power of the SOI, are 50.5, 60.1, 56.5, 68.8, and 59.5 dB above the level of SOI, respectively. Each of these arrives at the array at the following angles of incidence, with respect to the x-axis: 93°, 49°, 98°, 160°, 100° each. The SOI is incident at an angle of 90° at a frequency of 300 MHz. The Jammer signals are assumed to be amplitude modulated waveforms as indicated in Eq. (2), with the following parameters:

$m_1 = 0.5$, $f_{m1} = 1000$ Hz, $\phi_{m1} = 30°$,
$f_{c1} = 300$ MHz, $\phi_{c1} = 40°$
$m_2 = 0.1$, $f_{m2} = 2000$ Hz, $\phi_{m2} = 20°$,
$f_{c2} = 300$ MHz, $\phi_{c2} = 99°$
$m_3 = 0.8$, $f_{m3} = 1500$ Hz, $\phi_{m3} = 120°$,
$f_{c3} = 300$ MHz, $\phi_{c3} = 160°$
$m_4 = 0.35$, $f_{m4} = 900$ Hz, $\phi_{m4} = 90°$,
$f_{c4} = 300$ MHz, $\phi_{c4} = 90°$
$m_5 = 0.6$, $f_{m5} = 800$ Hz, $\phi_{m5} = 60°$,
$f_{c5} = 300$ MHz, $\phi_{c5} = 46°$.

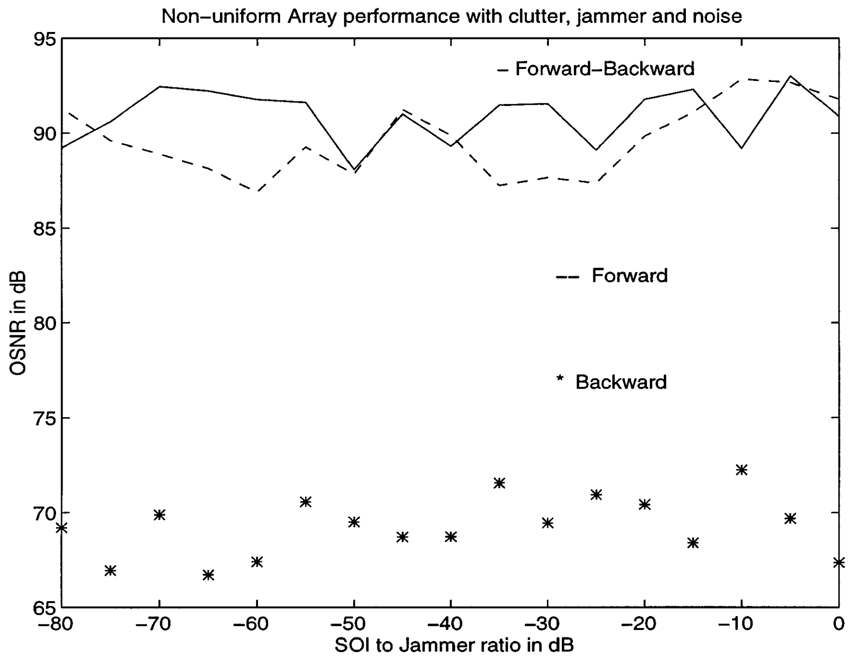

FIG. 5. Nonuniform array.

For this input, the signal strength was estimated correct to four decimal places. The sampling frequency was chosen to be 10 times the frequency of the signal. All three methods, forward, backward, and forward–backward, were used and similar results were achieved. Also the methods were applied to all four antenna configurations and the estimate was obtained correct to four decimal places in each of these cases. In all the examples in this paper, the number of antenna elements in the nonuniform, circular, hexagonal, and sinusoidal configurations has been taken to be 12. The number of time samples has been taken to be one less than the number of antenna elements, for the three deterministic meth-

