FIXED POINT ITERATION FOR ASYMPTOTICALLY QUASI-NONEXPANSIVE MAPPINGS IN BANACH SPACES

SOMYOT PLUBTIENG AND RABIAN WANGKEEREE

Received 21 October 2004 and in revised form 20 April 2005

Suppose that C is a nonempty closed convex subset of a real uniformly convex Banach space X. Let $T : C \to C$ be an asymptotically quasi-nonexpansive mapping. In this paper, we introduce the three-step iterative scheme for such map with error members. Moreover, we prove that if T is uniformly L-Lipschitzian and completely continuous, then the iterative scheme converges strongly to some fixed point of T.

1. Introduction

Let C be a subset of normed space X, and let T be a self-mapping on C. T is said to be *nonexpansive* provided that $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$; T is called *asymptotically nonexpansive* if there exists a sequence $\{k_n\}$ in $[0,\infty)$ with $\lim_{n\to\infty} k_n = 0$ such that $\|T^n x - T^n y\| \leq (1 + k_n)\|x - y\|$ for all $x, y \in C$ and $n \geq 1$. T is said to be an *asymptotically quasi-nonexpansive map* if there exists a sequence $\{k_n\}$ in $[0,\infty)$ with $\lim_{n\to\infty} k_n = 0$ such that $\|T^n x - p\| \leq (1 + k_n)\|x - p\|$ for all $x \in C$ and $p \in F(T)$, and $n \geq 1$ ($F(T)$ denotes the set of fixed points of T, that is, $F(T) = \{x \in C : Tx = x\}$).

From the above definitions, if $F(T) \neq \emptyset$, then asymptotically nonexpansive mapping must be asymptotically quasi-nonexpansive mapping.

The concept of asymptotically nonexpansiveness was introduced by Goebel and Kirk in 1972 [2]. In 2001, Noor [5, 6] introduced the three-step iterative scheme and he studied the approximate solutions of variational inclusions (inequalities) in Hilbert spaces. The three-step iterative approximation problems were studied extensively by Noor [5, 6], Glowinski and Le Tallec [1], and Haubruge et al. [3].

Recently, Xu and Noor [8] introduced the three-step iterative scheme for asymptotically nonexpansive mappings and they proved the following strong convergence theorem in Banach spaces.

Theorem 1.1 (see [8, Theorem 2.1]). *Let X be a real uniformly convex Banach space, let C be a nonempty closed, bounded convex subset of X. Let T be a completely continuous and asymptotically nonexpansive self-mapping with sequence $\{k_n\}$ satisfying $k_n \geq 0$ and*

International Journal of Mathematics and Mathematical Sciences 2005:11 (2005) 1685–1692
DOI: 10.1155/IJMMS.2005.1685

$\sum_{n=1}^{\infty} k_n < \infty$. *Let* $\{\alpha_n\}, \{\beta_n\}$, *and* $\{\gamma_n\}$ *be real sequences in* $[0,1]$ *satisfying*

(i) $0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1$,

(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$.

For a given $x_0 \in D$, *define*

$$\begin{aligned} z_n &= \gamma_n T^n x_n + (1-\gamma_n) x_n, \\ y_n &= \beta_n T^n z_n + (1-\beta_n) x_n, \\ x_{n+1} &= \alpha_n T^n y_n + (1-\alpha_n) x_n. \end{aligned} \tag{1.1}$$

Then $\{x_n\}, \{y_n\}$, *and* $\{z_n\}$ *converge strongly to a fixed point of* T.

In this paper, we will extend the iterative scheme (1.1) to the iterative scheme of asymptotically quasi-nonexpansive mappings with error members. Moreover, we will prove the strong convergence of iterative scheme to a fixed point of T (C need not to be a bounded set), requiring T to be uniformly L-Lipschitzian and completely continuous. The results presented in this paper generalize and extend the corresponding main results of Xu and Noor [8].

2. Preliminaries

For the sake of convenience, we first recall some definitions and conclusions.

Definition 2.1 (see [2]). A Banach space X is said to be *uniformly convex* if the modulus of convexity of X

$$\delta_X(\epsilon) = \inf\left\{1 - \frac{\|x+y\|}{2} : \|x\| = \|y\| = 1,\ \|x-y\| = \epsilon\right\} > 0 \tag{2.1}$$

for all $0 < \epsilon \leq 2$ (i.e., $\delta_X(\epsilon)$ is a function $(0,2] \to (0,1)$).

