Order Bi-spectrum For Bearing Fault Monitoring and Diagnosis Under Run-up Condition

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Abstract—Varying speed machinery condition detection and fault diagnosis are more difficult due to non-stationary machine dynamics and vibration. Therefore, most conventional signal processing methods based on time invariant carried out in constant time interval are frequently unable to provide meaningful results. This paper deals with the detection of bearing faults in gearbox under non-stationary run-up of gear drives. In order to process the non-stationary vibration signals such as run-up or run-down vibration signals effectively, the order bi-spectrum technique is presented. This new method combines computed order tracking technique with bi-spectrum analysis. First, the vibration signal is sampled at constant time increments during run-up of gearbox and then uses numerical techniques to resample the data at constant angle increments. Therefore, the vibration signals are transformed from the time domain transient signal to angle domain stationary one. Second, the re-sample signal is processed by bi-spectrum analysis method. The procedure is illustrated with the experimental vibration data of a gearbox. The experimental results show that order bi-spectrum technique can effectively diagnose and diagnosis the faults of bearing.

Index Terms—fault diagnosis, gearbox, bearing, vibration, signal processing, order tracking, bi-spectrum

I. INTRODUCTION

Rotating machine fault diagnosis is typically based on vibration. The spectral contents of emitted vibration signals are analyzed to ascertain the current condition of the monitored process. At present, for the fault diagnosis of rotating machinery, many research outcomes have been obtained in the stationary process. However, little research has been done for monitoring the vibrations of varying speed condition such as the run-up or run-down process. The reason why we stress the run-up or run-down process is that non-stationary vibrations signals from varying speed machinery may include more abundant information about its condition. Some phenomena, which are usually not obvious at constant speed operation, may become more apparent under varying speed conditions. Therefore, the behavior characteristics of the run-up or run-down process have a distinct diagnostic value, and the fault diagnosis of run-up or run-down process has owed its distinct standing in the fault diagnosis of rotating machinery. In the last decade vibration analysis and condition monitoring techniques for varying speed machinery have attracted the attention of scientists and engineers. Lopatinskaia et al. [1,2] presented the application of recursive filtering and angle domain analysis to non-stationary vibration analysis. The approach is implemented and validated through computer simulation and experiments. Meltzer [3,4] dealt with the recognition of faults in gear tooth during non-stationary start-up and run-down of planetary gear drives using the time-frequency approach and the time-quefrency approach. Wu et al. [5] presented the application of adaptive order tracking fault diagnosis technique based on recursive Kalman filtering algorithm to gear-set defect diagnosis and engine turbocharger wheel blades damaged under various conditions. Li et al. [6] presented the hidden Markov model-based fault diagnosis method in speed-up and speed-down process for rotating machinery.

However, the vibration signal of the run-up or run-down process is more complex than that of the stationary process. Conventional signal processing methods, which were developed for constant speed machinery monitoring, are based on digital sampling carried out in equal time intervals. If the machine operates under varying speed or load, its dynamic and vibrations become non-stationary. The vibration signal sampled from the rotating machinery is a non-stationary signal, whose amplitudes and frequencies both vary with time. Fixed time sampling cannot cope with the varying rotational frequency of the machine, resulting in increasing leakage error and spectral smearing [1,2]. Therefore, most of the conventional methods for signal processing become inappropriate when monitoring the vibrations of varying speed machinery [1,2]. Some progress has been made in the theoretical analysis [7,8], the signal processing methodology [9,10], measurements and practical applications of varying speed machinery monitoring [11,12,13].

