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Description of the growth curve for Angus pasture-fed cows under extensive systems[1]

V. Goldberg[2] and O. Ravagnolo

Instituto Nacional de Investigación Agropecuaria, INIA Las Brujas,
Ruta 48, km 10, Rincón del Colorado, Canelones, CP 90200, Uruguay

ABSTRACT: Growth curves are useful for describing the sigmoid shape of an animal's weight pattern over time. The purpose of the present study was to describe, for the first time, a complete growth curve for pasture-fed Angus cows in Uruguay and to analyze the major fixed effects that affect the estimation of mature cow weight (CW). A total of 22,743 records from 5,284 cows belonging to 73 herds were used in the statistical analysis. Five nonlinear models (Brody, Gompertz, Von Bertalanffy, logistic, and Richards) and a 3-knot cubic spline function were fitted to model weight change across age. Body weights were recorded at birth, weaning, and 18 mo and as an adult (at 2.3–3.6, 3.6–4.6, 4.6 to 5.6, 5.6 to 6.6, and 6.6 to 8.1 yr). After preliminary analyses, the fixed effects included in the model to estimate CW were BCS, herd, year and month of measurement, and age of dam. The herd and BCS were the major significant sources of variation. The Richards model had the lowest values for Akaike information criterion, Bayesian information criterion, and –2 log likelihood, with the asymptotic weight being 542 kg. The spline function was the model that fitted the data most closely to the observed weights followed by the Richards function. The Richards model gave better predictions of weights from birth to maturity than the other 4 nonlinear models, because these functions were shown to over- or underestimate weights at different ages in this beef cattle data set. The predicted curve showed that cows reach mature CW after 5 yr of age, which is the age commonly assumed as adult weight in beef cattle. Results form this analysis allowed the estimation of CW and rate of maturing and to take knowledge of the shape of growth curve for Angus females in Uruguay under extensive conditions, helping breeders to take selection decisions. In this way, breeders can optimize the management and efficiency for each production system.

Key words: Aberdeen Angus, beef cattle, growth models, mature cow weight

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INTRODUCTION

Uruguayan economy is strongly dependent on beef export, and meat also constitutes a basic component of the yearly population diet (60 kg/person); INAC, 2014). Most of the beef production uses extensive rearing systems consisting primarily of grazing pasture (Resconi et al., 2010).

Animal growth is one of the most important aspects for evaluating animal productivity in beef cattle enterprises. Efficiency of growth is more important in cattle than in other meat animals because of the low rate of reproduction and high maternal maintenance cost per animal marketed (Arango and Van Vleck, 2002). Due to this, the beef cattle industry has been taking a great interest in modeling animal growth for many years to provide a mathematical summary of weight evolution with age and use it thereafter to compare or predict animals' performance (Venot et al., 2004).

Additionally, mature cow weight (**CW**) has been considered an important trait in genetic improvement programs due to its effects on economically relevant

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[2]Corresponding author: vgoldberg@inia.org.uy

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traits such as maintenance requirements, reproduction, and other physiological traits (Costa et al., 2011). In accordance with this, since 2013, CW EBV (EPD) have been published in the National Genetic Evaluation for Aberdeen Angus breed in Uruguay. Despite having EPD for various weight records along animal life, a complete growth curve for cows in Uruguay has not yet been described.

Knowledge of the growth curve for pasture-fed cows is of huge economic importance because it will allow to identify the final live weight" into "because it will allow the identification of the final live weight and the rate at which the animal reaches this target. Moreover, it will lead to a better quantification of the maintenance energy requirements for productive use and also for development of breeding objectives. Due to the relevance of modeling the weight pattern over time, the aim of the current study was to describe a complete growth curve for Angus females under extensive conditions and to analyze the major fixed effects affecting CW.

MATERIAL AND METHODS

Data

Field records used for the present study were extracted from the Instituto National de Investigación Agropecuaria (National Institute of Agricultural Research) of Uruguay performance database. The data comprised 8 records of BW of Aberdeen Angus cows, born from 1992 to 2011, from a total of 84 breeders distributed throughout the country. Body weights were recorded at birth (**BiW**), weaning (**WW**), 18 mo (**W18**), 2.3 to 3.6 (**CW1**), 3.6 to 4.6 (**CW2**), 4.6 to 5.6 (**CW3**), 5.6 to 6.6 (**CW4**), and 6.6 to 8.1 yr (**CW5**). Mature CW was taken whenever they weaned a calf at the moment of weaning, at which time BCS was also recorded. A BCS scale was subjectively assessed by each breeder, using a scale from 1 to 8 points (Vizcarra et al., 1986). Although the initial scale was in units, half scores were accepted to avoid data loss. Only cows with their own WW and at least 1 CW with BCS were used in the analysis.

