On the Fundamental Limits of Broadcasting in Wireless Mobile Networks

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Abstract—In this paper, we investigate the fundamental properties of broadcasting in mobile wireless networks. In particular, we characterize broadcast capacity and latency of a mobile network, subject to the condition that the stationary node spatial distribution generated by the mobility model is uniform. We first study the intrinsic properties of broadcasting, and present a broadcasting scheme that simultaneously achieves asymptotically optimal broadcast capacity and latency, subject to a weak upper bound on the maximum node velocity. We then investigate the broadcasting problem when the burden related to selecting relay nodes is taken into account, and present a combined distributed leader election and broadcasting scheme achieving a broadcast capacity and latency which is within a poly-logarithmic factor from optimal.

Index Terms—wireless networks; mobile networks; broadcast capacity; broadcast latency; SINR interference model.

I. INTRODUCTION

Investigation of fundamental properties of wireless networks has received considerable attention in the research community, starting from the seminal Gupta and Kumar [6] work that characterized the capacity of a wireless multi-hop network for unicast transmissions. Since then, fundamental properties of wireless multi-hop networks have been investigated for a variety of communication patterns including unicast [5], [12], broadcast [7], [15], [20], multicast [19], and convergecast [10]. It has been shown that wireless multi-hop network scaling laws significantly change depending on network parameters such as node deployment (e.g., random vs. arbitrary), traffic pattern, and node mobility. Node mobility in particular has been shown to have considerable effects on wireless network scaling laws: for instance, per-node capacity of unicast transmission has been shown to be asymptotically vanishing with the number \( n \) of network nodes independently on the node deployment (see [6]), but to become constant (i.e., asymptotically optimal) in case network nodes are mobile [5] (under the assumption that very large delays in packet delivery can be tolerated).

To the best of our knowledge, none of the existing papers has investigated the effect of mobility on broadcasting scaling laws. Broadcasting scaling laws have been recently characterized in a series of papers [7], [20], including our work [15], [16] showing that, contrary to what happens for unicast transmission, asymptotically optimal capacity and latency can be achieved simultaneously for broadcast communication. However, all these results are based on the assumption that network nodes are static. An implicit consequence of this assumption is that the communication burden induced by the need of selecting broadcast relaying nodes within the network (called the coordination burden in the following) is consistently ignored in the analysis. While this assumption is reasonable in static networks, if relay nodes are mobile, a change in their position might cause an incomplete coverage of the broadcast packets, which must be received by all network nodes. Thus, the role of broadcast relay node must be continuously rotated amongst network nodes in a mobile network, in order to ensure broadcast coverage in spite of node mobility. Given this, evaluating the coordination burden cost becomes an integral part of the characterization of broadcasting scaling laws in mobile networks.

In this paper, we make a first step towards gaining a better understanding of the effect of mobility on the broadcasting communication paradigm. We first show that broadcasting is not inherently capacity nor latency limited by node mobility: we present a simple cell-based broadcasting scheme, called RIPPLECAST, that simultaneously achieves optimal broadcast capacity and latency under the assumption that: \( i \) nodes move in a bounded region according to a mobility model whose stationary node spatial distribution is uniform; and \( ii \) maximum node velocity is upper bounded by a (very large) constant. However, when the cost related to the coordination burden is taken into account the picture changes considerably: broadcasting capacity and latency degrade of a factor \( \Theta((\log n)^{1+\frac{\alpha}{2}}) \) with respect to optimal - \( n \) is the number of network nodes and \( \alpha > 2 \) is the path loss exponent - , and the upper bound on maximum node velocity becomes asymptotically vanishing as \( n \to \infty \). We thus formally prove that what limits broadcast performance in a mobile network are not the inherent properties of broadcast communication, but the coordination burden induced by the need of frequent re-selection of relay nodes within the network.