Definition 2.2. A mapping $T : C \to C$ is called *uniformly L-Lipschitzian* if there exists a constant $L > 0$ such that for all $x, y \in C$,

$$\|T^n x - T^n y\| \leq L\|x-y\|, \quad \forall n \geq 1. \tag{2.2}$$

In what follows, we will make use of the following lemmas.

Lemma 2.3 (see [4]). *Let the nonnegative number sequences* $\{a_n\}, \{b_n\}$, *and* $\{d_n\}$ *satisfy that*

$$a_{n+1} \leq (1+b_n)a_n + d_n, \quad \forall n = 1,2,\ldots, \sum_{n=1}^{\infty} b_n < \infty,\ \sum_{n=1}^{\infty} d_n < \infty. \tag{2.3}$$

Then,

(1) $\lim_{n\to\infty} a_n$ *exists;*

(2) *if* $\liminf_{n\to\infty} a_n = 0$, *then* $\lim_{n\to\infty} a_n = 0$.

Lemma 2.4 ([7], J. Schu's Lemma). *Let X be a real uniformly convex Banach space, $0 < \alpha \le t_n \le \beta < 1$, $x_n, y_n \in X$, $\limsup_{n\to\infty} \|x_n\| \le a$, $\limsup_{n\to\infty} \|y_n\| \le a$, and $\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = a$, $a \ge 0$. Then, $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

3. Main results

In this section, we prove our main theorem. First of all, we will need the following lemmas.

Lemma 3.1. *Let X be a real uniformly convex Banach space, C a nonempty closed convex subset of X. Let T be an asymptotically quasi-nonexpansive mapping with sequence $\{k_n\}_{n\ge1}$ such that $\sum_{n=1}^{\infty} k_n < \infty$ and $F(T) \ne \emptyset$. Let $x_0 \in C$ and*

$$\begin{aligned} z_n &= \alpha_n'' T^n x_n + \beta_n'' x_n + \gamma_n'' u_n, \\ y_n &= \alpha_n' T^n z_n + \beta_n' x_n + \gamma_n' v_n, \\ x_{n+1} &= \alpha_n T^n y_n + \beta_n x_n + \gamma_n w_n, \end{aligned} \tag{3.1}$$

where $\{\alpha_n\}$, $\{\alpha_n'\}$, $\{\alpha_n''\}$, $\{\beta_n\}$, $\{\beta_n'\}$, $\{\beta_n''\}$, $\{\gamma_n\}$, $\{\gamma_n'\}$, and $\{\gamma_n''\}$ are real sequences in $[0,1]$ and $\{u_n\},\{v_n\}$, and $\{w_n\}$ are three bounded sequences in C such that

(i) $\alpha_n + \beta_n + \gamma_n = \alpha_n' + \beta_n' + \gamma_n' = \alpha_n'' + \beta_n'' + \gamma_n'' = 1$,

(ii) $\sum_{n=1}^{\infty} \gamma_n < \infty$, $\sum_{n=1}^{\infty} \gamma_n' < \infty$, $\sum_{n=1}^{\infty} \gamma_n'' < \infty$.

If $p \in F(T)$, then $\lim_{n\to\infty} \|x_n - p\|$ exists.

Proof. Let $p \in F(T)$. Since $\{u_n\},\{v_n\}$, and $\{w_n\}$ are bounded sequences in C, put

$$M = \sup_{n\ge1} ||u_n - p|| \vee \sup_{n\ge1} ||v_n - p|| \vee \sup_{n\ge1} ||w_n - p||. \tag{3.2}$$

