A Proposed New Biomechanical Head Injury Assessment Function - The Maximum Power Index

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ABSTRACT

Recently, several cases of mild traumatic brain injury to American professional football players have been reconstructed using instrumented Hybrid III anthropomorphic test dummies ATDs. The translational and rotational acceleration responses of injured and uninjured players’ heads have been documented. The acceleration data have been processed according to all current head injury assessment functions including the GSI, HIC and GAMBIT among others.

A new hypothesis is propounded that the threshold for head injury will be exceeded if the rate of change of kinetic energy of the head exceeds some limiting value. A functional relation is proposed, which includes all six degrees of motion and directional sensitivity characteristics, relating the rate of change of kinetic energy to the probability of head injury. The maximum value that the function achieves during impact is the maximum power input to the head and serves as an index by which the probability of head injury can be assessed.

INTRODUCTION

Closed head injury occurs when the head experiences a change in its motion that exceeds its capacity to accommodate such change. Typically, the kinematics of the head have been characterized by its acceleration in time. Many functional relationships between the severity/probability of brain injury and acceleration, have been proposed. These brain injury assessment functions are all based upon the observed impact responses of cadavers, animals, volunteers, or accident victims. The limitations of each of these data sources are well known in the biomechanical engineering community.

Several head injury assessment functions have evolved over the past 40 years. A review of many such functions has been provided by Newman [1]. Among such kinematic head injury functions is the Severity Index SI, [2].

\[
SI = \int_{T} a(t)^{2.5} dt \tag{1}
\]

where \(a(t)\) is the linear acceleration of the head (in gravitational units) and \(T\) is the time duration of the head impact (in seconds). The maximum value achieved by this function during an impact test of a football helmet is used in the NOCSAE performance standard for football helmets [3].

Another well known head injury assessment function is the HIC [4].

\[
HIC = \left[ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t)dt \right]^{2.5} (t_2 - t_1) \tag{2}
\]

Both equation 1 and equation 2 have their origin in the Wayne State Concussion Tolerance Curve [5]. In fact, equation 2 comes directly from Versace’s [6] consideration of the appropriateness of equation 1.

Gadd [2] first approximated the WST curve with an empirical expression for which the slope of the Wayne State curve when plotted in log-log coordinates, was approximated by –2.5. Hence the 2.5 that appears in the above equations. His equation relating the average acceleration to time duration was simply:

\[
a_{ave}^{2.5} T = 1000 \tag{3}
\]

It has been argued that none of the above expressions are meaningful from an engineering point of view because, if for no other reason, they have units that do not relate to any known measure of impact severity.

Early on, Versace [6] proposed several other empirical fits to the Wayne State data some of which were considera-
bly better in some time domains than the Gadd approximation. One such approximation that seems not to have been considered is one in which the exponent was set not to 2.5 but simply to 2. One such function:

\[ a_{\text{ave}}^2 T = 6737 \]  \hspace{1cm} (4)

where \( a_{\text{ave}} \) is expressed in units of m/s\(^2\), is shown in Figure 1 along with the WST curve and the Gadd approximation.

Figure 1: Comparison of the Wayne State Tolerance Curve with Approximations

It will be observed that equation 1 is actually a better fit in the 5 to 30 msec range than the Gadd equation. Importantly this expression may have a significant physical meaning. Equation 4 can be rewritten as:

\[ \frac{V^2}{T} = 6737 \]  \hspace{1cm} (5)

where \( V \) is the change of velocity of the head. This expression has units that are proportional to the rate of change of kinetic energy or power. This observation provides the basis for an hypothesis that head injury severity/probability correlates to the magnitude of the rate of change of kinetic energy that the head undergoes during an impact. This very suggestion was first made nearly 25 years ago by DiLorenzo [7]. However at that time, insufficient data was available to derive any new head injury assessment function. In fact the concept was employed chiefly to examine hypothetically optimum acceleration waveforms. He nevertheless concluded, “The probability of bodily injury in a crash appears predictable on the rate at which energy is transferred to the body.”

In consideration of the effect of kinetic energy changes at the tissue level, a brief examination of the predictions of the “Viscous Criterion”, is provided in Appendix A.

