New Paths in the String Theory Landscape

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Based on
D. Chialva, U. Danielsson, N. Johansson, M.L. and M. Vonk, hep-th/0710.0620
U. Danielsson, N. Johansson and M.L., hep-th/0612222

2008-01-18

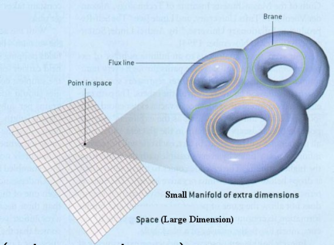
Motivation

String theory lives in 10D, we live in 4D.

Compactify.

Fluxes.

Branes.



(universe-review.ca)

⇒ string theory landscape of vacua

Motivation

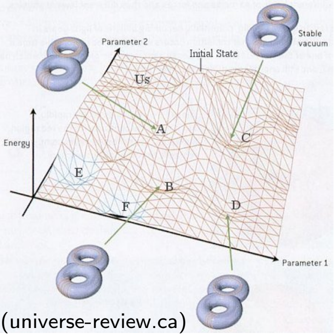
Natural questions:

How many vacua?

Distribution?

Continuously connected?

Barriers?



(universe-review.ca)

Effects from topography

Tunneling, domain walls, inflation, finiteness...

Flux compactifications 1

Flux vacua

Ingredients: manifolds, fluxes, branes... enormous landscape!

A landscape model

Type IIB SUGRA on (conformal) CY 3–fold.
3–fluxes through 3–cycles: fix CS moduli.
Generic. Rich structure.

Calabi–Yau manifolds

Complex, Kähler, Ricci flat.
$h^{2,1}$ Complex structure (CS) moduli.
$h^{1,1}$ Kähler moduli.

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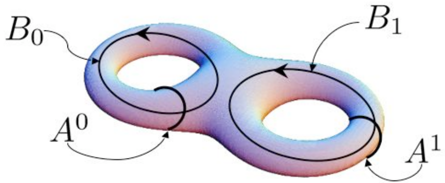
Fixing the complex structure

Complex structure $\sim$ holomorphic 3-form $\Omega(z)$.



3–cycles

3-cycles basis A_I, B_J.

$\int_{A_I} \alpha_J = \int_{B_I} \beta_J = \int \alpha_I \wedge \beta_J = \delta_{IJ}$

- Periods $\Pi_i(z) = \int_{C_i} \Omega(z)$
- z: CS moduli
- $\Pi(z) = (\Pi_1(z), \Pi_2(z), ...\Pi_N(z))$

3–flux

- IIB: RR F and NS $H \Rightarrow G = F - \tau H$
- Quantized: $\int_{C_i} F \sim F_i, \int_{C_i} H \sim H_i, F_i, H_i \in \mathbb{Z}$
- D3 tadpole condition: $\int_{CY} F \wedge H = \mathcal{N}_{D3}$

The potential for CS moduli

- **Fluxes** wrapping non-trivial cycles $\rightarrow$ **potential** V.
- $V = e^K \left(||DW||^2 - 3|W|^2 \right)$
  - Kähler potential $e^K = \frac{1}{Im(\rho)^3 Im\tau} \Pi^\dagger \cdot Q \cdot \Pi$
  - Superpotential $W = G \cdot \Pi(z)$
- CS moduli and τ fixed at minima of potential.
- No-scale: Kähler moduli unfixed perturbatively.

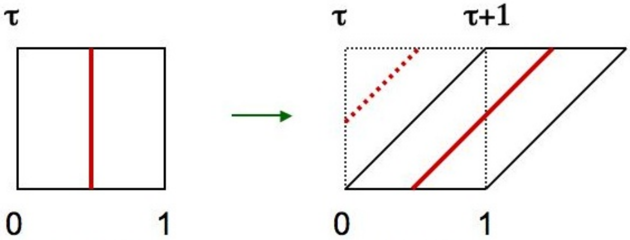
Paths between vacua

Paths between flux vacua (hep-th/0612222)

CS moduli space is complicated:

- singularities, branch cuts, non–trivial loops
- monodromies of 3-cycles.