At present, two techniques are mainly used to process the non-stationary signal: time frequency analysis (such as the short time Fourier transform (STFT), wavelet transform (WT) [14], Wigner-Ville distribution (WVD) [15,16,17] and Hilbert-Huang transform [18,19,20]) and order tracking technique [11,12,13]. The time frequency analysis involves three-dimensional functions that allow for visualizing the frequency and amplitude variations of the spectral components [14]. However, when the
analyzed vibration signal is composed of many spectral components and with large changes of the machine speed during measurement, they become very difficult to analyze. Recently, order tracking has become one of the important methods for fault diagnosis in rotating machinery [11,12,13]. Vibration signals produced from rotating machinery are speed dependent and hence orders as opposed to absolute frequencies are preferred as the frequency base. Orders represent the number of cycles per revolution and are thus ideal for representing speed-dependent vibrations. Therefore, order tracking normally exploits a vibration or a noise signal supplemented with the information of shaft speed for fault diagnosis of rotating machinery. The order spectrum gives the amplitude of the signal as a function of harmonic order and shaft speed in rotating machinery [11].

In this work, the computed order tracking approach and bi-spectrum analysis are introduced and applied specifically to gearbox fault diagnosis during run-up. This method is based on the re-sampling technique and the bi-spectrum estimation of the re-sampling signal, which is a function of the angle of the input shaft of the gearbox. This re-sampling signal can be obtained by resampling of the vibration signal that has been previous sampled in the time domain. The order power spectrum and order bi-spectrum techniques are based on the signal processing of the angle domain signal, where the resample signal is in accordance with the shaft angle of the gearbox. The order power spectrum and order bi-spectrum are then evaluated for the vibration signal re-sampled constantly in angle at equidistant phases of the input shaft of the gearbox. In this case, the results of the order power spectrum or order bi-spectrum are expressed as results of order analysis where the frequency axes are changed to the axes of orders independent of the input shaft speed. The usefulness of this approach will be shown by experimental example in Section VI.

To address the issues discussed above, this paper is organized as follows. Section I gives a brief introduction of the order tracking analysis technology. Section II briefly describes the bi-spectrum. Section III presents the principles and procedure of the computed order tracking. Section IV gives the method and procedure of the fault diagnosis based on computed order tracking and order bi-spectrum. Section V looks at the experimental set-up. Section VI gives the applications of the method based on computed order tracking and order bi-spectrum to faults diagnosis of bearing faults. Finally, the main conclusions of this paper are provided in Section VII.

II. A BRIEF INTRODUCTION OF BI-SPECTRUM

Let \( \{x(n)\} \) be a real, discrete, zero-mean stationary process with third-order cumulant \( R_{xx}(\tau_1, \tau_2) \) defined as [21]

\[
R_{xx}(\tau_1, \tau_2) = E[x(n)x(n + \tau_1)x(n + \tau_2)]
\]

Then the bi-spectrum of \( \{x(n)\} \) is given by the expression

\[
B_{xx}(\omega_1, \omega_2) = \sum_{\tau_1 = -\infty}^{\infty} \sum_{\tau_2 = -\infty}^{\infty} R_{xx}(\tau_1, \tau_2) e^{-j(\omega_1 \tau_1 + \omega_2 \tau_2)}
\]  

(2)

where \( |\omega_1| \leq \pi, |\omega_2| \leq \pi, |\omega_1 + \omega_2| \leq \pi \).

Therefore, in the same way that the power spectrum decomposes the power of a signal, the bi-spectrum decomposes the third-order cumulant. The bi-spectrum is a function of two frequency variables, \( \omega_1 \) and \( \omega_2 \), and whilst the power spectrum includes the contribution of each individual frequency component independently, the bi-spectrum analyses the frequency interaction between the frequency components at \( \omega_1 \), \( \omega_2 \) and \( \omega_1 + \omega_2 \) [22-23].

III. THE PRINCIPALS OF COMPUTED ORDER TRACKING

There are two popular techniques for producing synchronously sampled data: the traditional approach that uses special hardware to dynamically adapt the sample rate and a technique where the vibration signals and a tachometer signal are synchronously sampled, that is, they are sampled conventionally at equal time increments. From the synchronously sampled tachometer signal resample times required to produce synchronous sampled data are calculated. This process is referred to as computed order tracking and is particularly attractive, as it requires no special hardware. Also, this approach is more flexible than the traditional method, as for example different sample rates may be synthesized. The computed order tracking is considerably more flexible than the traditional approach. It may be organized to produce equally accurate or more accurate results than the traditional method. An added benefit is that computed order tracking requires no specialized hardware, which is an important factor in many conditions monitoring applications. Therefore, computed order tracking techniques are introduced and applied in this paper.