The initial data contained 5,544 animals with 23,969 records. Several quality controls on performance records were performed to exclude logical inconsistencies and biological incompatibilities. Animals were included in the analysis if they were raised on pastures without supplementation and were born from mothers of age 2 to 10 yr. Edits also included eliminating records of animals outside the range of 3 SD from the overall mean of the contemporary group (**CG**) for each weight. Age edits for WW and W18 were 120 to 305 and 410 to 700 d, respectively, with no more than 90 d difference within CG. These age ranges between animals are due to differences in management strategies under grazing conditions in our country. Measures of CW after 8 yr of age were discarded for being scarce. Records corresponding with BCS less than 3 or more than 6 points were also discarded.

For BiW, WW, and W18, the fitted models included CG (same herd, year, season, and management group) and age of dam (4 levels: 1 [2–3 yr], 2 [4–5 yr], 3 [6–7 yr], and 4 [8–10 yr]) as fixed effects, according to the National Genetic Evaluation criteria (Ravagnolo et al., 2014).

A linear model was used to study the major fixed effects affecting CW1, CW2, CW3, CW4, and CW5 through the GLM procedure of the SAS statistical package (SAS Inst. Inc., Cary, NC). The effects of the cow's own weaning CG and the sex, age, and weight of the weaned calf were also tested but were ignored because there appeared to be no benefit in including them. After preliminary analyses, the fixed effects included were BCS (6 levels: 3, 3.5, 4, 4.5, 5, and 5.5–6), herd, year and month of measurement, and age of dam (4 levels as previously described).

The final data set contained 22,743 observations from 5,284 Angus cows belonging to 73 breeders. A summary of the observed edited data is given in Table 1.

Curve Fitting

Five of the most widely nonlinear models (Brody, Gompertz, Von Bertalanffy, logistic, and Richards functions) were used to describe animal growth curves and to model weight change across age. Residuals of the 8 linearly preadjusted weight–age measurements were obtained for curve fitting for each animal. The equations of nonlinear models evaluated were the following:

a. Brody: $y_{it} = A_i\left[1 - B_i^{(-K_i \times t)}\right] + u_i + \varepsilon_{it}$
(Brody, 1945),

b. Gompertz: $y_{it} = A_i \exp\left[-B_i^{(-K_i \times t)}\right] + u_i + \varepsilon_{it}$
(Gompertz, 1825),

c. Von Bertalanffy: $y_{it} = A_i\left[1 - B_i^{(-K_i \times t)}\right]^3 + u_i + \varepsilon_{it}$
(Von Bertalanffy, 1957),

d. Logistic: $y_{it} = A_i\left[1 + B_i^{(-K_i \times t)}\right]^{-1} + u_i + \varepsilon_{it}$
(Verhulst, 1838), and

e. Richards: $y_{it} = A_i\left[1 - B_i^{(-K_i \times t)}\right]^m + u_i + {}_{it}$
(Richards, 1959),

Table 1. Descriptive statistics for live weights

Trait[1]	Mean	SD	Minimum	Maximum	No. records
BiW, kg	32.8	4.1	18	50	5,035
WW, kg	181.5	37.9	76	321	5,276
Age at weaning, d	195.4	29.5	121	302	5,284
W18, kg	288.2	57.5	150	565	5,263
Age at W18, d	547.9	32.1	414	692	5,284
CW1, kg	396.8	59.2	250	660	1,761
Age at CW1, d	1,218.3	112.5	843	1,296	1,866
CW2, kg	421.2	60.7	248	690	2,193
Age at CW2, d	1,454.9	148.5	1,297	1,662	2,300
CW3, kg	453.6	62.3	251	680	1,491
Age at CW3, d	1,838.8	147.7	1,663	2,028	1,614
CW4, kg	468.2	64.6	263	689	937
Age at CW4, d	2,208.3	148.2	2,029	2,394	981
CW5, kg	483.9	69.3	310	700	787
Age at CW5, d	2,609.6	166.1	2,395	2,969	679

[1]BiW = birth weight; WW = weaning weight; W18 = weight at 18 mo; CW1, CW2, CW3, CW4, and CW5 = adult cow weight measured at 2.3–3.6, 3.6–4.6, 4.6–5.6, 5.6–6.6, and 6.6–8.1 yr, respectively.