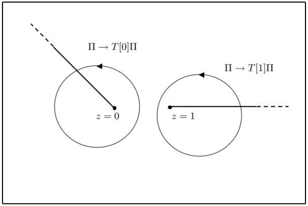
Idea: Use **monodromies** to continuously connect vacua.

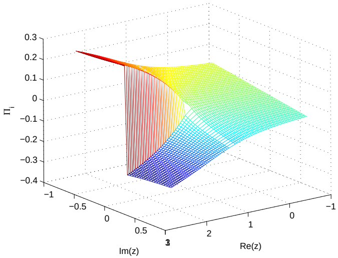


Paths between vacua

Monodromies

- Period monodromies $\Pi(z) \to T \cdot \Pi(z)$
- $T \in \mathcal{M} \subset Sp(N, \mathbb{Z})$
- E.g. **Mirror Quintic** $h^{2,1} = 1$ CS modulus $h^{1,1} = 101$ Kähler moduli





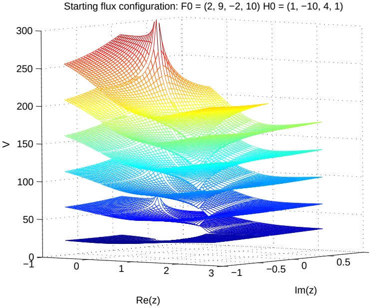
Paths between vacua

Recall: $V = e^K \left(||DW||^2 - 3|W|^2 \right)$, $W = G \cdot \Pi(z)$
Thus $\Pi(z) \to T \cdot \Pi(z)$
$\to V$ has branch cuts in CS moduli space.

Traverse cuts $\to$ paths between minima.

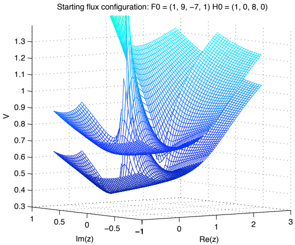
$\Pi \to T \cdot \Pi$ or $G \to G \cdot T$
$T \in \mathcal{M} \subset Sp(N, \mathbb{Z}) \to$

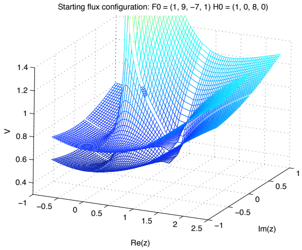
$\int F \wedge H$ unchanged.



Series of minima

Several **continuously connected minima** found:



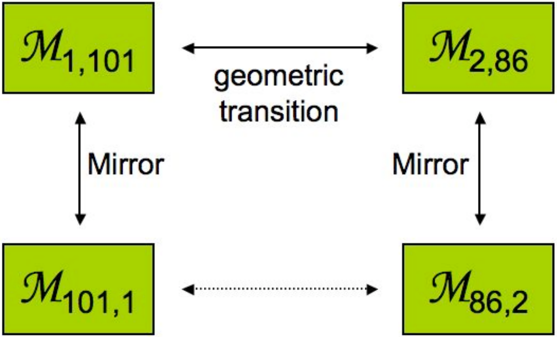


No **infinite** series of minima found.
What about flux minima **not** related by monodromies ("islands")?

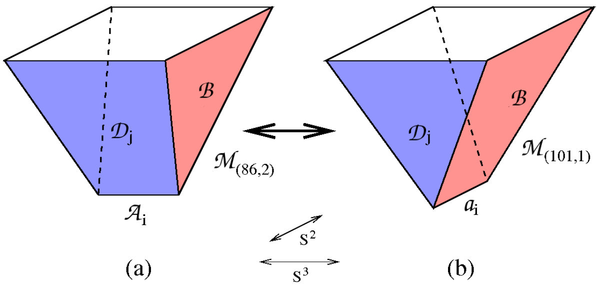
Extend moduli space

An extended landscape model 0710.0620

Monodromies: important for topography.
Larger moduli space $\Rightarrow$ more monodromies.
Geometric transitions:
Moduli spaces of different Calabi–Yau 3-folds are connected
Idea: extend $\mathcal{M}_{101,1}$ CS moduli space. Connect it to what?