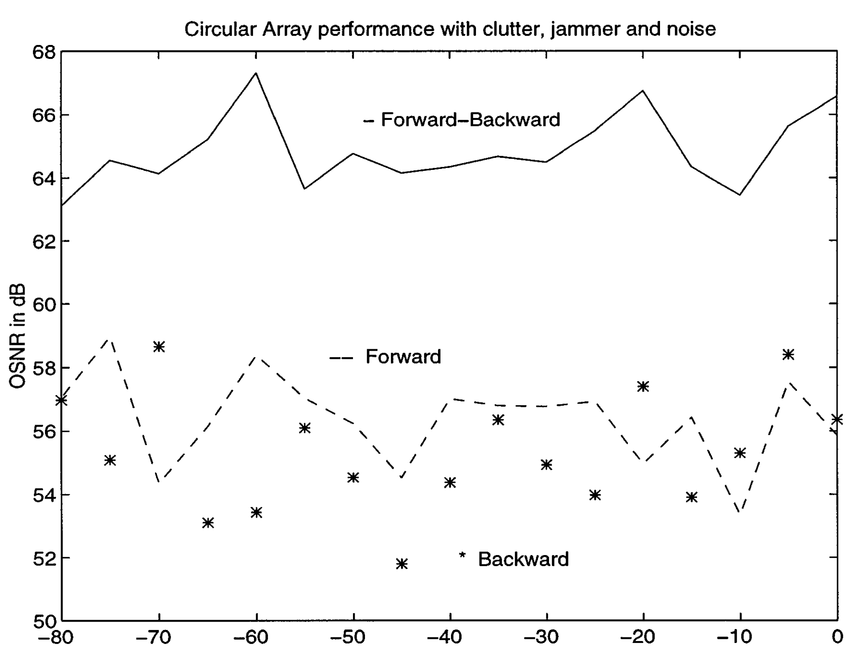

FIG. 6. Circular array.

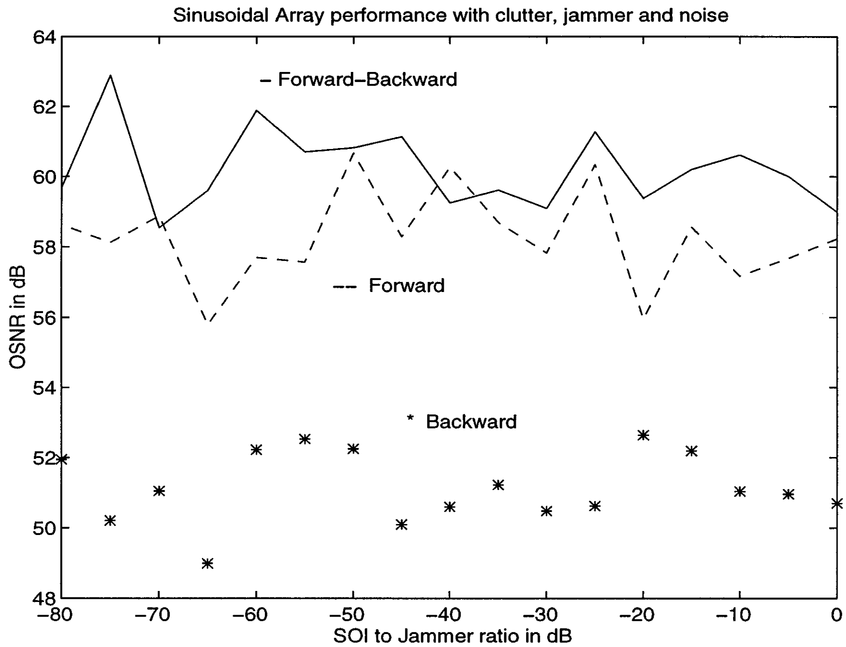

FIG. 7. Sinusoidal array.

ods based on a direct solution of the linear system of equations.

Example 2: Clutter and Thermal Noise Added

Now we consider the contribution of clutter and thermal noise in addition to the jamming signals and analyze the performance of the estimation method.

Thermal noise is added to each antenna output and is taken to be of the form of a complex signal whose magnitude is a uniformly distributed random variable between 0 and 1, and whose phase is a uniformly distributed random variable between 0 and 2π. The signal-to-total-thermal-noise ratio is fixed at 20 dB. The clutter is modeled as explained