The objective of computed order tracking (COT) [9] is a calculation of the vibration signal sampled constant in angle from sampled constant in time. From the mathematical point of view, this task could be solved by interpolation theory.

To determine the resample times, it will be assumed that the shaft is undergoing constant angular acceleration. With this basis, the shaft angle \( \theta(t) \) can be described by a quadratic equation of the following form [9]:

\[
\theta(t) = b_0 + b_1 t + b_2 t^2
\]

(3)

where \( b_0, b_1 \) and \( b_2 \) are unknown coefficients, which are found by fitting three successive key-phaser arrival times ( \( t_1, t_2 \) and \( t_3 \) ) which occur at known shaft angle increments \( \Delta \phi \). This can be obtained by the following conditions:
\[
\begin{align*}
\theta(t_1) &= 0 \\
\theta(t_2) &= \Delta \phi \\
\theta(t_3) &= 2\Delta \phi
\end{align*}
\]

Substituting these conditions into Eq. (3) and arranging in a matrix format gives,

\[
\begin{bmatrix}
0 \\
\Delta \phi \\
2\Delta \phi
\end{bmatrix} =
\begin{bmatrix}
1 & t_1 & t_1^2 \\
1 & t_2 & t_2^2 \\
1 & t_3 & t_3^2
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
b_2
\end{bmatrix}
\]

This set of equations is then solved for the unknown \( \{b_i\} \) components. Once these values are known, Eq.(3) may be solved for \( t \), yielding

\[
t = \frac{1}{2b_2} \left[ 4b_2(k\Delta \theta - b_0) + b_1^2 - b_1 \right]
\]

where \( k \) is the interpolation coefficient which can be obtained as follow

\[
\theta = k\Delta \theta
\]

where \( \theta \) is the shaft angle and \( \Delta \theta \) is the desired angular spacing between re-samples.

Once the resample times are calculated, the corresponding amplitudes of the signal are calculated by interpolating between the sampled data. After the amplitudes are determined, the re-sample data are transformed from the angle domain to the order domain by means of an FFT.

The order spectrum and order bi-spectrum techniques are based on the signal processing of the angle domain signal, where the resample signal is in accordance with the shaft angle of the gearbox. The order spectrum and order bi-spectrum are then evaluated for the resample signal. The usefulness of this approach will be shown with an experimental example in Section VI.

IV. PROPOSED ORDER BI-SPECTRUM METHOD FOR FAULTS DETECTION OF BEARING

The procedure of proposed order bi-spectrum method is given as follows:

1) Non-stationary vibration signal under run-up condition is sampled using a constant time increment;
2) Non-stationary vibration signal is re-sampled at a constant angle increment. Then the non-stationary vibration signal in time domain is transformed into stationary one in angle domain;
3) To demodulate the constant angle increment signal using Hilbert transform;
4) The order bi-spectrum is calculated according to Eq. (2);
5) The diagnostic conclusions are drawn according to the order bi-spectrum.

V. EXPERIMENTAL SET-UP

The test apparatus used in this study is shown in Fig.1 [24,15]. The experimental set-up consists of a single-stage gearbox, driven by a 4.5 kW AC governor motor. The driving gear has 30 teeth and the driven gear has 50 teeth. Therefore, the transmission ratio is 50/30, which means that an decrease in rotation speed is achieved. The module of the gear is 2.5 mm. The monitoring and diagnostic system is composed of three accelerometers, amplifiers, a speed and torque transducer, B&K 3560 spectrum analyzer and a computer. The sampling span is 3.2 kHz, the sampling frequency is 8192 Hz and the sampling time is 2 seconds. This time included one speed up of the gearbox from idle speed up to steady. After sampling, the measured vibration signals were loaded into MATLAB from data-files. Then, the vibration signals were re-samples. For their re-sampling, the algorithm described in the previous section was used. As a result of experiment, the vibration signals generated by the tested gearbox were obtained sampled constant in time as well as sampled constant in angle.