Then M is a finite number. So for each $n \ge 1$, we note that

$$\begin{aligned} ||x_{n+1} - p|| &= ||\alpha_n T^n y_n + \beta_n x_n + \gamma_n w_n - p|| \\ &\le \alpha_n ||T^n y_n - p|| + \beta_n ||x_n - p|| + \gamma_n ||w_n - p|| \\ &\le \alpha_n (1+k_n) ||y_n - p|| + \beta_n ||x_n - p|| + \gamma_n ||w_n - p||, \end{aligned} \tag{3.3}$$

$$\begin{aligned} ||y_n - p|| &= ||\alpha_n' T^n z_n + \beta_n' x_n + \gamma_n' v_n - p|| \\ &\le \alpha_n' ||T^n z_n - p|| + \beta_n' ||x_n - p|| + \gamma_n' ||v_n - p|| \\ &\le \alpha_n' (1+k_n) ||z_n - p|| + \beta_n' ||x_n - p|| + \gamma_n' ||v_n - p||, \end{aligned} \tag{3.4}$$

$$||z_n - p|| \le \alpha_n'' (1+k_n) ||x_n - p|| + \beta_n'' ||x_n - p|| + \gamma_n'' ||u_n - p||. \tag{3.5}$$

Substituting (3.5) into (3.4),

$$\begin{aligned}||y_n - p|| &\le \alpha'_n\alpha''_n(1+k_n)^2||x_n - p|| \\ &\quad + \alpha'_n\beta''_n(1+k_n)||x_n - p|| + \alpha'_n\gamma''_n(1+k_n)||u_n - p|| + \beta'_n||x_n - p|| + \gamma'_n||v_n - p|| \\ &\le (1-\beta'_n-\gamma'_n)\alpha''_n(1+k_n)^2||x_n - p|| + \beta'_n||x_n - p|| \\ &\quad + (1-\beta'_n-\gamma'_n)\beta''_n||x_n - p|| + m_n \\ &\le \beta'_n(1+k_n)^2||x_n - p|| + (1-\beta'_n)\alpha''_n(1+k_n)^2||x_n - p|| \\ &\quad + (1-\beta'_n)\beta''_n(1+k_n)^2||x_n - p|| + m_n \\ &= \beta'_n(1+k_n)^2||x_n - p|| + (1-\beta'_n)(\alpha''_n+\beta''_n)(1+k_n)^2||x_n - p|| + m_n \\ &\le \beta'_n(1+k_n)^2||x_n - p|| + (1-\beta'_n)(1+k_n)^2||x_n - p|| + m_n \\ &= (1+k_n)^2||x_n - p|| + m_n,\end{aligned} \tag{3.6}$$

where $m_n = \gamma''_n(1+k_n)M + \gamma'_nM$. Substituting (3.6) into (3.3) again, we have

$$\begin{aligned}||x_{n+1} - p|| &\le \alpha_n(1+k_n)\big((1+k_n)^2||x_n - p|| + m_n\big) + \beta_n||x_n - p|| + \gamma_n||w_n - p|| \\ &= \alpha_n(1+k_n)^3||x_n - p|| + \alpha_n(1+k_n)m_n + \beta_n||x_n - p|| + \gamma_n||w_n - p|| \\ &\le (\alpha_n+\beta_n)(1+k_n)^3||x_n - p|| + (1+k_n)m_n + \gamma_n||w_n - p|| \\ &\le (1+k_n)^3||x_n - p|| + (1+k_n)m_n + \gamma_n||w_n - p|| \\ &\le (1+k_n)^3||x_n - p|| + (1+k_n)m_n + \gamma_nM \\ &= (1+d_n)||x_n - p|| + b_n,\end{aligned} \tag{3.7}$$

where $d_n = 3k_n + 3k_n^2 + k_n^3$ and $b_n = (1+k_n)m_n + \gamma_nM$. Since $\sum_{n=1}^{\infty} d_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, by Lemma 2.3, we have that $\lim_{n\to\infty}\|x_n - p\|$ exists. This completes the proof. □

Lemma 3.2. *Let X be a real uniformly convex Banach space, C a nonempty closed convex subset of X. Let T be an asymptotically quasi-nonexpansive mapping with sequence $\{k_n\}_{n\ge1}$ such that $\sum_{n=1}^{\infty} k_n < \infty$ and $F(T) \ne \emptyset$. Let $x_0 \in C$ and for each $n \ge 0$,*

$$\begin{aligned}z_n &= \alpha''_nT^nx_n + \beta''_nx_n + \gamma''_nu_n, \\ y_n &= \alpha'_nT^nz_n + \beta'_nx_n + \gamma'_nv_n, \\ x_{n+1} &= \alpha_nT^ny_n + \beta_nx_n + \gamma_nw_n,\end{aligned} \tag{3.8}$$

