In the past few decades, there has been a great deal of fruitful interaction between the theory of algebraic curves and the theory of linear error correcting codes. A link between higher dimensional projective varieties and codes can be set up in an analogous manner using the language of projective system due to Tsfasman and Vladut. More recently some attempts have been made to exploit this connection. In this talk I will describe a number of examples involving classical projective varieties such as the Schubert varieties in Grassmannians which illustrate this connection. It will be seen that this leads to several interesting problems that are predominantly of a combinatorial nature. I will first outline some background and explain some recent results and open questions. No prior knowledge of Coding Theory or Schubert varieties may be expected from the audience.