Signal Processing Algorithm of STC Waveforms for the Phased array MIMO Radar: Overview on Target Localization

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Abstract: The MIMO phased array radar is a theoretical concept for multi-sensor radar planning that combines elements of traditional phased-array radar with the developing technology of multiple-input multiple output (MIMO) radar. Space-time coding (STC) has been shown to play a key role in the design of Phased Array MIMO radar where MIMO radar can achieve improved localization performance by employing a coherent processing approach with proper antenna positioning. Coherent processing, however, entails the challenge of ensuring phase coherence of the carrier signals from different distributed radar elements. In recent years, studies have shown how the performances of conventional phased-array radar can be improved by using the same approach. We consider the use of STC to mitigate the waveform cross-correlation effects in MIMO radar. In addition, we also extend the model to partial waveform cross-correlation removal based on waveform set division. Numerical results demonstrate the effectiveness of STC in MIMO radar for waveform decorrelation. In this paper, the aim is to shed some light on the optimum transmit policy as the radar is to detect a target at an unknown location: To this end, at first the Cramer–Rao bounds as a function of the STC matrix are computed to providing a systematic treatment of the phase synchronization problem in coherent MIMO radar systems. This paper introduces the signal processing issued for the phased array MIMO radar without and with STC waveforms and also studied phased array MIMO radar with STC waveforms improved target detection and recognition performance.

Index Terms: MIMO, STC, Phased array, Radar, Probability detection and SNR.

I. INTRODUCTION

Phased array system has been proposed in the 1950’s and is widely used in communication and radar applications. For communication applications, phased array systems have been employed on either the transmitter or receiver side, or both. Phased array antennas are finding their way more frequently into radar systems [1], [2]. These types of antennas are constructed by assembling radiating elements in a geometrical configuration which are shortly referred as an array. In most cases, the elements of the array are identical and they are placed in a regularly arranged planar grid. The total field of the array is determined as the superposition of the fields radiated by the individual elements. In the receiver side of phased array systems, strong interferes from different directions can be placed in the nulls of a radiation pattern to nullify interfere with the desired signals. Another fundamental merit is the improvement of the effective signal-to-noise ratio (SNR) at the output of the receiver by 10log (N) (dB) because the time-delayed signals from the antenna array add coherently while the noise generated by each receiver chain for the antenna elements adds incoherently, hence increasing a channel capacity. For the transmitter side, higher special power efficiency for the phased array systems creates less interference to nearby communication system. To compensate for the time delay between the antenna elements, a phase shifter can be used instead of a variable time delay element for narrow-band applications. The spacing between the antennas is usually chosen to λ/2 be close to for high antenna directivity and no grating lobes. STC for waveform design has been introduced in [3] to cope with detection under possibly correlated clutter and, more generally, to introduce a further degree of freedom at the transmitter side. STC is a revolutionary development for
exploiting the MIMO channel by using antenna array processing technology, which is currently stimulating considerable interest across the wireless industry. STC ‘concept’ builds on the significant work by winter’s in the mid-80’s which highlighted the importance of antenna diversity on the capacity of wireless systems [4]. The use of multiple antennas at both the transmitter and receiver is essential for the STC concept to work effectively, since STC exploits both the temporal and spatial dimensions for the construction of coding designs which effectively mitigate fading (for improved power efficiency) and are able to capitalize upon parallel transmission paths within the propagation channel (for improved bandwidth, efficiency). Consider two radar scenarios: a phased array, where the same waveform is transmitted from each antenna; and orthogonal MIMO radar [5], whereby independent waveforms are transmitted from each antenna. A major benefit of phased array MIMO radar is the increment of signal power incident on the target due to beam forming gain; whereas the conventional phased array radar cannot beam form on transmit resulting in a comparative reduction in SNR.

In this paper, the problem of target detection in co-located MIMO phased array radar is considered. A pulse-train signaling [6] is assumed to be used in this system. STC [7] is a MIMO technique that is designed for use with multiple transmitter antennas. We focus on the signal processing strategy for target detection performance [8] of Phased array MIMO radar with STC waveforms over conventional phased array MIMO radar [9]. The received signal modeling and problem formulations are presented. Scaled versions of a single waveform are transmitted by phased-array MIMO radar antenna. It is shown that due to unknown values of Doppler frequency and target DOA, a compound hypothesis testing problem is confronted. The standard technique for compound hypothesis tests when the Probability Distribution Function (PDF) of the unknown parameters is not known is the Generalized Likelihood Ratio (GLR) detection. Full exploitation of these potentials can result in significant improvement in target detection, parameter estimation, target tracking and recognition performance. This is the motivation for MIMO phased array radar using pulse-train signaling.