II. RELATED WORK

The fundamental properties of broadcasting in wireless multi-hop networks have been investigated only very recently. In [20], Zheng investigated the broadcast capacity of random networks with single broadcast source under the generalized physical interference model, and presented a broadcast scheme providing asymptotically optimal capacity. The author also presented a different broadcast scheme, and proved its asymptotically optimal performance with respect to information diffusion rate, which is closely related to latency. The authors of [7] confirmed that optimal broadcast capacity can be achieved in a more general network model, in which arbitrary node positions are allowed, an arbitrary subset of the network nodes is assumed to generate broadcast packets, and accurate SINR-based interference models are used. In [15], we have shown that asymptotically optimal broadcast capacity and latency can be simultaneously achieved in a static network, under the assumption of single broadcast source. This result has
been recently extended to the case of an arbitrary number of broadcast sources in [16].

While several papers have proposed broadcasting schemes for mobile networks (see, e.g., [13], [14]), to the best of our knowledge none of them attempted at characterizing the fundamental properties of broadcasting in mobile networks. The work that is closest to our is [2], where the authors present a location-based broadcasting protocol for mobile ad hoc networks, and formally characterize the number of communication steps needed to deliver a broadcast packet to all network nodes. However, the authors in [2] are concerned with delivering a single broadcast packet, while in this paper we are interested in characterizing the maximum rate at which broadcast packets can be sent by the source. Furthermore, the results of [2] are valid under a simplistic interference model based on the notion of conflict graph, while ours hold under the more realistic, SINR-based physical interference model.

III. NETWORK MODEL AND PRELIMINARIES

We consider a wireless network composed of \( n + 1 \) wireless nodes distributed in a square region \( R \) of side \( L = L(n) \). One of the nodes is stationary, and is located in the center of the deployment region. This node, denoted \( s \) in the following, is the broadcast source. The remaining \( n \) nodes are mobile, and move within \( R \) according to some mobility model \( M \). Model \( M \) is such that the induced stationary node spatial distribution (which is assumed to exist) is uniform. In other words, a snapshot taken at time \( t \) of the positions of \( n \) nodes moving according to \( M \), for a sufficiently large \( t \), is statistically equivalent to a uniform random distribution of \( n \) nodes into \( R \). Examples of mobility models satisfying this assumption are random walks, brownian motion, random direction model with proper border rules, etc (see [9] and references therein).

We assume nodes communicate through a shared wireless channel of a certain, constant capacity \( W \), and that the nodes transmission power is fixed to some value \( P \). Correct message reception at a receiver node is subject to an SINR-based criterion, also known as physical interference model [6]. More specifically, a packet sent by node \( u \) is correctly received at a node \( v \) (with rate \( W \)) if and only if

\[
\frac{P_r(u)}{N + \sum_{i \in T} P_r(i)} \geq \beta ,
\]

where \( N \) is the background noise, \( \beta \) is the capture threshold, \( T \) is the set of nodes transmitting concurrently with node \( u \), and \( P_r(x) \) is the received power at node \( v \) of the signal transmitted by node \( x \).

We also make the standard assumption that radio signal propagation obeys the log-distance path loss model, i.e. the received signal strength at distance \( d \) from the transmitter (for sufficiently large \( d \), say, \( d \geq 1 \)) equals \( P \cdot d^{-\alpha} \), where \( \alpha \) is the path loss exponent. In the following, we make the standard assumption that \( \alpha > 2 \), which is often the case in practice. We then have\(^1\) \( P_r(x) = P \cdot d(x,v)^{-\alpha} \), where \( d(x,v) \) is the Euclidean distance between nodes \( v \) and \( x \).

For given values of \( P \), \( \beta \), \( \alpha \), and \( N \), we define the transmission range \( r_{\text{max}} \) of a node as the maximum distance

\[\text{up to which a receiver can successfully receive a message in absence of interference. From the definition of physical interference model, we have } r_{\text{max}} = \sqrt{\frac{W}{\beta N}}.\]

The maximal communication graph is a graph \( G = (V,E) \) representing all possible communication links in the network, i.e., \( V \) is the set of the \( n + 1 \) nodes, and (undirected) edge \((u,v) \in E\) if and only if \( d(u,v) \leq r_{\text{max}} \).