Table 2. Analysis of variance for birth weight (BiW), weaning weight (WW), weight at 18 mo (W18), and the 5 adult cow weights (CW1–CW5[1])

		SS[2] (type III)				
Trait	R^2	CG[3]	Age of dam			
BiW	0.39	27,290***	1,864***			
WW	0.60	4,182,287***	60,897***			
W18	0.77	12,190,168***	3,865 ns[4]			
		BCS	Herd	Year	Month	Age of dam
CW1	0.55	687,669***	1,218,806***	87,494***	56,016**	18,389*
CW2	0.56	1,072,437***	1,644,477***	86,419***	69,947***	26,790**
CW3	0.56	733,782***	1,319,892***	60,813***	80,980***	35,654**
CW4	0.54	559,379***	769,155***	49,665**	47,105**	22,375*
CW5	0.53	555,328***	597,675***	51,704**	58,279**	1,259 ns

[1]CW1, CW2, CW3, CW4, and CW5 = adult cow weight measured at 2.3–3.6, 3.6–4.6, 4.6–5.6, 5.6–6.6, and 6.6–8.1 yr, respectively.

[2]SS = sums of squares.

[3]CG = contemporary group.

[4]ns = not significant.

*$P < 0.05$; **$P < 0.01$; ***$P < 0.0001$.

in which y_{it} is the observed BW of the *i*th animal at age *t*, *Ai* is the asymptotic weight for the *i*th animal, B_i is the integration constant, K_i is the maturing rate of the *i*th animal, *t* is the age in days, *m* is a shape parameter that allows for a variable inflection point, *ui* is the random effect for animal *i*, and ε*it* is the fitting error.

The analyses were performed with the NLMIXED procedure of SAS (SAS Inst. Inc.). The predicted parameters *A* (asymptotic weight), *B* (integration constant), *K* (maturing rate), and *M* (inflection parameter) were estimated using dual quasi-Newton as the optimization technique and adaptive Gaussian quadrature as the integration method, and limits were imposed to maintain biological sense.

The models were compared based on the –2 log likelihood, Akaike information criterion (**AIC**), and Bayesian information criterion (**BIC**).

A spline curve was also modeled as a reference to have an additional criterion to compare the 5 nonlinear models, because the shape of the curve is not being forced when splines are applied. After preliminary analyses, cubic splines with 3 knots were adjusted for each animal using the GLIMMIX procedure of SAS statistical package (SAS Inst. Inc.).

RESULTS AND DISCUSSION

Fixed Effects

The estimates of fixed effects fitted for the 8 live weights are presented in Table 2. The BCS had a highly significant effect for the 5 CW analyzed and it was an important source of variation The least squares means differences between the levels of BCS were significant with the exception of BCS 4.5 and 5, where differences were not statistically significant for any of the 5 CW analyzed ($P > 0.05$; Table 3). This result is probably due to the subjective scale; most likely these cows correspond to the same value of BCS.

According to Williams (2007), in addition to their subjective nature, BCS are also not routinely collected in the field and are, therefore, scarce, making it difficult to obtain a large number of records. Due to the limited number of observations available as well as their subjectivity, measures of body condition are somewhat unreliable, leading to much debate over whether or not they should be included in analyzing mature weight. In our country, BCS has been used as a management criterion and also have been measured for more than 10 yr, providing a large amount of data for this trait. On the other hand, although BCS is subjective, it was able to pick up significant differences of 20 to 50 kg between consecutive half scores, which are relevant differences under grazing conditions.

The CG for BiW, WW, and W18 and the herd, year, and month of measurement for the 5 CW analyzed had also a highly significant effect. In accordance to these results, Northcutt et al. (1992) also found that for Angus CW, the effects of herd, year–month that the record was taken, and cow age were highly significant sources of variation. Additionally, BCS was significant, accounting for 16% of the total variation in weight.

