Basic Study of Improving Efficiency of Wireless Power Transfer via Magnetic Resonance Coupling Based on Impedance Matching

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Abstract—Wireless power transfer is essential for the spread of Electric Vehicle (EV) usage as it provides a safe and convenient way to charge the EVs. Recently, a highly efficient mid-range wireless power transfer technology using electromagnetic resonance coupling, WiTricity, was proposed. Studies show that the resonant frequencies of the two antennas change according to the air gap in between the antennas. To achieve maximum efficiency using this system, the resonance frequencies of the antennas and the frequency of the system has to be matched. However, when this technology is applied in the MHz range (which allows small sized antennas), the usable frequency is bounded by the Industrial, Scientific, and Medical (ISM) band. Hence a method to fix the resonance frequency within the ISM band is required. In this paper, the possibility of using impedance matching (IM) networks to adjust the resonance frequency of a pair of antennas at a certain distance to 13.56 MHz is studied. We studied the electrical characteristics of the antenna with equivalent circuits, electromagnetic analysis and experiments. The equivalent circuits are used as reference to calculate the parameters of the IM circuits. The simulations and experiments shows that the IM circuits can change the resonance frequency to 13.56 MHz for different air gaps, thus improving the power transfer efficiency.

Key Words—Wireless power transfer, Resonance, Coupling, Magnetic, Impedance matching,

I. INTRODUCTION

Nowadays, with the development of mobile appliances such as mobile phones and laptops, as well as the recent boom of EVs, the need for a technique to wirelessly charge these applications has increased. Wireless power transfer is essential in the spread of EV usage as it provides a safe and convenient way of charging EVs. An EV has to be charged a lot more frequently than a regular gasoline engine car as its energy storage medium (battery or capacitor) has a much lower energy density compared to petroleum. Therefore, a wireless power transfer system can be used in the automatic charging system which makes charging the EVs more convenient. Also, as the process of plugging the power cord into the socket will be unnecessary, the danger of being electrocuted due to the wear and tears of an old cord, or rain will be avoided, making the charging process safer. For example, the power transmitting antenna can be placed in the car park, and the receiving antenna can be placed in the car. This enables the EV to be charged wirelessly just by being in the parking box. To achieve this, the wireless power transfer system must satisfy these three conditions: high efficiency, large air gaps, and high power.

Presently, the most popular wireless power transfer technologies are the electromagnetic induction, and the microwave power transfer. However, the electromagnetic induction has a short range[5], and microwave power transfer has a low efficiency as it uses radiation. Recently, a highly efficient mid-range wireless power transfer technology using electromagnetic resonance coupling, WiTricity, was proposed. It satisfies all three conditions to make wirelessly charging EVs possible as it has a high efficiency at mid range (approximately 90% at 1m and 50% at 2m [1] at 60W).

Until now, this phenomenon was explained using the Mode Coupling theory. However, this theory is often complicated, and inconvenient when it comes to designing the circuits around the system.

In this paper, we study this phenomenon using antenna design theories and circuit design theories. The characteristics of the antennas are explained using equivalent circuits, electromagnetic analysis, and experiments. The frequency characteristics of the antennas and the relation to its efficiency are studied, and a tuning circuit is inserted to increase its efficiency by changing the resonance frequency to match the frequency of the power source. The parameters of the tuning circuit are calculated based on the equivalent circuit and impedance matching theories. Its effects are studied with experiments and simulations.

II. WIRELESS POWER TRANSFER SYSTEM USING MAGNETIC RESONANCE COUPLING

Wireless Power Transfer System

As shown in Fig. 1, the wireless power transfer system involves resonating two identical antennas with a high frequency power source. The power is transmitted through magnetic resonance coupling in between the two antennas at the resonance frequencies. The power transmitted to the receiving antenna is then rectified and used to charge energy
storage mediums such as batteries and electric double layer capacitors (EDLC).