We define the broadcast capacity of the network as the maximum possible rate \( \lambda(n) \) such that all packets generated by source \( s \) are received by the remaining \( n \) nodes within a certain time \( T_{\text{max}} \), with \( T_{\text{max}} < \infty \). The broadcast latency of the network is the minimal time \( T(n) \) such that the packet generated by \( s \) at time \( t \) is received by \( n \) nodes within time \( t + T(n) \). It is clear that, in order to have meaningful values of broadcast capacity and latency, the maximal communication graph of a network must be connected\(^2\). Thus, the assumption of connected maximal communication graph is made throughout this paper. More specifically, we assume that graph \( G \) is connected w.h.p. under the assumption that nodes are distributed according to the asymptotic node spatial distribution resulting from mobility model \( M \) which, we recall, is assumed to be uniform\(^3\).

Given the assumption of stationary uniform node spatial distribution of the mobility model \( M \), the critical transmission range for connectivity is [3]:

\[\text{ctr}(n) = \Theta \left( \frac{L(n)}{n} \sqrt{\frac{\log n}{n}} \right).\]

We recall that the critical transmission range for connectivity is the minimal common value of the transmission range such that the resulting maximal communication graph is connected.

Assume the deployment region \( R \) is divided into non-overlapping square cells of side \( l \), with \( l = \frac{r_{\text{max}}}{\beta W^{1/2}} \), for some constant \( h > 1 \). In turn, each of these cells is partitioned into 9 square mini-cells of side \( \frac{l}{3} \) (see Figure 1). The following proposition defines a value of \( L(n) \) such that several properties of the resulting node deployment hold, w.h.p.

**Proposition 1**: Assume \( L(n) = \frac{r_{\text{max}}}{\beta W^{1/2}} \sqrt{\frac{n}{\log n}} \) for some constant \( h > 1 \), and assume \( n \) nodes are distributed uniformly

\(^2\)Note that in general broadcasting is possible even if graph \( G \) is never instantaneously connected, but it is indeed connected when considering its evolution over time. However, in this low-density conditions, sustaining a non-zero broadcasting rate is hardly possible, due to the orders-of-magnitude lower speed of human mobility as compared to the speed of radio transmissions.

\(^3\)Given the probabilistic characterization of mobile node positions assumed in this paper, most of the properties proved in this paper hold with high probability (w.h.p.), i.e., with probability approaching 1 as \( n \) goes to infinity.
at random in a square region of side $L(n)$. Then, the following properties hold w.h.p.:

(a) the minimally occupied mini-cell contains at least one node;
(b) the maximally occupied mini-cell contains $\Theta(\log n)$ nodes;
(c) the maximum transmission range $r_{\text{max}}$ is asymptotically minimal to ensure network connectivity.

Finally, we introduce the notion of cell distance, which will be extensively used in the following. Given any two cells $A$ and $B$ in the deployment region, the cell distance between $A$ and $B$, denoted $d(A, B)$, is the minimum number of adjacent cells (horizontal, vertical, and diagonal adjacency) that must be traversed to reach $A$ starting from $B$ (and vice versa).

Due to lack of space, in the rest of the paper we omit proofs, and only briefly mention the results accounting for the coordination burden. Interested reader is referred to [17].

IV. BOUNDS ON BROADCAST CAPACITY AND LATENCY

The following upper bound on the broadcast capacity trivially follows by observing that the maximum rate at which any receiver can receive broadcast packets is $W$ [7]. The bound holds for an arbitrary network.

Claim 1: In any network with $n$ nodes, we have $\lambda(n) \leq W$. Define $D(n)$, the diameter of the network (relative to the broadcast source), as the maximum Euclidean distance between a network node $u$ and the source $s$. Given that nodes are mobile, the diameter of the network changes over time. However, Proposition 1 implies an invariant property of network diameter under our deployment assumptions, as stated in the following proposition:

Proposition 2: Let $D(n, t)$ be the network diameter at time $t$. If $t$ is sufficiently large, $L(n) = \frac{r_{\text{max}}}{2h\sqrt{2}} \sqrt{\frac{n}{\log n}}$ for some constant $h > 1$, $n$ nodes move according to a mobility model with stationary uniform node spatial distribution in a square region of side $L(n)$, and the source node is located in the center of the deployment region, then $D(n) \geq \sqrt{\frac{2}{n}}(L(n) - \frac{2}{n}) = \Omega(L(n))$, w.h.p.