The age of dam had a significant effect for BiW, WW, CW1, CW2, CW3, and CW4 but was not significant for W18 and CW5. In all cases, the effect of cow's dam age on live weights was small, accounting for a small part of the total variance. Additionally, when least squares

Table 3. Least squares means for the 5 mature cow weight analyzed (CW1, CW2, CW3, CW4, and CW5[1]) by BCS, with the SE in parentheses

BCS	CW1	CW2	CW3	CW4	CW5
3	368.8[a] (5.7)	394.5[a] (6.5)	429.0[a] (5.9)	451.7[a] (7.0)	446.3[a] (7.9)
3.5	388.7[b] (5.9)	413.5[b] (6.5)	446.8[b] (5.9)	465.1[b] (7.5)	473.0[b] (8.7)
4	406.9[c] (5.6)	436.9[c] (6.3)	471.4[c] (5.4)	490.9[c] (6.5)	495.9[c] (7.4)
4.5	446.3[d] (7.2)	471.1[d] (7.3)	500.0[d] (6.8)	523.8[d] (8.5)	525.9[d] (9.2)
5	444.5[d] (6.5)	474.5[d] (6.9)	508.7[d] (6.4)	534.9[d] (7.9)	541.5[d] (8.4)
5.5–6	498.2[e] (11.8)	527.7[e] (9.6)	532.3[e] (9.8)	559.0[e] (11.3)	576.5[e] (13.2)

[a–e]Different letters between rows indicate significant differences ($P < 0.05$).

[1]CW1, CW2, CW3, CW4, and CW5 = adult cow weight measured at 2.3–3.6, 3.6–4.6, 4.6–5.6, 5.6–6.6, and 6.6–8.1 yr, respectively.

means for age of dam were estimated (not shown), only level 1 had statistically significant differences with level 2 for CW1, CW2, CW3, and CW4; with level 3 for CW3 and CW4; and with level 4 for CW3.

Age of dam effects on postweaning weights have generally been considered to be important. For instance, in the study of Brown et al. (1972), age of dam has a significant effect for CW but they observed that it declined in importance as the animal matured. These authors explain that the persistency of dam effects in heifers may be the result of compensatory growth during the postweaning phases being related to preweaning maternal effects. In the study of Evans (2001), results also showed that there was a significant age of dam effect on mature weight of the cow and fixed effect estimates were greater and more important for weight observations when the cow's dam was younger.

Curve Fitting

The estimates of the growth model parameters are presented in Table 4. Based on model selection criteria, the Richards model had the best adjustment, having the lowest values for AIC, BIC, and –2 log likelihood followed by the Brody, Von Bertalanffy, Gompertz, and logistic functions (Table 5). As is shown in Fig. 1E, the asymptotic weight of the Richards curve (542 kg) was not reached until 3,000 d of age. When data was fitted with the spline curve, the adult weight is approximately 475 kg and is reached at around 2,700 d

Table 5. Values of criteria used for comparing models

Model	–2 log likelihood	AIC[1]	BIC[2]
Brody	193,944	193,954	193,986
Von Bertalanffy	199,617	199,627	199,659
Gompertz	201,847	201,857	201,889
Logistic	206,267	206,277	206,309
Richards	189,138	189,150	189,189

[1]AIC = Akaike information criterion.

[2]BIC = Bayesian information criterion.

Table 4. Estimates of asymptotic weight (*A*), integration constant (*B*), maturing rate (*K*), and inflection parameter (*M*) for the 5 models evaluated, with the SE in parentheses

Model	*A*	95 CI[1]	*K*	*B*	*M*	Age[2]
Brody	464.9 (0.68)	463.6–466.3	0.0017 (0.00001)	0.907 (0.001)	–	2,800
Von Bertalanffy	448.9 (0.65)	447.7–450.2	0.0025 (0.00001)	0.495 (0.001)	–	2,500
Gompertz	444.7 (0.64)	443.4–445.9	0.0029 (0.00002)	1.939 (0.006)	–	2,100
Logistic	437.1 (0.62)	435.9–438.3	0.0043 (0.00002)	4.897 (0.034)	–	1,700
Richards	541.6 (5.56)	530.7–552.5	0.0007 (0.00003)	0.995 (0.001)	0.525 (0.008)	–

[1]CI = confidence interval.