In the system that involves changing the resonance frequency of the antennas, a directional coupler is installed before the transmitting antenna to measure the transmitted power and reflected power in between the antennas. The measured values are input into a computer (PC) which is used to control the parameters of the IM circuit. The IM circuit functions as a tuner to change characteristics of the antennas so that the resonance frequency can be adjusted.

**Equivalent circuits**

Electromagnetic resonance coupling involves creating an LC resonance, and transferring the power with electromagnetic couplings without radiating electromagnetic waves. Hence, the magnetic coupling and electric coupling can be represented as mutual inductance and mutual capacitance respectively as shown in Fig. 2.

$Z_{\text{source}}$ in Fig. 2 represents the characteristic impedance, and $Z_{\text{load}}$ is the impedance of the load. In this system, they are both considered to be the same at $Z_0$, 50$\Omega$ the default characteristic of most high frequency systems. The ohm loss and the radiation loss of the antennas are represented by $R$. In this paper, the power is transferred via magnetic coupling. Therefore the coupling can be represented by mutual induction $L_m$.

Next the resonance frequency is calculated based on the equivalent circuit. To satisfy the resonance condition, the reactance of Fig.2 must be 0, as in equation (1). The condition in equation (1) can be satisfied by two resonant frequencies as calculated in equation (2) and (3). The coupling coefficient $k$ can be calculated from equation (2) and (3) to become equation (4).

\[
\frac{1}{\omega L_m} + \frac{2}{\omega (L - L_m)} - \frac{1}{\omega C} = 0 \tag{1}
\]

\[
\omega_m = \frac{\omega_0}{\sqrt{(1+k)}} = \frac{1}{\sqrt{L(L + L_m)C}} \tag{2}
\]

\[
\omega_r = \frac{\omega_0}{\sqrt{(1-k)}} = \frac{1}{\sqrt{(L - L_m)C}} \tag{3}
\]

\[
k = \frac{L_m}{L} = \frac{\omega_0^2 - \omega_m^2}{\omega_r^2 + \omega_m^2} \tag{4}
\]

Next, the efficiency of the power transfer is calculated based on the equivalent circuit. The ratio of power reflection $\eta_{11}$ and transmission $\eta_{21}$ can be defined by equations (5) and (6), where $S_{11}$ is the reflected wave and $S_{21}$ is the transmitted wave. To simplify the calculations, $R$ is considered to be 0$\Omega$.

Here, $S_{21}$ can be calculated with equation (7)[2].

\[
\eta_{11} = S_{11}^2 \times 100\% \tag{5}
\]

\[
\eta_{21} = S_{21}^2 \times 100\% \tag{6}
\]

\[
S_{21} = \frac{2 \cdot jL_m Z_0 \omega}{L_m \omega^2 - (\omega L - \frac{1}{\omega C})^2 + 2 jZ_0 (\omega L - \frac{1}{\omega C}) + Z_0^2} \tag{7}
\]

**Frequency Characteristics**

As the air gap between the antennas increases, the coupling in between the antennas weaken, and the coupling coefficient will be smaller. Therefore, the impedance of the circuit will change as the air gap changes, thus affecting the power transfer efficiency.

Fig. 3 shows the ratio of power reflection $\eta_{11}$ and transmission $\eta_{21}$, and the frequency characteristics of the system when the air gap(g), is changed in between 100mm-250mm. The antenna used here is a self resonating 5 turn, 150mm radius, 5mm pitch, open type helical antenna. Fig. 4 shows the relationship of the coupling factor and the air gap.
As shows in the figures above, when the gaps are small and the coupling is strong, there exist two resonance frequencies that permit maximum efficiency power transfer. As the gap becomes larger, the two resonance frequencies moves closer to each other and eventually becomes one. If the gap gets even larger, the maximum efficiency will drop.