We are now ready to prove a lower bound for broadcast latency in mobile networks, subject to an upper bound on node velocity.

Theorem 1: Suppose the same assumptions of Proposition 2 hold, and the maximum node velocity is $\tilde{v} = \frac{r_{\text{max}}}{\tau}$, where $\tau$ is the (constant) time required to send and correctly receive a packet. Then, the broadcast latency is $\Omega\left(\sqrt{\frac{n}{\log n}}\right)$, w.h.p.

V. MATCHING CAPACITY AND LATENCY BOUNDS

In this section we present a broadcasting algorithm achieving asymptotically optimal capacity and latency bounds in mobile networks, under the assumption that broadcast relaying nodes are somewhat magically selected within the network. This assumption, although admittedly not realistic, is made with the purpose of separately studying the fundamental properties of broadcasting in mobile networks from those of electing leaders (i.e., relay nodes).

A. Algorithm overview

While broadcasting in mobile networks is apparently a very complex task due to mobility of individual nodes, this apparent complexity can be tamed by observing that the identity of a specific node within the network is not relevant to a broadcasting scheme, as long as reception of each broadcast packet by each of the (mobile) nodes can be guaranteed. In other words, what is relevant to a broadcasting scheme is not the identity of a node, but its position within the network. Thus, instead of selecting specific nodes to relay broadcast packets, a smart broadcasting scheme for mobile networks should focus on invariant properties of the node spatial distribution generated by the mobility model, and use such properties to select relay nodes based on their location within the network.

The broadcasting scheme, which we call RIPPLECAST, is based on the following assumptions:

- a spatial TDMA approach is assumed at the MAC layer;
- the deployment region is divided into cells and mini-cells, as described in Section III. Cell subdivision is used to virtualize the broadcasting task from a node-related process to a cell-related process. In particular, broadcast relaying nodes (leaders) are chosen within the central mini-cell of each cell, and the broadcasting process becomes one of propagating broadcast packets between cells. Without loss of generality, we assume that the source node $s$ is in the central cell.

RIPPLECAST is based on a cell coloring scheme, as in Figure 2, composed of a constant number $k^2$ of colors, which is used to spatially separate simultaneously active transmissions. In particular, the coloring scheme ensures that, under the assumption that at most one transmitter is active in each cell with the same color, all transmitted packets are correctly received by all the nodes located in the cells adjacent to the transmitter cell. A round of transmission is composed of $k^2$ transmission slots, one for each color. The color of a cell $A$ is denoted $\text{col}(A)$ in the following. Similarly, $\text{col}(u)$ denotes the color of the cell to which node $u$ belongs. With RIPPLECAST, propagation of broadcast packets occurs along concentric “waves” (ripples, whence the name RIPPLECAST): in the first round, a packet is transmitted to nodes located in cells at cell distance one from $s$; in the second round, the
Algorithm for source node $s$:

1. Let $i$ be the color of the current time slot; $ID$ is the current packet ID.
2. If $col(s) = i$ then
   1. transmit new packet; $ID = ID + 1$

Algorithm for a generic node $v$:

1. Let $i$ be the color of the current time slot; if $v$ is a leader node, let $j$ be the ID of the last packet received by node $v$.
2. listen to the channel.
3. if new packet arrive then
   1. receive the packet.
   2. let $j'$ be the ID of the received packet.
   3. if $(j' = j + 1)$ then
      1. store packet in transmit buffer.
      2. if $buffer(v)$ is not empty then
         1. transmit packet and empty transmit buffer.

Fig. 4. The RIPPLECAST broadcasting scheme.

packet is propagated to nodes located in cells at cell distance two from $s$, and so on, till the packet is propagated to the furthest cells in the deployment region (see Figure 3). Since a new packet is generated by source $s$ at each round, the propagation proceeds in a pipelined fashion, and eventually at each round each ripple of leaders is propagating a different packet.