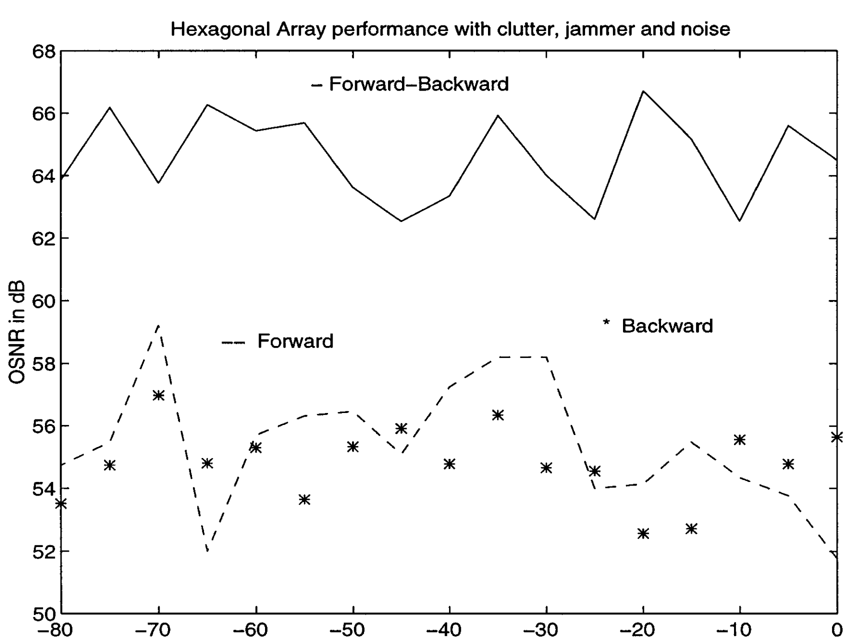

FIG. 8. Hexagonal array.

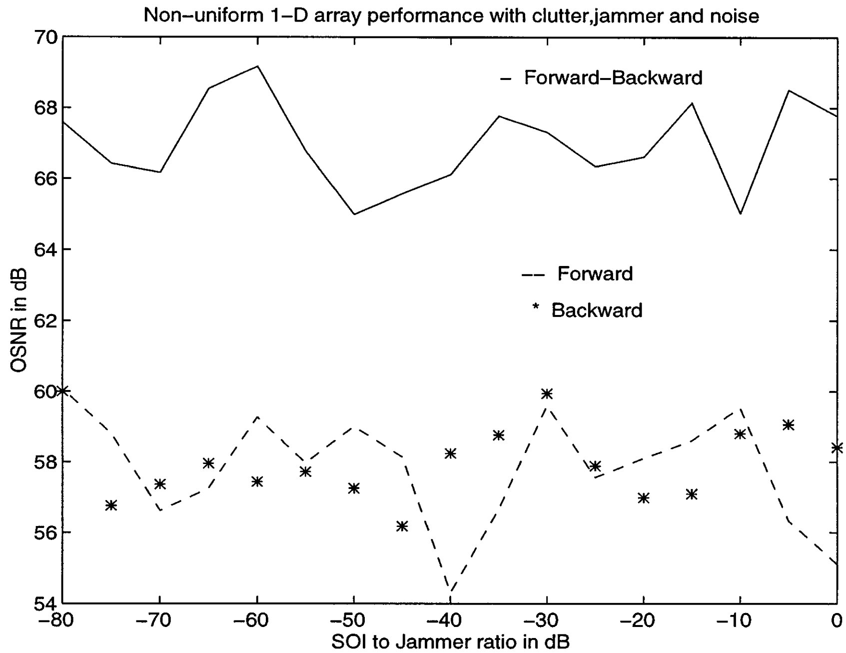

FIG. 9. Nonuniform 1-D array.

earlier, and the clutter-to-signal (SOI) ratio is fixed at 20 dB; i.e., the clutter is 20 dB above the level of the SOI.

The SOI is taken to arrive at the array at an angle of 90° to the *x*-axis. The clutter signals are assumed to arrive at the array at intervals of 0.1°, at all angles between 120° and 130°. The jamming signal is taken to be a single source, arriving at an angle of 49° with respect to the *x*-axis. The power level of the jammer is varied from SOI level, to 80 dB above SOI level, at intervals of 0.5 dB. For a particular jammer power level, 25 iterations were carried out for each method, and the average output signal-to-noise ratio was computed. The number of adaptive weights has been taken to be one less than the total number of