[2]Age in days at which the animal's asymptotic weight is approximately reached.

of age (Fig. 1F). The estimation of *A* with the Brody, Von Bertalanffy, Gompertz, and logistic models was lower (between 437 and 465 kg), and the estimation of *K* with the 5 models was in the range of 0.001 and 0.004. As is shown in Table 4, there is no overlap in *A* estimates among the 5 models analyzed.

The 5 nonlinear models analyzed in the current study are the most used to describe growth patterns in beef cattle (Johnson et al., 1990; Arango and Van Vleck, 2002). The Brody equation has been the most used in beef cattle studies because it is easy to compute and interpret and allows for missing data points and because of goodness of fit (e.g., Brown et al., 1972; Beltran et al., 1992; Kaps et al., 1999). In some studies, the Richards function was reported to fit data better than the Brody equation, and it has the advantage of providing an inflection point that can be useful when evaluating environmental effects on growth (DeNise and Brinks, 1985; Johnson et al., 1990; Beltran et al., 1992). However, there is a higher computational requirement, because 4 parameters are estimated instead of 3 parameters.

Table 6 shows the predicted values at each age where the real measurements were taken. The 3-knot cubic spline function was the model that fitted the data most closely to the observed weights, because the shape of the curve is not being forced when splines are applied. Meyer (2005) reported that random regression models fitting basis functions of B-splines of age at recording as covariables are well suited to modeling growth trajectories of beef cattle and their dispersion structure. In particular, they were less susceptible to the "end-of-range" problems frequently observed for such analyses when fitting polynomial regressions, commonly used to model trajectories of function-valued traits recorded in livestock improvement programs.