B. RIPPLECAST

The RIPPLECAST algorithm is reported in Figure 4. The algorithm for the source node $s$ is very simple: when the transmission slot corresponding to $col(s)$ is scheduled, the source node transmits a new packet, and increments the packet ID by one. Any non-source node $v$ acts as follows. Independently of the color of the scheduled slot, node $v$ listens to the channel, and receives new packets. Note that a node in general receive packets with the same ID several times; only new packets are received at step 3. of the algorithm. If the ID of the new received packet equals the ID of the most recently received packet increased by one, then the new packet is stored in the transmit buffer. If the color of the current slot equals $col(v)$, $v$ is the cell leader, and the transmit buffer is not empty, the packet is transmitted and the transmit buffer emptied.

Function $cellLeader()$ at step 7 checks whether node $v$ is currently a leader node. Leader selection obeys the following rules. During round $t$, a node (call it $v$) currently (more specifically, at the beginning of the round) located within a central mini-cell is selected as leader node for that cell (call it $A$) for that round. More specifically, during round $t$ node $v$ will be in charge of transmitting the packet received by the cell it belonged to at round $t - 1$. As we shall see in the next section, the fact that leader nodes are selected amongst the nodes in the central-mini cell, coupled with an upper bound on node velocity, ensures that node $v$ was in cell $A$ also during the entire round $t - 1$, thus guaranteeing a correct propagation of broadcast packets. If node $v$ is still in the central mini-cell of cell $A$ at the beginning of round $t + 1$, it keeps the leader role also in the next round, otherwise a new node amongst the ones currently present in the central mini-cell is selected as leader for round $t + 1$.

The actual rule used to selected leaders in case more than one nodes are present in a mini-cell is irrelevant.

C. Analysis

We start borrowing a result from [15], which shows that cells can be colored using $k^2 = \Theta(1)$ colors, in such a way that the packet transmitted by a leader node is correctly received by all nodes located in neighboring cells (horizontal, vertical, and diagonal adjacency), under the assumption that at most one node per cell with the same color is transmitting. The coloring scheme depicted in Figure 2 assigns the same color to cells at cell distance $k$ along the horizontal and vertical direction (details can be found in [15]). The following result has been proved in [15].

Proposition 3: Given a deployment region divided into square cells of side $l = \sqrt{\frac{858}{\beta}}$, for some constant $h > 1$, it is possible to devise a coloring scheme with $k^2$ colors, where $k \geq \bar{k} = \left[2 + 2^{\frac{3}{\beta}} \left(\beta (\alpha - 1) h^n / (h^n - 1)\right)^{\frac{1}{\alpha}}\right]$, and $\zeta$ is the Riemann’s zeta function, such that the packets transmitted by leader nodes with the same color are received by all nodes located in cells adjacent to the cell of a transmitter node (horizontal, vertical, and diagonal adjacency), under the assumption that at most one node per cell with the same color is transmitting.

Note that, being $h$, $\alpha$ and $\beta$ constants, the number of colors $\bar{k}^2$, which coincides with the number of transmission slots in a round, is $\Theta(1)$.

The next Lemma states that source node $s$ generates new packets at rate $\frac{W}{L} = \Omega(W)$, which is asymptotically optimal.

Lemma 1: Assume algorithm RIPPLECAST is used to broadcast packets in the network. The source node $s$ generates packets at rate $\frac{W}{L} = \Omega(W)$.

We next show that each packet generated by the source is correctly received all network nodes within time $T(n) = O\left(\sqrt{\frac{n}{\log n}}\right)$.

Lemma 2: Assume $n$ nodes move within a square region of side $L(n) = \frac{r_{max}}{\alpha h \sqrt{2}} \sqrt{\frac{n}{\log n}}$ according to a mobility model $\mathcal{M}$ with: $i)$ uniform stationary node spatial distribution, and $ii)$ maximum node velocity equal to $\bar{v} = \frac{l}{3\beta\tau}$, where $\tau$ is the duration of a transmission slot and $l$ is the side of a cell. Furthermore, assume algorithm RIPPLECAST is used to broadcast packets. Then, a packet generated by the source node at round $t$ is received by all network nodes within round $t + O\left(\sqrt{\frac{n}{\log n}}\right)$, w.h.p.