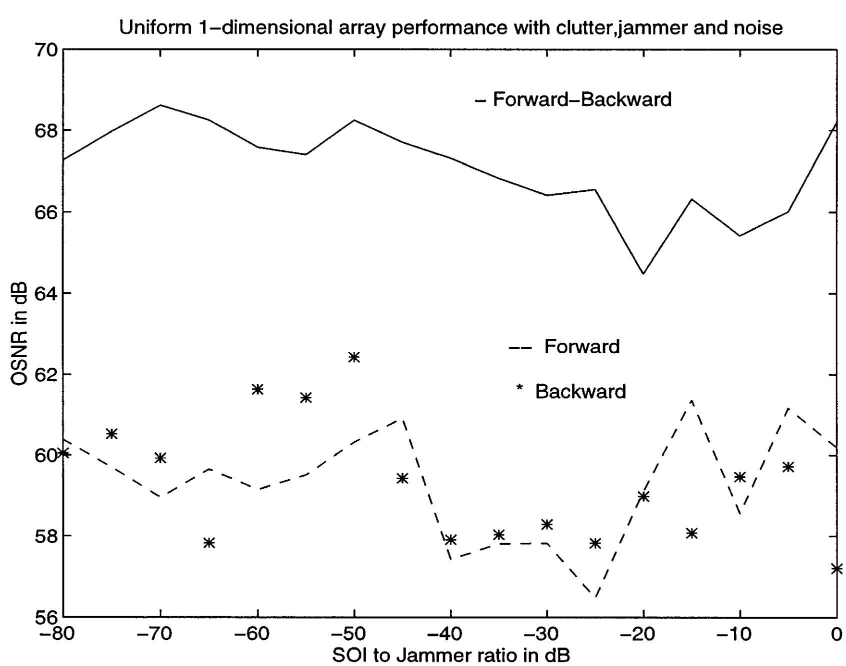

FIG. 10. Uniform 1-D array.

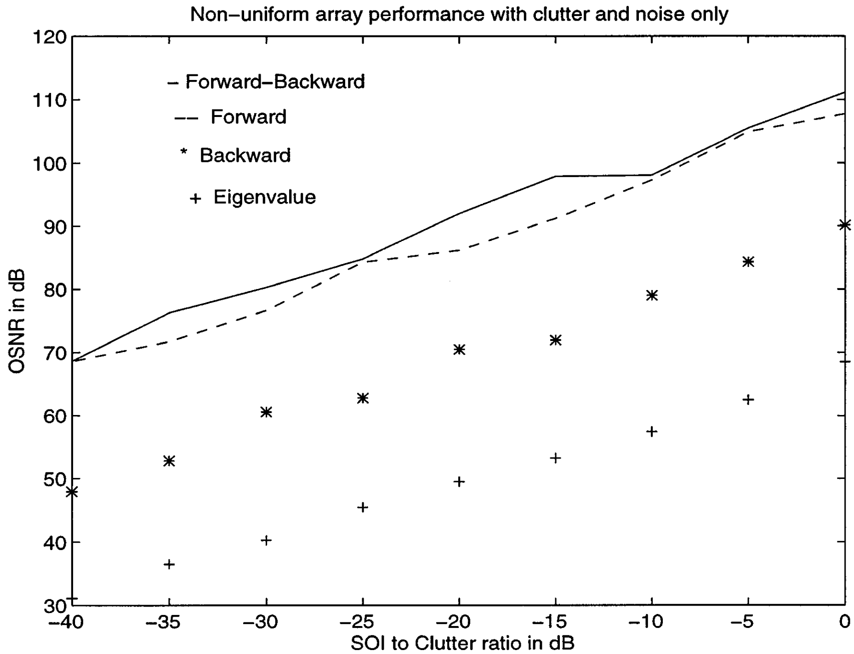

FIG. 11. Nonuniform array.

antenna elements, and the number of time samples is equal to the number of adaptive weights.

The output signal-to-noise ratio is an indicator of the accuracy of our estimate. It is defined as

$$OSNR = 20 \log_{10} \frac{|S|}{|S - \tilde{S}|}, \tag{28}$$

where S is the actual signal strength, and $\tilde{S}$ indicates the estimate. $|\cdot|$ indicates absolute value.