VI. BEARING FAULTS DIAGNOSIS BASED ON ORDER BI-SPECTRUM

In this section, the order power spectrum and order bi-spectrum will be applied to vibration signal analysis measured from a gearbox during speed-up process.

Ball bearings are installed in many kinds of machinery. Many problem of those machines may be caused by defects of the ball bearing. Generally, localized defects may occur on inner race, outer race or rollers of bearing. A local fault may produce periodic impacts, the size and the repetition period which are determined by the shaft rotation speed, the type of fault and the geometry of the bearing. The successive impacts produce a series of impulse response, which maybe amplitude modulated because of the passage of fault through the load zone. The spectrum of such a signal would consists of a harmonics series of frequency components spaced at the component fault frequency with the highest amplitude around the resonance frequency. These frequency components are flanked by sidebands if there is an amplitude modulation due to the load zone. According to the period of the impulse, we can judge the location of the defect using characteristic frequency formulae.

![Figure 1. Experimental set-up](image-url)
The tested bearing was used to study only one kind of surface failure: the bearing was damaged on the inner race or outer race. The ball bearing tested has a groove on the inner race or outer race. Localized defect was seeded on the inner race or outer race by an electric-discharge machine to keep their size and depth under control. The size of the artificial defect was 1 mm in depth and the width of the groove was 1.5 mm. The type of the ball bearing is 206. There are 9 balls \( z=9 \) in a bearing and the contact angle \( \alpha = 0^\circ \), ball diameter \( d = 9.5 \text{mm} \), bearing pitch diameter \( D = 41.75 \text{mm} \). Then the characteristic frequency of the inner race or outer race defect can be calculated according to the Eq.(8), Eq.(9), respectively.

\[
f_{\text{inner}} = \frac{z}{2} \left(1 + \frac{d}{D} \cos \alpha \right) f_r \tag{8}
\]

\[
f_{\text{outer}} = \frac{z}{2} \left(1 - \frac{d}{D} \cos \alpha \right) f_r \tag{9}
\]

where \( f_r \) is the rotating frequency of the input shaft.

Therefore, according to Eq.(8) and Eq.(9), the characteristic frequency of the inner race and outer race defect are given as follows:

\[
f_{\text{inner}} = 5.42 f_r \tag{10}
\]

\[
f_{\text{outer}} = 3.58 f_r \tag{11}
\]

Then the characteristic order of the inner race and outer race are obtained as follows:

\[
O_{\text{inner}} = 5.42 \tag{12}
\]

\[
O_{\text{outer}} = 3.58 \tag{13}
\]

A. Application of Order Bi-spectrum to Fault Diagnosis of Inner Race

The rotating speed signal of the input shaft for the tested gearbox is displayed in Fig.2. Fig.2 (a) represents the sampling pluses of the input shaft from the optical encoder (60 pulses per rotational period). The encoder signals consist of 16384 points and have a total duration of 2 seconds. To obtain approximate values of rotational speed for every data point, polynomial curve fitting was used. It was found that linear approximate was sufficient for this research. polynomial coefficients were determined for each data and analytical descriptions of the rotational speed were obtained. Fig.2 (b) is the calculated instantaneous rotating speed using interpolating method. Fig.2 (b) clearly shows that the rotating speed of the input shaft runs up from idle to steady speed about 700 rpm.