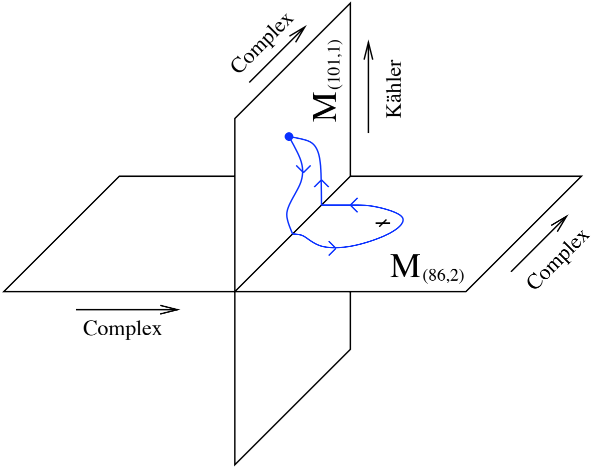
Geometric transitions



Mirror symmetry $\rightarrow \mathcal{M}_{(86,2)} \overset{GT}{\longleftrightarrow} \mathcal{M}_{(101,1)}$

$\mathcal{M}_{(86,2)}$	$\mathcal{M}_{(101,1)}$
shrink 16 3-cycles $\mathcal{A}_i$	blow up 16 2-cycles a_i
$\mathcal{A}_1 - \mathcal{A}_2 = \delta\mathcal{D}_1$	
...	$\sum a_i = \delta\mathcal{B}$
$\mathcal{A}_{15} - \mathcal{A}_{16} = \delta\mathcal{D}_{15}$	$\delta\mathcal{D}_i = 0$

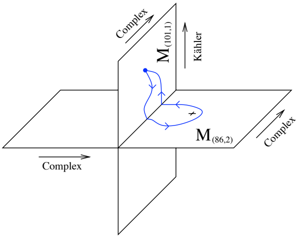
Extend moduli space



Extend moduli space

We need to:

- Construct $\mathcal{M}_{(86,2)}$ (using toric geometry)
- Compute **periods** of $\mathcal{M}_{(86,2)}$
- **Embed** $M_{(101,1)}$ in $M_{(86,2)}$
- Compute new **monodromies**



Geometric transitions with flux?
Infinite series of string theory vacua?

Toric geometry

Toric geometry

Construct CY: zero locus of polynomial equations on toric variety.
Toric variety: $\frac{(C^*)^n - Z}{G}$
Fans and polytopes $\leftrightarrow$ toric variety and equations.

Batyrev's mirror construction

$\mathcal{M}_{(2,86)}$: CI in $\mathbb{P}^1 \times \mathbb{P}^4 \Rightarrow$ Polytope for $\mathcal{M}_{(2,86)}$: $\nabla = \nabla_1 + \nabla_2$
Mirror construction:
Polytope for $\mathcal{M}_{(86,2)}$: $\Delta = \Delta_1 + \Delta_2$, $\langle \nabla_k, \Delta_j \rangle \geq -\delta_{k,j}$

$\mathcal{M}_{(86,2)}$

$f_1 \equiv 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5 / t_2 t_3 t_4 t_5$
$f_2 \equiv 1 - a_6 / t_1 - a_7 t_2$,
CS moduli $\sim a_i$:
$\phi_1 = a_1 a_6$, $\phi_2 = a_2 a_3 a_4 a_5 a_7$.

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$\mathcal{M}_{(86,2)}$ periods: 1

The fundamental period

$$\omega_0 = \frac{1}{(2\pi i)^5} \int_\gamma \frac{1}{f_1 f_2} \frac{dt_1}{t_1} \wedge \ldots \wedge \frac{dt_5}{t_5}$$

Near $\phi_1 = \phi_2 = 0$

$\omega_0(\phi) = \sum_{n_1,n_2} c(n_1,n_2)\phi_1^{n_1}\phi_2^{n_2}$, where $c(n_1,n_2) = \frac{(n_1+4n_2)!(n_1+n_2)!}{(n_1!)^2(n_2!)^5}$

Picard–Fuchs equations

Recall $\Pi_i = \oint_{C_i} \Omega$

$H^3 = H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$ is finite

$\Rightarrow L_k\Omega = d\eta$

$\Rightarrow L_k\Pi_i = \oint_{C_i} L_k\Omega = \oint_{C_i} d\eta = 0$

We get: 2 DE of degree 2 and 3

$\longrightarrow$ 6 linearly indep. solutions $\sim$ 6 periods.