**ISM Band**

As stated in the section above, the resonance frequency changes as the coupling factor changes, and the maximum efficiency power transfer occurs at the resonance frequency. Therefore, a system that matches the frequency of the power source and the resonance frequency is needed. There are two ways to achieve resonance. One is to match the frequency of the power source to the resonance frequency, and the other is to match the resonance frequency to the power source. The former can be easily achieved by applying a feedback control for the efficiency to the power source. However, when this power transfer system is applied in the MHz range, the usable frequency range is bounded by the Industrial, Scientific, and Medical (ISM) band. According to the ISM band, the usable frequency ranges are extremely small. For example at 13.56MHz, the range is 13.56MHz ± 7kHz.

As a result, to apply this technology in the MHz range, the frequency of the power source must be fixed at a usable range, and a tuner must be used to match the resonance frequency of the antennas to the frequency of the power source. In this
paper, a tuner based on the impedance matching theory is used to match the resonant frequency of the antennas to the frequency of the power source that is fixed at 13.56MHz.

**IMPE DANCE MATCHING**

Impedance matching is a technique commonly used in power transfer systems and communication systems to improve the efficiency of the system. In theory, the resonance frequency of the antenna can be matched to a particular frequency by using this technique. This section briefly explains the theory behind this technique, and studies its effect using simulation.

In Fig. 6, when the impedance of the power source is defined as $Z_{\text{source}}$ and that of the load is defined as $Z_{\text{load}}$, the power transferred to the load is written in equation (8). The power transferred to the load reaches its maximum when $Z_{\text{source}} = Z_{\text{load}}$, as in equation (9). Therefore, when the impedance of the load in the source’s point of view matches $Z_{\text{source}}$, the circuit is considered matched and the maximum efficiency is achieved.

$$P = I^2Z = \frac{V^2}{Z_{\text{source}}} \left( \frac{1}{Z_{\text{source}} + 2 + \frac{Z_{\text{load}}}{Z_{\text{source}}}} \right)$$

$$P_{\text{max}} = \frac{V^2}{4Z_{\text{source}}}$$

The impedance matching circuit can be considered as a two-port network that can be described with equation (10). The matching conditions are satisfied when the parameters are equal to equation (11).

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$Z_{\text{source}} = \frac{AB}{\sqrt{CD}}$$

$$Z_{\text{load}} = \frac{DB}{\sqrt{CA}}$$

There are three main types of impedance matching circuits which are L, π and T type. In this paper, the L type matching circuit was used to match the resonance frequency of the antennas to the frequency of the power source that was fixed at 13.56MHz.

A. **Simulation Results**

A simulation was conducted to test the effects of the impedance matching networks on the antennas. The equivalent circuit used in the simulation is shown in Fig. 7, where an impedance matching network is inserted in between the power source and the transmitting antenna. The antenna simulated here is a 5 turn, 15cm radius, 5mm pitch, open type spiral antenna that is self resonating at 13.56MHz. Here, both the input and output impedance, $Z_{\text{source}}$ and $Z_{\text{load}}$ is considered to be $Z_0$, 50Ω. For simplification purposes, the Ohm loss and radiation loss of the antenna, R is considered 0Ω in the simulation. Using the vector network analyzer (VNA), the L and C parameters of the antennas were calculated to be 10300nH and 13.26pF respectively. The inductor $L_1$ used in the impedance matching circuit is set at 6000nH, and combined with capacitor $C_1$ to make a variable inductor. The parameters of the tuning circuit satisfy equation (11), the matching condition of the circuit.

The coupling coefficient $k$ was varied from 0.03 to 0.20, and the frequency characteristics were calculated based on S parameter (Scattering Parameter) theories and confirmed using pSpice. The $S_{21}$ to frequency graphs are as shown in Fig. 8.