A POWER BASED HEAD INJURY ASSESSMENT FUNCTION

A general expression for the rate of change of translational and rotational kinetic energy for any rigid object, is of the form:

\[ \text{Power} = P = \sum m \ddot{\mathbf{a}} \cdot \mathbf{v} + \sum I \ddot{\mathbf{\alpha}} \cdot \mathbf{\omega} \]  \hspace{1cm} (6)

where:

\[ \ddot{\mathbf{a}} = \text{linear acceleration (m/s}^2\text{)} \]
\[ I = \text{mass moment of inertia (Nms}^2\text{)} \]
\[ m = \text{mass (kg)} \]
\[ \ddot{\mathbf{\alpha}} = \text{angular acceleration (rad/s}^2\text{)} \]
\[ \mathbf{\omega} = \text{angular velocity (m/s)} \]

When the coefficients are set equal to the mass and appropriate mass moments of inertia for the human head, the expression becomes:

\[ \text{HIP} = 4.50 a_x \int a_x \, dt + 4.50 a_y \int a_y \, dt + 4.50 a_z \int a_z \, dt + 0.016 \alpha_x \int \alpha_x \, dt + 0.024 \alpha_y \int \alpha_y \, dt + 0.022 \alpha_z \int \alpha_z \, dt \]  \hspace{1cm} (7)

where HIP is the Head Impact Power. The inertial reference frame is of course that of the head itself.

As with most injury assessment functions, HIP varies throughout the impact being 0 when \( t = 0 \), passing through a maximum and returning to 0 at the end of the event. At its maximum, the rate of change of kinetic energy of the head is at a maximum. As with VC, for example, it will be assumed that the probability of inertially induced injury to the brain will be highest when the rate of change of energy of the head is highest. That is, head injury will correlate with the maximum value achieved by equation 7 during the impact event.

To be completely general, one must acknowledge that the brain likely has a different tolerance to power absorption in each of the different directions and for each axis of rotation. It is not known at this time what are the precise differences in these sensitivities, however an INDEX could be derived in which the values of the coefficients reflected these differences. Such an equation would be of the form.
\[ P_I = C_1a_x \int a_x dt + C_2a_y \int a_y dt + C_3a_z \int a_z dt + C_4\alpha_x \int \alpha_x dt + C_5\alpha_y \int \alpha_y dt + C_6\alpha_z \int \alpha_z dt \]  

Where \( P_{I_{\text{max}}} \) is the maximum value of the Power Index. The coefficients could be chosen so as to normalize the index with respect to some selected failure level. In fact, some of the coefficients may well be numerically different depending on whether the acceleration is positive or negative. For the present these coefficients are unknown, however, some considerations are later explored to examine the sensitivity of equation 7 to different assumptions.

METHODS

MILD TRAUMATIC BRAIN INJURY DATABASE

For the past few years, professional football players in North America have been engaged in a research program whereby impacts to the heads of athletes who may have sustained a concussion, are re-constructed with Hybrid III dummies. Concussion is a form of mild traumatic brain injury MTBI, where a loss of consciousness may or may not occur. Though various degrees of concussion severity are acknowledged, a quantifiable scale has yet to be fully developed. In the present situation, the data is gathered from team physicians who treat the athletes when they are injured. The data is thus subject to the interpretation of the physician and indeed some concussions may go undiagnosed and others may be misinterpreted. At best, we are able to segregate the data into two groups; affirmatively diagnosed MTBI and MTBI not observed. Video recordings of over 100 cases were examined in the course of this investigation.

RE-ENACTMENT METHODOLOGY

The details by which any incident is reconstructed are discussed in reference 12. The laboratory-based reconstruction of football helmeted head impacts is achieved using Hybrid III ATDs. Though MTBI is associated with head contact with many surfaces, including the ground, knees, elbows and other heads, the primary focus of the current study is head-to-head collisions. This has two advantages over other configurations. First, the impacting surfaces, being certified helmets, are well defined and characterized. Second, in head-to-head collisions, two data sets can be collected for each reconstruction: the injured and non-injured players. A common thread that was noted among most of such cases was that the injured (usually the struck) head was impacted laterally, and the non-injured (usually the striking) head was impacted in approximately a vertical direction. As a consequence, the body mass of the struck player is not a significant factor in the collision.

From the kinematic analysis of game video as described in reference 12, the velocity of one player’s head relative to the other is determined.

The injured player headform is mounted on a standard Hybrid III neck, which is connected to a carriage on a vertical track. The carriage has provision to adjust the orientation of the head and neck to match that of the player at collision. The non-injured player headform is mounted on a full Hybrid III test dummy neck and torso that is held in static suspension by spring-loaded tethers at the hips and shoulders. The arms and legs are removed. Both headforms are instrumented with nine linear accelerometers arranged to allow the calculation of triaxial linear and rotational accelerations following the NHTSA protocol.¹

Alignment of the headforms is achieved through frame-by-frame analysis of the game video, and the respective camera positions. The direction of the carriage travel represents that of the calculated relative velocity vector of collision. The carriage is raised to a height that upon release in free-fall yields the intended impact speed.² High-speed video cameras, capturing 500 frames per second, are positioned at the same relative angles to the point of collision as the game cameras. When the set-up appears correct from multiple views, the case is documented and the test is run.³

The correctness of the test set-up is verified by examining the head rebound kinematics. If the headforms do not move in the same way as the players’ heads did, the initial set-up is adjusted and the test repeated.