Is the upper bound on node velocity imposed by Lemma 2 restrictive? The answer, for typical values of the network parameters, is no, owing to the very high packet propagation speed within the network. For instance, assuming an outdoor propagation environment with path-loss $\alpha = 3$, channel parameters typical of an 802.11a/g network with 54/9 M/bs data rate (more specifically, $\beta = 22dB$, $P = 100mW$, and $\nu = -90dBm$), a packet size of $1KB$, and setting $h = 2$ in the cell partitioning scheme, we have that $\bar{k} = 50$, $r_{max} = 858m$, $l = 151m$, $\tau = 180\mu sec$ (leaving adequate margin for radio signal propagation time), and the upper bound on velocity is $\bar{v} = 111.852 m/sec \approx 403 km/h$.

We are now ready to prove the main result of this section:

Theorem 2: Assume $n$ nodes move within a square region of side $L(n) = \frac{r_{max}}{\alpha h \sqrt{2}} \sqrt{\frac{n}{\log n}}$ according to a mobility model...
\( M \) with: i) uniform stationary node spatial distribution, and ii) maximum node velocity equal to \( v = \frac{l}{3k\tau} \), where \( \tau \) is the duration of a transmission slot and \( l \) is the side of a cell. Algorithm RIPPLECAST provides asymptotically optimal broadcast capacity and latency.

VI. BROADCASTING WITH LEADER ELECTION

In this section, we revisit the broadcasting problem taking into account also the burden incurred by leader election. Distributed leader election is one of the most investigated problems in the distributed computing literature. Though, the leader election problem we face is non-standard: although each single leader election in a mini-cell corresponds to the classical single-hop leader election problem [11], we have to perform several such elections: one for each of the \( \frac{n}{\log n} \) mini-cells in the deployment region. To speed up this process, we propose running as many simultaneous leader elections as possible, combining the ID-based leader election scheme proposed in [1] for network-wide election of a single leader node in a wireless multi-hop network, and the carrier sense based technique used in [18] to distributely build a dominating set in a wireless multi-hop network.

The main idea is to run interleaved leader election and broadcasting phases. The broadcasting phase is as before, while in the leader election phases several leader elections are run in parallel in the cells colored with the same color. In order to avoid mutual interference between concurrent leader elections, we have to use a non-constant number of colors, which leads to a broadcast capacity and latency within a factor \( \Theta((\log n)^{1+\frac{1}{2}}) \) from optimal.

Unfortunately, the burden related to the leader election process considerably strengthen the upper bound on node velocity, which becomes asymptotically vanishing as \( n \) grows to infinity. Thus, the larger the network, the more stationary the nodes must be in order to achieve near-optimal broadcast capacity and latency. However, the actual bound on maximal node velocity is shown to be only marginally influenced by the number of network nodes (see [17] for details).

VII. DISCUSSION AND FUTURE WORK

In this paper, we have investigated the fundamental limits of broadcasting in mobile wireless networks, and we have shown that, while broadcasting is not inherently limited (in terms of both capacity and latency) by node mobility, the coordination burden caused by the need of repeatedly selecting broadcast relay nodes does indeed reduce broadcast performance of a poly-logarithmic factor.

What are the implications of our findings for the design of practical broadcasting protocols for mobile networks? The main implication is that network designers should focus their design on identifying invariant properties of the mobile network (e.g., node spatial distribution), and then build their protocol exploiting these properties. Clearly, location-awareness is likely to be a key feature in designing efficient broadcasting protocols for mobile wireless networks.

It is interesting also to discuss the relative effect of node mobility in case of unicast and broadcast communications: in unicast communication, node mobility can be used as a mean to suppress (or considerably reduce) the relaying burden, thus bringing capacity up to the optimal value; on the contrary, in case of broadcast, node mobility introduces the need of frequently re-selecting broadcast relay nodes, thus inducing a coordination burden which causes a poly-logarithmic capacity and latency degradation with respect to optimal. However, it is important to observe that this performance degradation is not inherently due to the broadcast communication pattern, but rather to a “common practice” of performing broadcast communications based on the selection of broadcast relay nodes. Hence, a promising research direction is to investigate whether alternative broadcasting approaches, such as cooperative communications, can be used to reach the capacity and latency limits.

REFERENCES