where $\{u_n\}, \{v_n\}$, and $\{w_n\}$ are three bounded sequences in C and $\{\alpha_n\}$, $\{\alpha'_n\}$, $\{\alpha''_n\}$, $\{\beta_n\}$, $\{\beta'_n\}$, $\{\beta''_n\}$, $\{\gamma_n\}$, $\{\gamma'_n\}$, and $\{\gamma''_n\}$ are real sequences in $[0,1]$ which satisfy the same assumptions as Lemma 3.1 and the additional assumption that $0 \le \alpha < \alpha_n, \beta_n, \alpha'_n, \beta'_n \le \beta < 1$ for some α, β in $(0,1)$. Then $\lim_{n\to\infty}\|T^ny_n - x_n\| = 0 = \lim_{n\to\infty}\|T^nz_n - x_n\|$.

Proof. For any $p \in F(T)$, it follows from Lemma 3.1, that $\lim_{n\to\infty} \|x_n - p\|$ exists. Let $\lim_{n\to\infty} \|x_n - p\| = a$ for some $a \geq 0$. From (3.6), we have

$$||y_n - p|| \leq (1+k_n)^2 ||x_n - p|| + m_n. \tag{3.9}$$

Taking $\limsup_{n\to\infty}$ in both sides, we obtain

$$\limsup_{n\to\infty} ||y_n - p|| \leq \limsup_{n\to\infty} ||x_n - p|| = \lim_{n\to\infty} ||x_n - p|| = a. \tag{3.10}$$

Note that

$$\begin{aligned}
\limsup_{n\to\infty} ||T^n y_n - p|| &\leq \limsup_{n\to\infty} (1+k_n) ||y_n - p|| = \limsup_{n\to\infty} ||y_n - p|| \leq a, \\
a = \lim_{n\to\infty} ||x_{n+1} - p|| &= \lim_{n\to\infty} ||\alpha_n T^n y_n + \beta_n x_n + \gamma_n w_n - p|| \\
&= \lim_{n\to\infty} \left\| \alpha_n \left[T^n y_n - p + \frac{\gamma_n}{2\alpha_n}(w_n - p) \right] + \beta_n \left[x_n - p + \frac{\gamma_n}{2\beta_n}(w_n - p) \right] \right\| \\
&= \lim_{n\to\infty} \left\| \alpha_n \left[T^n y_n - p + \frac{\gamma_n}{2\alpha_n}(w_n - p) \right] + (1-\alpha_n) \left[x_n - p + \frac{\gamma_n}{2\beta_n}(w_n - p) \right] \right\|.
\end{aligned} \tag{3.11}$$

By J. Schu's Lemma 2.4, we have

$$\lim_{n\to\infty} \left\| T^n y_n - x_n + \left(\frac{\gamma_n}{2\alpha_n} - \frac{\gamma_n}{2\beta_n} \right)(w_n - p) \right\| = 0. \tag{3.12}$$

Since $\lim_{n\to\infty} \|(\gamma_n/2\alpha_n - \gamma_n/2\beta_n)(w_n - p)\| = 0$, it follows that

$$\lim_{n\to\infty} ||T^n y_n - x_n|| = 0. \tag{3.13}$$

Finally, we will prove that $\lim_{n\to\infty} \|T^n z_n - x_n\| = 0$. To this end, we note that for each $n \geq 1$,

$$||x_n - p|| \leq ||T^n y_n - x_n|| + ||T^n y_n - p|| \leq ||T^n y_n - x_n|| + (1+k_n) ||y_n - p||. \tag{3.14}$$

Since $\lim_{n\to\infty} \|T^n y_n - x_n\| = 0 = \lim_{n\to\infty} k_n$, we obtain that

$$a = \lim_{n\to\infty} ||x_n - p|| \leq \liminf_{n\to\infty} ||y_n - p||. \tag{3.15}$$

It follows that

$$a \leq \liminf_{n\to\infty} ||y_n - p|| \leq \limsup_{n\to\infty} ||y_n - p|| \leq a. \tag{3.16}$$