The results are shown in Figs. 5–8 for all four different configurations, for the methods indicated. In each of these figures, the results obtained from the three direct methods have been plotted. The x-axis corresponds to the SOI-to-jammer power level,

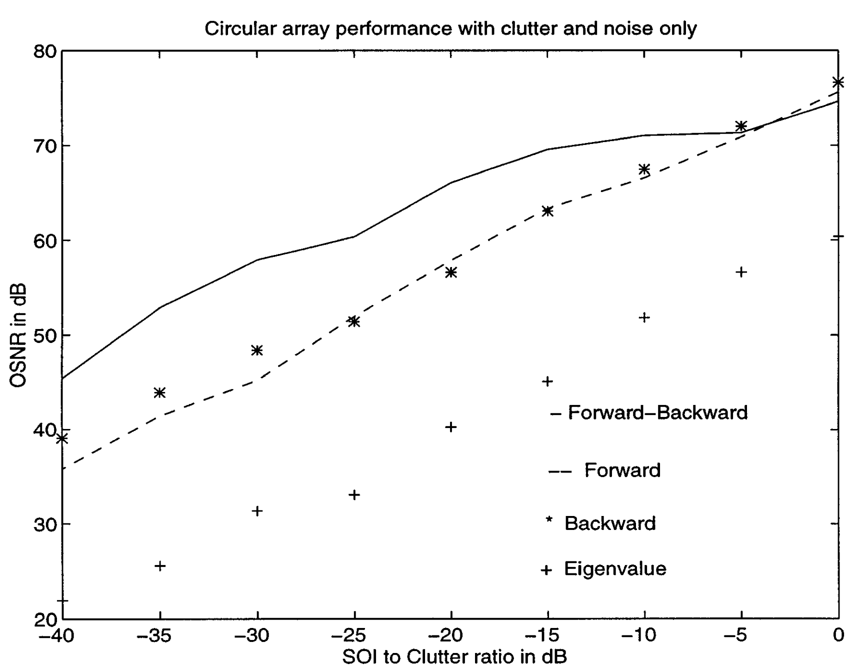

FIG. 12. Circular array.

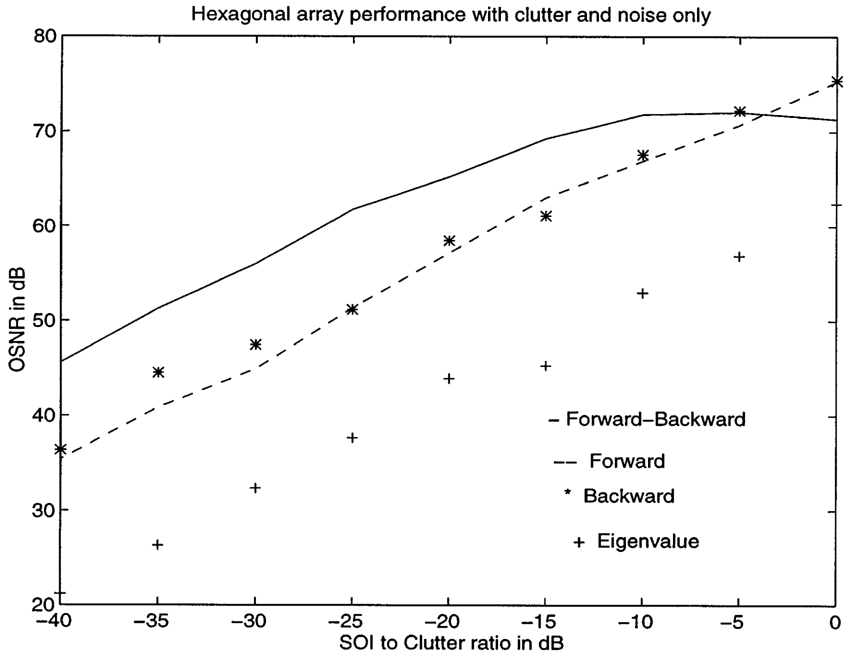

FIG. 13. Hexagonal array.

while the y-axis corresponds to the output signal-to-noise ratio, as defined above.

The eigenvalue method did not yield meaningful OSNR for this case, probably due to numerical instabilities in the MATLAB generalized eigenvalue routine that we used. Use of better algorithms, like the QZ algorithm, could alleviate this problem.

It can be observed in these figures that the forward–backward method shows much better performance than either the forward or the backward method. For obtaining the results plotted in these figures, we have used an overdetermined system of equations for the FB method and consequently used an SVD method to solve the resulting least squares problem.