The original vibration signal with inner race fault is displayed in Fig.3 (a). Fig.3 (a) shows that the vibration signals are non-stationary which the amplitude of the vibration is increasing during the input shaft speed up. The result of applying conventional spectral analysis (FFT) to the specified non-stationary signal is given in Fig.3 (b). Fig.3 (b) displays the FFT of the vibration signals with inner race fault. It is very clear that the resulting spectrum is significantly obscured by spectral smearing. Besides, traditional spectral averaging cannot be applied to the non-stationary signal during the input shaft run-up process. Fig.3 (b) clearly shows that spectral smearing substantially affects the result of conventional analysis based on time sampling. Therefore, classical Fourier analysis has some limitation such as being unable to process non-stationary signals.
The angular re-sampling technique is applied to the vibration signal of Fig.3 (a). Fig.4 displays the re-sample vibration signal with uniform angular increment of 0.008722 rad. It is clear that there are periodic impacts in the angle domain vibration signal. There are significant fluctuations in the peak amplitude of the signal. However, it is hardly possible to evaluate the bearing fault condition only through such angle domain vibration signal. Fig.5 shows the order power spectrum of the re-sample vibration signal. The order power spectrum, as shown in Fig.5, is dominated by the repetition order of the gear mesh order and its harmonics. It can be seen from Fig.5, that the order power spectrum represents the complicated quantities. This complexity of the order power spectrum follows from the frequency smearing and modulation effects. Therefore, the conventional order power spectrum was not capable of revealing the characteristic order of inner race fault that was corrupted by the modulation and noise.

The order bi-spectrum was evaluated according to the conventional direct method [2] after the re-sample signal has been demodulated by Hilbert transform. The order bi-spectrum is depicted in Fig.6 (contour plot) and Fig.7 (mesh plot). From Fig.6 we can see that the graphs of the order quantities are much simple than that of the order power spectrum of Fig.5. In case of the order bi-spectrum, it can be identified that the characteristic order ($O_{inner}$) of inner race fault and its harmonics are represented clearly in the order bi-spectrum. The simplicity of the order quantity representation can be put down to the ability of the order signal processing method to eliminate undesirable spectral smearing and modulation effects. Fig.6 and Fig.7 demonstrate the advantage of the order quantity application for the analysis vibration signals generated by gearbox under running up condition. Especially, the order bi-spectrum better identifies the order components and consequently leads to a better understanding of the transient vibration characteristics than that of the order power spectrum.
Figure 8(a) shows the original vibration signal with outer race fault during the input shaft speed-up. Fig. 8(b) displays the FFT of the vibration signal with outer race fault. It is clear that the resulting spectrum is the same as the inner race fault that is significantly obscured by spectral smearing.

Figure 9 displays the re-sample vibration signal with uniform angular increment. Fig. 10 is the order power spectrum of the re-sample vibration signal. The conventional order power spectrum was not capable of revealing the characteristic order of outer race fault in the same way. The order bi-spectrum is depicted in Fig. 11 (contour plot) and Fig. 12 (mesh plot), respectively. It can be seen clearly from Fig. 11 and Fig. 12 that there are the characteristic order ($O_{outer}$) of outer race fault and its harmonics. Therefore, the outer race fault can be easily detected by using order bi-spectrum. Fig. 11 and Fig. 12 demonstrate the advantage of the order bi-spectrum for the analysis of vibration signals generated by gearbox during run-up process.

VII. CONCLUSIONS

A method for fault diagnosis of bearing under run-up condition was presented based on a newly developed signal processing technique termed as computed order-tracking and order bi-spectrum. Using computed order-tracking technique, the non-stationary vibration signals of bearing faults in time domain can be transformed into stationary ones in the angle domain. The definition of the order bi-spectrum for analysis of vibration signals generated by rotating machinery was introduced. This method is based on the bi-spectrum estimation from the vibration signal sampled constant in angle with respect to the shaft speed of the gearbox. The order bi-spectrum method assists in the elimination of spectral smearing and modulation effects caused by the variation in shaft speed. The experimental results show that order bi-spectrum can be effectively used as a diagnostic feature for bearing faults.

ACKNOWLEDGMENT

The authors are grateful to the National Natural Science Foundation of China (No. 50975185), Zhejiang Provincial Natural Science Foundation (No. Y1080040). The authors are also grateful to the editors and reviewers for their constructive comments.

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