All acceleration data were collected at 10kHz following SAE J211 protocol. Acceleration data were pre-processed according to CFC 1000 requirements, and then later re-filtered digitally at CFC 180. This secondary filtering was found to remove spurious noise from the rota-

¹ The stationary ATD was additionally instrumented with a six-axis upper neck load cell to facilitate possible future investigation of neck loading issues.
² In some cases where the impact speed was higher than possible by gravity alone, elastic shock cords were used to boost the carriage speed.
³ It should be noted that game video is captured at only 30 frames per second, and that there is rarely a video frame of the actual instant of impact, only one before and one after. In these cases, some subjective interpolation is required for the set-up.
tional acceleration calculations without unduly affecting the overall signal.

RESULTS

Twelve cases involving 24 players have been re-enacted and the full 3 dimensional responses of their helmeted heads have been observed. Details of this work are described in Newman et al [11,12]. Data from that study have been processed according several current head injury assessment functions. In addition to calculating SI, HIC and GAMBIT [13], the data have been employed to calculate the value of HIP$_m$ (from equation 7). Logistic regression analysis has also been performed and various statistical parameters evaluated.

Data tables and logist plots for the various head injury assessment functions have been previously presented in Reference 11. The various risk curves are reproduced in Appendix B. Each of these, based upon this data set, represents the probability of a mild traumatic brain injury being associated with a specific value of the injury measure. The MTBI-HIP$_m$ risk curve is provided in Figure 2.

![Figure 2: Probability of Concussion Based on HIP$_m$](image)

This curve demonstrates a gradual increase in the probability of MTBI with increasing HIP$_m$ with a 50% probability at 12.8kW. The slope of the curve is relatively shallow as the computed results are spread over a wide range. These results however are based solely on equation 7 as shown. However, if certain assumptions are made regarding directional sensitivity, the form and significance of this curve can change.

DIRECTIONAL SENSITIVITY CONSIDERATIONS

If one were able to assign specific degrees of sensitivity to each of the various degrees of head motion as suggested by equation 8, it would likely be possible to increase the predictability of the Power Index.

Even in the absence of detailed directional sensitivity data, one can presume that the head is less sensitive to impact in the A-P direction than for lateral impacts. To examine the possible influence of such a premise, equation 7 was recalculated and the logistic analysis conducted again for the case where it is assumed that the head is actually completely insensitive to impacts in the A-P direction. This was achieved simply by setting the linear x and angular y power terms equal to zero. The results of this approach are shown in Figure 3.

![Figure 3: Probability of Concussion Based on HIP$_m$ Without Frontal Impact Terms](image)

It will be observed that this risk curve is much steeper. That is, it is a more sensitive predictor of MTBI. This strongly supports the idea that the head indeed is less sensitive to frontal impacts than to lateral. Eliminating the A-P terms also has the expected effect of lowering the maximum HIP to produce a specific injury probability. A similar exercise could be conducted eliminating the vertical components though this has not been undertaken at this time.

A second consideration in the present regard is the influence of the rotational components in the power equation. Generally for this data set, these components do not contribute substantially to the total power transmitted to the head. Additionally, current standards for head protection, such as SI and HIC, do not invoke rotational components. Thus equation 7 has been exercised for all rotational accelerations set to zero. The same logistic regression analysis procedure was repeated for this linear impact power function. The resulting probability curve is shown in Figure 4.
Without Angular Terms

It will be noted that this form of the linear HIP$_m$ risk curve is indeed rather similar to that of figure 2. Overall the risk, as would be expected, is higher for a given value of HIP$_m$. That is, disregarding the rotational terms, results in a lower value of the maximum head impact power to produce the same risk of MTBI.

The above cases are provided to illustrate the flexibility and generality of the HIP concept in that it can be “calibrated” to reflect different directional sensitivities of the head to impact loading. However, no matter how powerful the HIP concept may be, it will not be considered a serious candidate to replace the incumbent HIC unless it can be shown to be better in many ways.

The current data set has been analyzed with regard to HIC and the corresponding risk curve is shown in Figure 5.