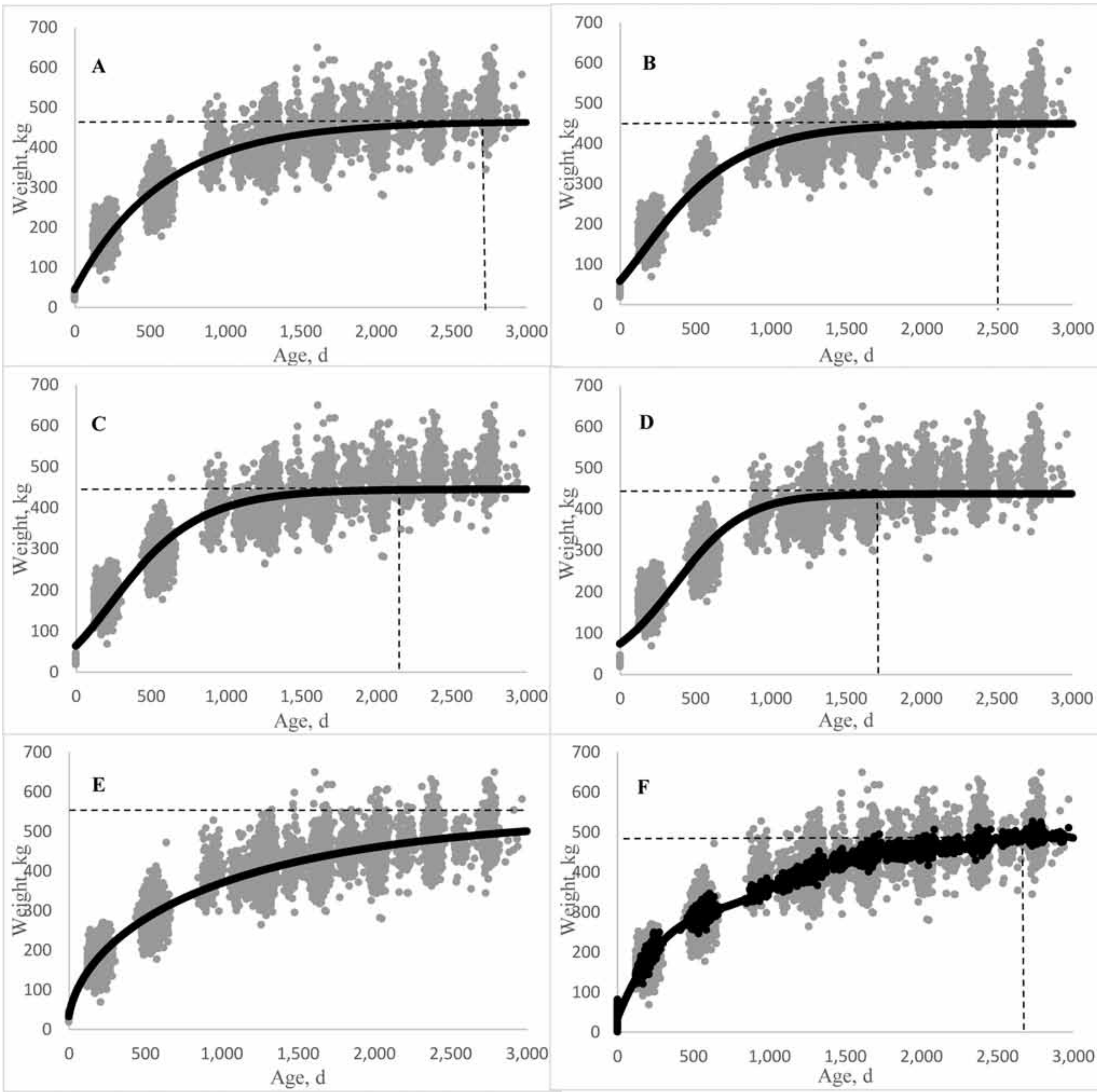

Figure 1. Observed (gray) and predicted (black) growth curves for Brody (A), Von Bertalanffy (B), Gompertz (C), logistic (D), Richards (E), and cubic spline (F) functions.

Table 6. Weights mean estimates for nonlinear models and spline function at the day in which each weight was recorded in average

Trait[1]	Day of measure	Observed weight	Brody model	Von Bertalanffy model	Gompertz model	Logistic model	Richards model	Spline function
BiW	0	32.8	44.1	58.4	64.3	74.4	35.2	35.6
WW	195	181.5	162.1	152.3	148.8	140.6	181.2	178.9
W18	548	288.2	298.5	301.5	301.4	299.8	290.3	291.2
CW1	1,218	396.8	411.5	418.7	421.1	426.3	396.3	392.8
CW2	1,455	421.2	429.2	432.1	432.7	433.1	419.9	426.2
CW3	1,839	453.6	446.3	442.5	440.8	436.3	449.5	452.2
CW4	2,208	468.2	455.0	446.4	443.4	436.9	470.6	463.9
CW5	2,610	483.9	459.9	448.0	444.3	437.0	487.9	487.5

[1]BiW = birth weight; WW = weaning weight; W18 = weight at 18 mo; CW1, CW2, CW3, CW4, and CW5 = adult cow weight measured at 2.3–3.6, 3.6–4.6, 4.6–5.6, 5.6–6.6, and 6.6–8.1 yr, respectively.

The Brody, Von Bertalanffy, Gompertz, and logistic models overestimated BiW and underestimated WW. Additionally, they overestimated CW1 and underestimated CW4 and CW5. These problems were not present when data was fitted with the Richards and the spline functions. As reviewed by Evans (2001), Gompertz, logistic, and Von Bertalanffy functions are known to over- or underestimate weights at different ages in beef cattle data sets. Beltran et al. (1992) reported that the Brody function gave better predictions of weights from 18 mo to maturity than did the Richards function, but early weights were usually slightly overestimated.

The values for the curve's parameters in the present study were estimated modeling the growth curve using records from birth to 8 yr old. According to Morrow et al. (1978), mature weight estimated through growth curves is reliable only when weight data correspond to ages higher than 4.5 yr old. Additionally, they reported that with the inclusion of 2 more years of weights, up to 6.5 yr, the asymptotic weight should be a better estimator of average mature weight, free from seasonal fluctuation. In accordance with these authors, the *A* parameter estimated in the present study would be reliable and a good estimator of mature weight in Angus cows.

Conclusion

Taking into account that the Richards model was the best model to fit the data, cows would reach mature weight later than 5 yr, which is the commonly age assumed as adult weight in beef cattle. Results from this analysis allowed the estimation of mature weight and rate of maturing and to take knowledge of the shape of the growth curve for Angus pasture-fed females in Uruguay. This information should be helpful for researchers and breeders to optimize the management and efficiency for each production system in Uruguay.

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