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$\mathcal{M}_{(86,2)}$ periods: 2

The Frobenius method

$\phi_1 = \phi_2 = 0$ regular singular point
$\rightarrow$ five solutions with logarithmic singularities:
Using $\omega(\rho,\phi) = \sum_{n_1,n_2} c(n_1+\rho_1, n_2+\rho_2)\phi_1^{n_1+\rho_1}\phi_2^{n_2+\rho_2}$
we get all periods as: (hep-th/9406055)

$$\omega_1 = \partial_{\rho_1}\omega|_{\rho=0},$$
$$\omega_2 = \partial_{\rho_2}\omega|_{\rho=0},$$
$$\omega_3 = \kappa_{1jk}\partial_{\rho_j}\partial_{\rho_k}\omega|_{\rho=0},$$
$$\omega_4 = \kappa_{2jk}\partial_{\rho_j}\partial_{\rho_k}\omega|_{\rho=0},$$
$$\omega_5 = \kappa_{ijk}\partial_{\rho_i}\partial_{\rho_j}\partial_{\rho_k}\omega|_{\rho=0}$$

$\kappa_{ijk} = \int J_i \wedge J_j \wedge J_k$ classical intersection numbers.

Want integral and symplectic monodromies: change basis.
(No details here).

Embed $M_{(101,1)}$ in $M_{(86,2)}$

$\mathcal{M}_{(86,2)}$

Recall: $f_1 \equiv 1 - a_1 t_1 - a_2 t_3 - a_3 t_4 - a_4 t_5 - a_5/t_2 t_3 t_4 t_5 = 0$
$f_2 \equiv 1 - a_6/t_1 - a_7 t_2 = 0.$

$\mathcal{M}_{(101,1)}$

Substitute f_2 into f_1

$$b_0 + b_1 u_1 + b_2 u_2 + b_3 u_3 + b_4 u_4 + \frac{b_5}{u_1 u_2 u_3 u_4} + \frac{b_6}{u_1 u_2 u_3} = 0$$

Redefined CS moduli

$z_1 = \frac{b_1 b_2 b_3 b_6}{b_0^4} = \frac{\phi_2}{(1-\phi_1)^4}$ and $z_2 = -\frac{b_1 b_2 b_3 b_4 b_5}{b_0^5} = \frac{\phi_1 \phi_2}{(1-\phi_1)^5}$

Take $z_1 \to 0$: Mirror quintic equation!

Embed $M_{(101,1)}$ in $M_{(86,2)}$

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Take $z_1 \rightarrow 0$: Mirror quintic equation!

The mirror quintic locus

The MQ locus $z_1 \longrightarrow 0$:
Which period vanish?
Monodromy around the locus?

Analytically continue $\omega_0 \Rightarrow$

$$\omega_0 = \sum_{m_1,m_2=0}^{\infty} \frac{(4m_1+5m_2)!}{((m_1+m_2)!)^3 m_1!(m_2!)^2} z_1^{m_1} z_2^{m_2}.$$

$z_1 \longrightarrow 0$: MQ fundamental period.
Other periods:
Analytically continue ω_i: focus on derivatives.

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Embedded periods and monodromies

Periods

Integral and symplectic basis:

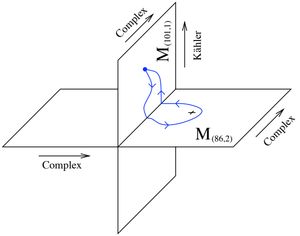
$$\Pi_{(86,2)} = \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \\ \Pi_5 \\ \Pi_6 \end{bmatrix} \xrightarrow{z_1 \to 0} \begin{bmatrix} \Pi_1^{MQ} \\ \Pi_2^{MQ} \\ \Pi_3^{MQ} \\ \Pi_4^{MQ} \\ 0 \\ \sum c_i \Pi_i^{MQ} \end{bmatrix}$$

New paths between vacua

4 new monodromies.