Figure 5: Probability of Concussion Based on HIC

The probability of concussion associated with this curve is somewhat higher for a given HIC than what is typically expected. This is most likely because the particular data set is dominated by lateral head impacts and if in fact the head is less tolerant to such impacts, this is not unreasonable. HIC is based solely upon the resultant linear acceleration of the head. Thus it cannot be “tuned” for different directional features nor can it incorporate the influence of rotational motion.

One way to compare the significance of the HIP variations and HIC is through statistical means. For the four cases considered here, the actual computed data upon which the regression analyses were based is provided in Table 1. The statistical variables of significance are provided in Table 2.

Table 1: Calculated MTBI Assessment Data

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Reported MTBI</th>
<th>HIP$_m$</th>
<th>HIP$_m$ nonA-P</th>
<th>HIP$_m$ Linear</th>
<th>HIC</th>
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<tbody>
<tr>
<td>07-2</td>
<td>0</td>
<td>6.7</td>
<td>5.7</td>
<td>5.0</td>
<td>93</td>
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<td>38-2</td>
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<td>39-2</td>
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<td>19.4</td>
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<tr>
<td>48-2</td>
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<td>7.0</td>
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<td>130</td>
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<td>57-2</td>
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<td>11.0</td>
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<tr>
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<td>9.3</td>
<td>187</td>
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Table 2: Comparison of Concussion Risk Functions

<table>
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<th></th>
<th>HIP$_m$</th>
<th>HIP$_m$ non A-P</th>
<th>HIP$_m$ Linear</th>
<th>HIC</th>
</tr>
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<tbody>
<tr>
<td>Probability</td>
<td>12.8</td>
<td>9.6</td>
<td>11.0</td>
<td>240</td>
</tr>
</tbody>
</table>

$^5$ All HICs were maximized in a time interval of less than 15msec.
The –2 Log Likelihood Ratio (-2LLR) provides a means of assessing whether adding an independent variable to the model improves the significance of the model. A smaller numerical value of –2LLR denotes a result of higher significance. An exact fit of the regression model to the data is associated with a zero value of –2LLR. A zero value of –2LLR is usually associated with dichotomous (1/0) data that do not overlap, i.e., those in which the highest observed value of the independent variable associated with no injury (0), is less than the lowest observed value associated with injury (1). A regression analysis of such data is of little practical value, since its outcome is readily predictable.

The significance (p-value) is often used as a screening tool in regression analyses, with a suggested threshold of \( p = 0.25 \) for the inclusion of an independent variable in the model. In common with –2LLR, lower numerical values are associated with higher significance.

Two important observations are made with respect to these numbers. The exclusion of the A-P power terms yields a function with an even lower –2LLR and a p-value that is still considered significant. This implies that judicious selection of directional sensitivity data can improve the significance and utility of the HIP\(_m\)-MTBI risk curve. The exclusion of all the rotational terms has the expected effect of reducing the significance of the predictions but -2LLR is still lower than that of HIC. Hence the linear HIP\(_m\) is still more significant than HIC.

**DISCUSSION**

**CLINICAL DATA ACCURACY**

As discussed above, the results herein are presented as dichotomous variables, i.e., as 1 when MTBI is diagnosed, or 0 when MTBI is not diagnosed. In the latter case it has been generally assumed that no MTBI in fact occurred. Following the calculation of all the injury parameters, each case was reviewed to verify, to the extent possible, the actual presence or absence of MTBI. In one case, for which all the calculated injury parameters were quite low and a concussion had been diagnosed, it was subsequently determined that the player did not actually suffer any neuropsychological deficits following impact and did not miss a single play. The following day during routine medical follow-up the player complained of a headache thus, in the interests of safety, the physician included him in the MTBI dataset. Following this review, we reclassified this case as a non-MTBI. In another case, the player had not been examined by medical staff at game time and was not included in the MTBI cases. However subsequent detailed review of the game tapes showed that the player exhibited decerebrate posturing immediately upon striking the ground and had to be helped off the field by training staff. This incident was reclassified from non-MTBI to MTBI.

**STATistical Relevance**

Examination of the probability curves in Figures 2 to 5 shows that the numerical values of injury parameters overlap the MTBI and non-MTBI cases. That is, there are always some non-MTBI cases with higher numerical injury assessment values than some MTBI cases (and vice versa). It is this feature that results in a smooth continuous curve for the concussion probability curves. Before deciding that the new injury assessment function HIP\(_m\) is a suitable replacement for HIC, or any of the others considered, it is instructive to note that if the data for the case of the highest injury value with no concussion and the one with the lowest value with a concussion had both been not collected, all the injury assessment functions would produce what would be essentially a step function. In such cases, regression analysis per se becomes meaningless as each function simply provides a distinct number above which a concussion is assured and for below which the probability of concussion is zero. This kind of speculation is provided simply to point out how sensitive these analyses are to the volume of data that is available. Any of these correlations have a better chance of being more meaningful the more data there is.