This implies that

$$\lim_{n\to\infty} ||y_n - p|| = a. \tag{3.17}$$

On the other hand, we note that

$$\begin{aligned} \|z_n - p\| &= \|\alpha''_n T^n x_n + \beta''_n x_n + \gamma''_n u_n - p\| \\ &\le \alpha''_n (1+k_n)\|x_n - p\| + \beta''_n \|x_n - p\| + \gamma''_n \|u_n - p\| \\ &\le \alpha''_n (1+k_n)\|x_n - p\| + (1-\alpha''_n)(1+k_n)\|x_n - p\| + \gamma''_n \|u_n - p\| \\ &\le (1+k_n)\|x_n - p\| + \gamma''_n \|u_n - p\|. \end{aligned} \tag{3.18}$$

By boundedness of the sequence $\{u_n\}$ and $\lim_{n\to\infty} k_n = 0 = \lim_{n\to\infty} \gamma''_n$, we have

$$\limsup_{n\to\infty} \|z_n - p\| \le \limsup_{n\to\infty} \|x_n - p\| = a, \tag{3.19}$$

and so

$$\begin{aligned} \limsup_{n\to\infty} \|T^n z_n - p\| &\le \limsup_{n\to\infty} (1+k_n)\|z_n - p\| \le a, \\ a = \lim_{n\to\infty} \|y_n - p\| &= \lim_{n\to\infty} \|\alpha'_n T^n z_n + \beta'_n x_n + \gamma'_n v_n - p\| \\ &= \lim_{n\to\infty} \left\| \alpha'_n \left[T^n z_n - p + \frac{\gamma'_n}{2\alpha'_n}(v_n - p) \right] + \beta'_n \left[x_n - p + \frac{\gamma'_n}{2\beta'_n}(v_n - p) \right] \right\| \\ &= \lim_{n\to\infty} \left\| \alpha'_n \left[T^n z_n - p + \frac{\gamma'_n}{2\alpha'_n}(v_n - p) \right] + (1-\alpha'_n) \left[x_n - p + \frac{\gamma'_n}{2\beta'_n}(v_n - p) \right] \right\|. \end{aligned} \tag{3.20}$$

By J. Schu's Lemma 2.4, we have

$$\lim_{n\to\infty} \left\| T^n z_n - x_n + \left(\frac{\gamma'_n}{2\alpha'_n} - \frac{\gamma'_n}{2\beta'_n} \right)(v_n - p) \right\| = 0. \tag{3.21}$$

Since $\lim_{n\to\infty} \|(\gamma'_n/2\alpha'_n - \gamma'_n/2\beta'_n)(v_n - p)\| = 0$, it follows that

$$\lim_{n\to\infty} \|T^n z_n - x_n\| = 0. \tag{3.22}$$

This completes the proof. □

Theorem 3.3. *Let X be a real uniformly convex Banach space, C a nonempty closed convex subset of X. Let T be uniformly L-Lipschitzian, completely continuous, and an asymptotically quasi-nonexpansive mapping with sequence $\{k_n\}_{n\ge 1}$ such that $\sum_{n=1}^{\infty} k_n < \infty$ and $F(T) \ne \emptyset$. Let $x_0 \in C$ and for each $n \ge 0$,*

$$\begin{aligned} z_n &= \alpha''_n T^n x_n + \beta''_n x_n + \gamma''_n u_n, \\ y_n &= \alpha'_n T^n z_n + \beta'_n x_n + \gamma'_n v_n, \\ x_{n+1} &= \alpha_n T^n y_n + \beta_n x_n + \gamma_n w_n, \end{aligned} \tag{3.23}$$

where $\{u_n\}, \{v_n\}$, and $\{w_n\}$ are three bounded sequences in C and $\{\alpha_n\}$, $\{\alpha'_n\}$, $\{\alpha''_n\}$, $\{\beta_n\}$, $\{\beta'_n\}$, $\{\beta''_n\}$, $\{\gamma_n\}$, $\{\gamma'_n\}$, and $\{\gamma''_n\}$ are real sequences in $[0,1]$ which satisfy the same assumptions as Lemma 3.1 and the additional assumption that $0 \le \alpha < \alpha_n, \beta_n, \alpha'_n, \beta'_n \le \beta < 1$ for some α, β in $(0,1)$. Then $\{x_n\}, \{y_n\}$, and $\{z_n\}$ converge strongly to a fixed point of T.