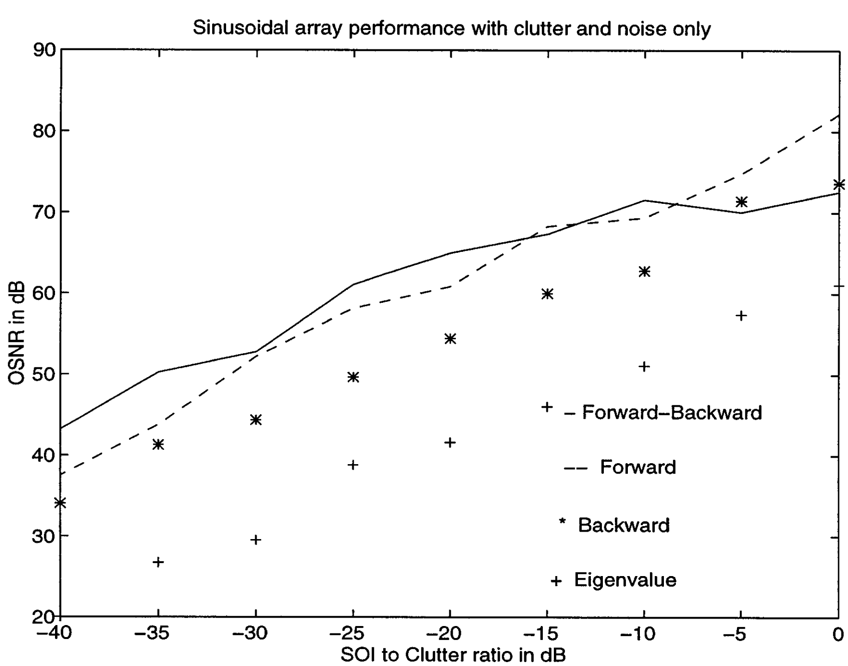

FIG. 14. Sinusoidal array.

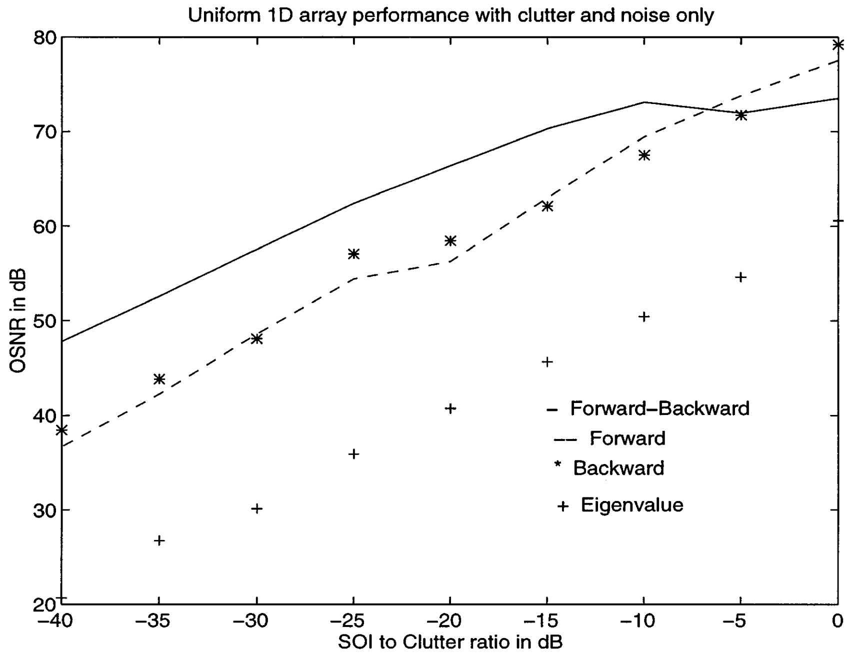

FIG. 15. Uniform 1-D array.

Another point to be noted from these figures is the consistently high level of performance that all three methods have achieved for the arrays under cond-sideration. In particular, these methods yield much higher levels of OSNR for the nonuniform array case.

Finally, for the sake of comparison we have shown a set of results for a one-dimensional uniform as well as nonuniform, array. Each of these arrays is of the same total length, and each contains 12 antenna elements. These results (Figs. 9 and 10) indicate that the performance of the deterministic methods is almost identical for both the uniform and the nonuniform 1-D case.