## II. PERFORMANCE ANALYSIS

### A. Phased array radar

Phased array radar uses antenna arrays for transmitting and receiving signals. These arrays may be linear or planar. In both the linear and planar arrays the separation between the elements is usually uniform. These arrays may be co-located and even transmit and receive functions can be performed by the same array. The two arrays may also be widely separated allowing the radar system to operate in bistatic mode.

#### System Model

The model of the multistatic phased array radar system is shown in fig. 1 that has M transmits and N receives elements. Assume that transmit and receive arrays are uniform linear arrays with inter element spacing of \(d_t\) and \(d_r\) respectively. Since the inter-element spacing of phased array radar antennas is small, the bistatic RCS seen by every transmit-receive pair in a phased array radar system is assumed to be the same.

If the transmit array performs transmit beam-forming in the direction of \(\theta\) the transmitted signal \(\tilde{x}(t)\) can be written in the vector form as

\[
\tilde{x}(t) = a(\theta) \sqrt{\frac{E_t}{M}} x(t) ... ... ... ... (1)
\]

Where, \(\sqrt{\frac{E_t}{M}} x(t)\) denote the discrete time baseband signal transmitted by the transmit antenna elements where \(x(t)\) is the input message signal, \(E_t\) is the total average transmitted energy and where \(a(\theta)\)is the transmitter steering vector.

![Fig. 1.Phased Array Radar Configuration.](image-url)
Then the received signal model becomes

\[ y = \frac{E_t}{M} NM \alpha + w \] \hspace{1cm} (2)

Where, \( w \) is a zero mean vector of complex random processes. Note that if \( \alpha \), where \( \alpha \) is a zero mean complex normal random variable, is small, the amplitude of the received signal will be small despite this processing gain and detection probability will decrease dramatically.

**Probability Detection**

The detection problem in phased array radar can be formulated as binary hypothesis testing problem:

\[ H_0 : \ y = \frac{E_t}{M} w \]
\[ H_1 : \ y = \frac{E_t}{M} MN \alpha + w \] \hspace{1cm} (3)

Where \( H_0 \) indicates absence of signal and \( H_1 \) indicates presence of signal.

Assume that \( \alpha \) is a zero mean complex normal random variable with a variance of \( \sigma_\alpha^2 = 1 \).

It is well known that the optimum solution to this hypothesis testing problem under Neyman-Pearson criterion is the Likelihood Ratio Test (LRT) as,

\[ \frac{p(y|H_1, \sigma_\omega^2, \sigma_\alpha^2)}{p(y|H_0, \sigma_\omega^2)} \gtrless \frac{H_1}{H_0} T \] \hspace{1cm} (4)

In this case the distributions of \( \alpha \) and \( w \) are known, So the likelihood ratio test [4] can be written as,

\[ |y|^2 \gtrless \frac{H_1}{H_0} T' \] \hspace{1cm} (5)

The false alarm rate \( (P_{fa}) \) can be defined in the term of threshold \( T' \) as,

\[ = \exp \left( \frac{-T'}{N\sigma_\omega^2} \right) \] \hspace{1cm} (6)

Where, \( T' = -N\sigma_\omega^2 \ln (P_{fa}) \)

The probability detection \( (P_d) \) can be calculated in terms of threshold \( T' \) as,

\[ = \exp \left( \frac{-T'}{E_t MN^2 + N\sigma_\omega^2} \right) \] \hspace{1cm} (7)

Equivalently probability detection \( (P_d) \) can be written in terms of \( P_{fa} \) and SNR as,

\[ P_d = \exp \left( \frac{\ln (P_{fa})}{(SNR) MN + 1} \right) \] \hspace{1cm} (8)

**Results and Observation**

To illustration the probability detection performance of phased array radar, the detector in equation (8) is implemented for which \( P_{fa} \) value is set to \( 10^{-2} \). The resulting \( P_d \) vs. SNR curve is represented in fig. 2.

**Fig. 2. Probability of detection for phased array radar, changing M.**

The detection performance of the phased array radar system enhances as the number of transmit antennas increases although the transmitted power is constant. The gain increases as the number of transmit antennae increases although the noise power in the received signal remains constant.