It will be noted further that, high values of the injury parameters do not correspond to more severe concussions. In addition, this database is of a rather select sample. All impacts are between the helmeted heads of opposing players. MTBIs occurred only to the players whose head sustained a significant lateral component. The head of the player with the highest HIP\(_m\) without a concussion underwent acceleration mainly in the fore-aft direction.

**CONCLUSIONS**

A new head injury assessment function, the maximum head impact power HIP\(_m\), has been postulated. By incorporating the scalar sum of the power terms for all six degrees of freedom, a function has been developed that, based upon a new set of mild traumatic brain injury data, appears to correlate better than existing head injury assessment functions.

The maximum head impact power index can be further refined by including directional sensitivity coefficients.
One approach toward achieving this would be to employ modern mathematical models such as the Wayne State finite element brain model [14]. By exercising the model for discrete and well-defined directional inputs, these effects will become apparent. It is planned to undertake such an analysis. When completed, an even better head impact power index may be possible.

As with all the injury assessment functions, the HIP is very sensitive to the amount of data available. The investigators are continuing to reconstruct cases of MTBI that occur to professional football players and expect to add to this database during the next two years. In addition however, data sets that are currently available for head injury research but which have only been subject to analysis through the HIC approach, can also be examined in light of this new proposal. It is hoped that those who have access to such data, might consider applying the power index approach.

ACKNOWLEDGMENTS

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REFERENCES

Appendix A: Rate of Change of Energy as a Possible Brain Injury Criterion

When speaking of the kinematics of the head, one is typically discussing the rigid body motion of the skull. How the change of kinetic energy of the skull might relate to the deformation of and injury to the non-rigid brain matter should be considered. One way to do this is by consideration of the “Viscous Criterion”.

Proposed by Lau and Viano in 1986 [8], this function states that a certain level or probability of injury will occur to a viscous organ if the product of its compression C and the rate of compression V exceeds some limiting value. The injury criterion hence takes the form

\[(VC)_{\text{max}} < K\]  
(A1)

This function has been validated for a number of different body regions including the thorax and the abdomen [9]. Viano has proposed that it could also apply to the brain [10].

For mild traumatic brain injury, it may be assumed that the distortions within the brain are small. We will assume further that for such distortions, a linear relation exists between force and displacement. That is

\[F = kx\]  
(A2)

If we assume that brain matter for these small displacements acts in a Newtonian manner, then the force acting upon an elemental mass of brain will cause it to accelerate according to

\[F = ma\]  
(A3)

Hence for this case, the Viscous Criterion simply takes the form

\[VdV/dt = P\]  
(A4)

or

\[d(V^2/2)/dt = P\]  
(A5)

Thus according to the viscous criterion, an injury severity/probability for brain tissue will be exceeded if the rate of change of kinetic energy of an elemental mass exceeds some limiting value. Alternately, equation A4 may be expressed simply as

\[V_\text{a} = P\]  
(A6)

or

\[a\text{d}t = P\]  
(A7)

As with the Viscous Criterion, we can compute the value of P as a function of time and require that the maximum value not exceed some limiting value. In other words, the power criterion simply becomes

\[(a\text{d}t)_{\text{max}} < I\]  
(A8)

Identical logic would prevail for rotational acceleration and power dissipated through that kind of motion would have a criterion function of the form.

\[(\alpha\text{d}t)_{\text{max}} < J\]  
(A9)

In spite of its name, there is no requirement that it apply only to viscous substances. Since V is simply the derivative of C(t), the non-dimensionalized displacement time history, (which can be generated by any mechanical system), C completely defines the value of the criterion function.
Appendix C: Measured and Computed Acceleration and Power Time Histories

Case: 07-1

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 38-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case 39-1

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case 39-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 48-1

- Linear Acceleration X
- Angular Acceleration X
- Linear Acceleration Y
- Angular Acceleration Y
- Linear Acceleration Z
- Angular Acceleration Z
- Linear Resultant
- Angular Resultant
- Power Time History
Case: 57-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 69-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 71-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 77-1

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 77-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 84-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 92-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History
Case: 98-2

Linear Acceleration X

Angular Acceleration X

Linear Acceleration Y

Angular Acceleration Y

Linear Acceleration Z

Angular Acceleration Z

Linear Resultant

Angular Resultant

Power Time History