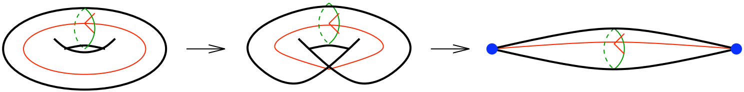
Geometric transitions.

$\longrightarrow$ New series of MQ vacua



Geometric transitions with fluxes

A: shrinking 3–cycle, B: torn 3–cycle.
Need to be careful:
$M_{(86,2)}$ monodromy might yield flux through A or B!



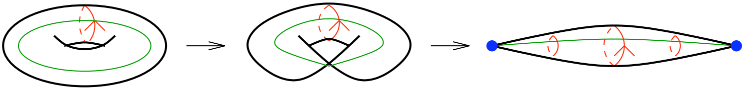
Flux through A hep-th/9811131, 0008142...

RR/NS–flux through **shrinking** 3–cycle A:
$\rightarrow$ D5/NS5–branes on new 2–cycles.
Positions of 5–branes $\sim$ new **open string moduli**.
$\hookrightarrow$ New period: $\Pi_B(t,z) = \int_B \Omega \rightarrow V_{MQ}(z) \rightarrow \tilde{V}_{MQ}(t,z)$.

Geometric transitions with fluxes

Flux through B 0709.4277, hep-th/0510042

RR/NS Flux through **torn** 3-cycle B → D1/F1-instantons?
No new terms in the MQ potential.



Geometric transitions with fluxes

Flux through both 3–cycles A and B

New **open string moduli**
Tadpole condition:
$\int F \wedge H$ might change $\rightarrow$ D3–branes.

Examples

N RR-flux through A, M NS-flux through B:
either N D5–branes and M F1–instantons
or compact **Klebanov–Strassler**: N D5–branes, MN D3–branes.

N RR-flux through A, M RR-flux through B:
N D5–branes, maybe M D1–instantons, no D3–branes.

Geometric transitions with fluxes

Flux potential at transition

Near transition point: $V_{(86,2)}(z_1, z_2) = V_1(z_1, z_2) + V_2(z_2)$;

$V_2(z_2) \xrightarrow{z_1 \to 0} V_{MQ}(z)$

With flux through A: $V_1(z_1, z_2) \xrightarrow{z_1 \to 0} \infty$

Without flux through A: $V_1(z_1, z_2) \xrightarrow{z_1 \to 0} 0$

No flux through shrinking cycle → geometric transition controlled.
Look for connected minima without such flux.

Infinite series of minima

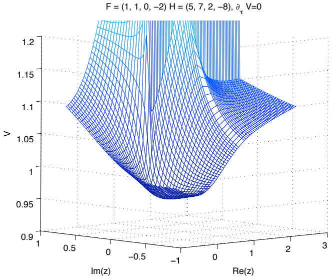
Requirements

Apply monodromy n times:
$F_0 \to F_0 T^n$
If $T = 1 + \Theta$, $\Theta^2 = 0$
$\hookrightarrow F_0 T^n = F_0 + nF_0 T$
Start flux F_0, H_0

- $F_0 T = F_L$

Limit flux F_L, H_L

- has minimum
- $F_L \wedge H_L = 0$
- $F_L T = F_L$



$F_0 = F_0 + nF_L$, $H_0 = H_0 + nH_L \Rightarrow$ infinite number of minima.
N.B. Kähler moduli not fixed.

Conclusions and Outlook

- Semi-discrete landscape.
- Topography $\rightarrow$ dynamics.
- Monodromies connect vacua.
- New, continuous paths.

The new paths allow us to

- connect more vacua continuously.
- find infinite series of minima.
- describe domain walls.
- use connected moduli spaces.

Outlook

- Kähler moduli dynamics. Back reaction.
- Transition with fluxes.
- Tunnelling between minima.
- Inflation.