**Fig. 3. Probability of detection for phased array radar, changing N.**

If the number of transmit elements is held constant at the value of 5 and the number of receive elements is increased, the \( P_d \) vs. SNR curve in fig. 3 is obtained. We
can see from the graph that as number of receiving antennas is increased the probability of detection increases, because the total received energy increases.

B. Phased array MIMO Radar with STC waveforms

In the detection problems studied so far for the phased array MIMO radar which employs linear or planer antenna arrays are may be co-located and even transmit and receive functions can be performed by the same array, are developed without including these STC signals explicitly.

In the transmitted signals are modeled as a train of rectangular pulses whose amplitudes are modulated by space time codes and the corresponding detectors are developed. With this approach, the transmitted signals can be further optimized to better a given performance metric. The STC Phased array MIMO radar configuration is shown in Fig. 4.

2.2.1. System Model

Consider a phased array MIMO radar with STC waveforms system that has transmit and a receive array consisting of M and N elements respectively. The received signal is also scaled so that the total received signal increases directly proportional to rectangular pulses. Then the received signal model becomes

\[ y = \sqrt{\frac{n \cdot E_{t}}{M}} NM\alpha + w \ldots \ldots \ldots \ldots \ldots \ldots (9) \]

2.2.2. Probability Detection

The detection problem here can be formulated as binary hypothesis testing problem as follows:

\[ H_0 : y = w \]
\[ H_1 : y = \sqrt{\frac{E_t}{M}} MN\alpha + \omega \]

Where \( H_0 \) indicates absence of signal and \( H_1 \) indicates presence of signal.

To see the performance limit of coherent MIMO radar, the vector \( \alpha \) become identical and coherent integration of the received samples becomes possible before detection process and \( w \) is now a complex number.

For phased array MIMO radar with STC waveforms, from the definition of SNR for the radar system is

\[ (SNR)_{STC} = \frac{n E_{t}}{\sigma^2} = n \cdot SNR \ldots \ldots \ldots \ldots (11) \]

Then probability detection(\( P_d \)) can be written in terms of \( P_{fa} \) and SNR as,

\[ P_d = \exp\left(\frac{\ln(P_{fa})}{(n \cdot SNR)M + 1}\right) \ldots \ldots \ldots (12) \]

So, the probability of detection of phased array MIMO radar depends on number of transmit and receive antennas, SNR and as well as number of rectangular pulse trains.

![Fig. 4: STC Phased array MIMO radar configuration](image)

![Fig. 5: Probability of detection plot for phased array MIMO radar with STC waveforms, changing M.](image)
**Results and Observation**

To compare with the detection performance of phased array MIMO radar with STC waveforms, the probability detection in equation (12) is implemented for which $P_d$ value is set to $10^{-2}$. If the number of receiving elements is held constant at the value of 5, and the number of transmitting elements is increased, the $P_d$ vs. SNR curve in Fig. 5 is obtained.

It is interesting to see that the detection performance increases with increasing number of transmitting antennas.

If the number of transmitting elements is held constant at the value of 5, and the number of receiving elements is increased, the $P_d$ vs. SNR curve in Fig. 6 is obtained. It is interesting to see that the detection performance increases as the number of receiving antennas increases. Because of these increases in the detection performance; using more widely separated receiving antennas instead of increasing the number of spatially diverse transmitting antennas seems more reasonable.

![Fig. 6: Probability of detection for phased array MIMO radar, changing N.](image)

The ROC of phased array MIMO radar versus phased array MIMO radar with STC waveforms and also comparison of probability detection of phased array MIMO radar with and without STC waveforms are is given in Fig. 7 and Fig. 8 respectively. These figures are obtained using the analytical expressions given in Equations (9) and (12) for $M = N = 5$. In the both figures, the blue lines belong to phased array MIMO radar with STC waveforms and the red lines belong to phased array MIMO radar without STC waveforms.

![Fig. 7: ROC- phased array MIMO radar with and without STC waveforms.](image)

![Fig. 8: Comparison of probability detection for phased array MIMO radar with and without STC waveforms.](image)

### III. Conclusions

In this paper, the analytical description of the coherent MIMO approach to phased-array radars for pulse-train signaling with coherent beam has been provided. With respect to the retrieved representation, a design procedure for phase array MIMO for pulse-train signaling with coherent beam structures was introduced. Practical realization of the coherent processing in MIMO radar systems requires the development of implementable techniques to ensure a common notion of phase among all the distributed radar elements. In this work, we proposed and studied effective approaches to achieve...
phase synchronization in coherent MIMO radar systems by using phase array radar for pulse-train signaling with coherent beam.

**REFERENCE**


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