Based on Fig. 8, it is observed that the matching circuit is theoretically capable of changing the resonance frequency of the antennas to 13.56MHz for different air gaps. Also, given that they are two identical antennas, when the coupling weakens, the resonance frequency of the antennas is equal to their self resonance frequency. Therefore, if the self resonance frequency of the antennas is set at 13.56MHz, there will be no need for impedance matching for that air gap. When the gap gets even larger and the coupling becomes weaker, the maximum efficiency of the system drops. This means that the system is at maximum efficiency up to a certain distance, (200mm for this antenna) and becomes a less ideal power transfer system after that distance.
k = 0.2, \( C_1 = 34\, \text{pF}, C_2 = 67\, \text{pF}, L_1 = 6000\, \text{nH} \)

k = 0.1, \( C_1 = 27\, \text{pF}, C_2 = 118\, \text{pF}, L_1 = 6000\, \text{nH} \)

k = 0.05, no matching needed

k = 0.03, no matching needed

**Fig. 8 Simulation results. The graphs of pre and post matching antenna frequency characteristics for each coupling coefficient**

**IMPEDEANCE MATCHING EXPERIMENT**

**A. Experiment Setup**

The experiment is set up according to Fig. 7. Each antenna is connected to the VNA with coaxial cables. The VNA is used to provide the power source, and to measure the \( S_{11} \) and \( S_{21} \) parameters of the system. The experiment is conducted at low power. The system is expected to function similarly in high power situations as stated in [2].

Fig. 9 shows a picture of the antenna used in the experiment with the tuning circuit attached to it. The inductor is made using a coil and a ferrite core, and multiple ceramic condensers connected in parallel are used to form the capacitors \( C_1 \) and \( C_2 \).

In this experiment, the air gap is fixed at 9cm with the antennas coaxial (no displacement along the x-y plane). The coupling coefficient \( k \) here is estimated to be 0.135 based on electromagnetic analysis and pre-match resonance frequencies of the system.

**B. Experiment Results and Comparisons with Simulations**

Fig. 10 shows the comparisons of the simulation results and the experiment results. Here, the parameters used in the matching system for both the simulation and the experiments are \( C_1 = 32\, \text{pF}, C_2 = 78\, \text{pF} \). With inductor \( L_1 \) fixed at approximately 5750\,\text{nH}.

The experiment results show that the resonance frequency can be tuned to match 13.56MHz with an impedance matching circuit, and the efficiency at that frequency is increased from 50% to 70%. Also, the equivalent circuit model can be used to estimate the frequency characteristics of the pre and post matched systems.

In the post matched system experiment result, the efficiency \( \eta_{21} \) at the resonance frequency is lower than that of the simulation result. However, it can be said that the loss is not due to a mismatched system as the power reflection ratio \( \eta_{11} \) is extremely low, meaning no power is reflected back to the power source. In other words, the loss is due to other factors such as radiation loss, the low Q value of the system (due to the usage of L type impedance matching networks), loss from the ferrite core used to make the inductor, and Ohmic loss of the inserted matching circuit.
Here, the parameters used in the matching system are $C_1=32\text{pF}$, $C_2=78\text{pF}$.

With inductor $L_1$ fixed at $5750\text{nH}$ and the gap at $9\text{cm}$

**CONCLUSION**

We studied the frequency characteristics and the power transfer efficiency of the antennas using equivalent circuits, electromagnetic analysis, simulations and experiments. The resonance frequency of the antennas changes as the air gap changes. Since the maximum power transfer efficiency occurs at the resonance frequency, the resonance frequency must match the frequency of the power source. When this is applied in the MHz range (which allows smaller size antennas), the usable frequency range is bounded by the ISM band. Therefore, a system which uses an impedance matching network to match the resonant frequency of the antennas to a power source fixed at $13.56\text{MHz}$ was proposed.

The tuning parameters of the impedance matching circuits were estimated using the equivalent circuits. The effects were analyzed with equivalent circuits, electromagnetic analysis, simulations and experiments. The experiment and simulation results show that the resonance frequency of the system can be changed using impedance matching circuits. They also show that the equivalent circuit model can be used to estimate the frequency characteristics.

**REFERENCES**