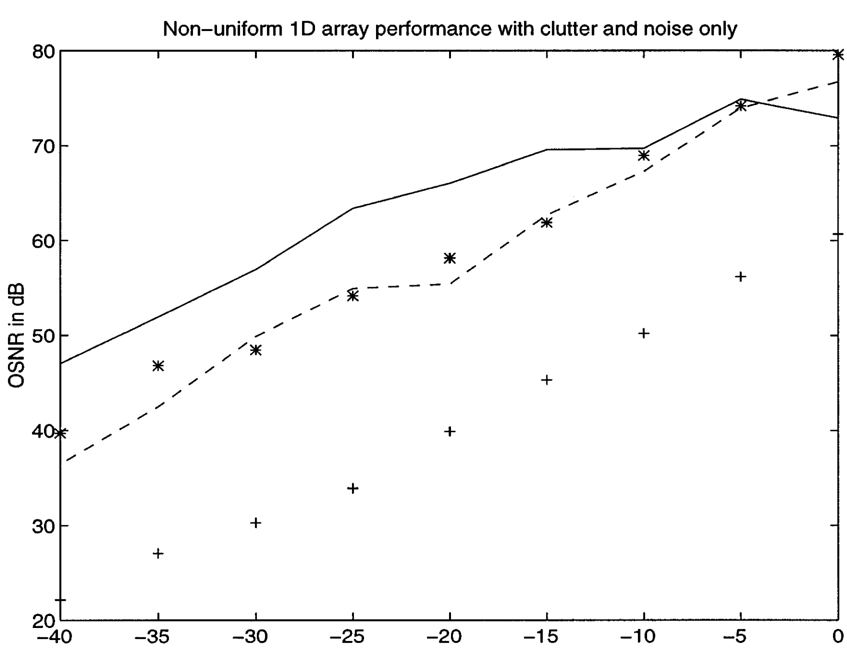

FIG. 16. Nonuniform 1-D array.

Example 3: Clutter and Thermal Noise Only (Jammer Absent)

In this example we consider the performance of the methods in the presence of the clutter and noise, but with the jammer being absent. In Figs. 11–14 we plot the OSNR versus the SOI-to-clutter power level, for each of the four antenna configurations. The SOI-to-noise power level, at a given SOI-to-clutter level is 40 dB above the given SOI-to-clutter power level.

It can be observed from the figures that the eigenvalue method does the worst among the deterministic methods. The forward and the backward method do almost the same, while the FB method yields the best results. Once again, the FB method used here utilizes an SVD solution for an overdetermined system of equations.

Again, for the sake of comparison we have shown a set of results for a one-dimensional, uniform as well as nonuniform, array. Each of these arrays is of the same total length, and each contains 12 antenna elements. These results (Figs. 15 and 16) again indicate the comparable performance in the two cases, as in the last example.

VI. CONCLUSIONS

The results obtained above indicate that the direct data domain method is an effective method for solving the signal strength estimation problem in an adaptive antenna framework. This approach can deal effectively with nonstationary jammer environments. Further, the computational complexity is reduced by using techniques such as the conjugate gradient method for solving the resulting system of equations.

Among the two broad deterministic methods discussed—eigenvalue and direct solution of a matrix equation—the latter is more robust to high jammer powers. Selected examples have been discussed to indicate the performance of these methods.

ACKNOWLEDGMENT

This work has been supported by DARPA and was monitored by Rome Laboratory under Grant F30602-95-1-0014.

REFERENCES

1. Sarkar, T. K., and Sangruji, N. An Adaptive nulling system for a narrow band signal with a look direction constraint utilizing the conjugate gradient method, *IEEE Trans. Antennas Propagation* **37** (1989).
2. Sarkar, T. K., Park, S., Koh, J., and Schneible, R. A. A deterministic least squares approach to adaptive antennas, *Digital Signal Process.* **6** (1996), 185–194.
3. Sarkar, T. K., Arvas, E., and Rao, S. M. Application of FFT and the conjugate gradient method for the solution of electromagnetic radiation from electrically large and small conducting bodies, *IEEE Trans. Antennas Propagation* **34** (1986), 635–640.