Proof. It follows from Lemma 3.2 that

$$\lim_{n\to\infty} ||T^n y_n - x_n|| = 0 = \lim_{n\to\infty} ||T^n z_n - x_n|| \tag{3.24}$$

and this implies that

$$||x_{n+1} - x_n|| \le \alpha_n ||T^n y_n - x_n|| + \gamma_n ||w_n - x_n|| \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{3.25}$$

We note that

$$\begin{aligned} ||T^n x_n - x_n|| &\le ||T^n x_n - T^n y_n|| + ||T^n y_n - x_n|| \le L||x_n - y_n|| + ||T^n y_n - x_n|| \\ &\le \alpha_n' L||x_n - T^n z_n|| + \gamma_n' L||v_n - x_n|| + ||T^n y_n - x_n|| \longrightarrow 0 \quad \text{as } n \longrightarrow \infty, \end{aligned} \tag{3.26}$$

$$\begin{aligned} ||x_n - Tx_n|| &\le ||x_{n+1} - x_n|| + ||x_{n+1} - T^{n+1}x_{n+1}|| + ||T^{n+1}x_{n+1} - T^{n+1}x_n|| + ||T^{n+1}x_n - Tx_n|| \\ &\le ||x_{n+1} - x_n|| + ||x_{n+1} - T^{n+1}x_{n+1}|| + (1 + k_{n+1})||x_{n+1} - x_n|| + L||T^n x_n - x_n||. \end{aligned} \tag{3.27}$$

It follows from (3.25), (3.26), and the above inequality that

$$\lim_{n\to\infty} ||x_n - Tx_n|| = 0. \tag{3.28}$$

By Lemma 3.1, $\{x_n\}$ is bounded. It follows from our assumption that T is completely continuous and that there exists a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$ such that $Tx_{n_k} \to p \in C$ as $k \to \infty$. Moreover, by (3.28), we have $\|Tx_{n_k} - x_{n_k}\| \to 0$ which implies that $x_{n_k} \to p$ as $k \to \infty$. By (3.28) again, we have

$$||p - Tp|| = \lim_{k\to\infty} ||x_{n_k} - Tx_{n_k}|| = 0. \tag{3.29}$$

This shows that $p \in F(T)$. Furthermore, since $\lim_{n\to\infty} \|x_n - p\|$ exists, we have $\lim_{n\to\infty} \|x_n - p\| = 0$, that is, $\{x_n\}$ converges to some fixed point of T. It follows that

$$\begin{aligned} ||y_n - x_n|| &\le \alpha_n' ||T^n z_n - x_n|| + \gamma_n' ||v_n - x_n|| \longrightarrow 0, \\ ||z_n - x_n|| &\le \alpha_n'' ||T^n x_n - x_n|| + \gamma_n'' ||u_n - x_n|| \longrightarrow 0. \end{aligned} \tag{3.30}$$

Therefore, $\lim_{n\to\infty} y_n = p = \lim_{n\to\infty} z_n$. This completes the proof. □

References

[1] R. Glowinski and P. Le Tallec, *Augmented Lagrangian and Operator-Splitting Methods in Nonlinear Mechanics*, SIAM Studies in Applied Mathematics, vol. 9, SIAM, Pennsylvania, 1989.

[2] K. Goebel and W. A. Kirk, *A fixed point theorem for asymptotically nonexpansive mappings*, Proc. Amer. Math. Soc. **35** (1972), 171–174.

[3] S. Haubruge, V. H. Nguyen, and J. J. Strodiot, *Convergence analysis and applications of the Glowinski-Le Tallec splitting method for finding a zero of the sum of two maximal monotone operators*, J. Optim. Theory Appl. **97** (1998), no. 3, 645–673.

[4] Q. Liu, *Iteration sequences for asymptotically quasi-nonexpansive mapping with an error member of uniform convex Banach space*, J. Math. Anal. Appl. **266** (2002), no. 2, 468–471.

[5] M. A. Noor, *New approximation schemes for general variational inequalities*, J. Math. Anal. Appl. **251** (2000), no. 1, 217–229.

[6] ———, *Three-step iterative algorithms for multivalued quasi variational inclusions*, J. Math. Anal. Appl. **255** (2001), no. 2, 589–604.

[7] J. Schu, *Iterative construction of fixed points of strictly pseudocontractive mappings*, Appl. Anal. **40** (1991), no. 2-3, 67–72.

[8] B. Xu and M. A. Noor, *Fixed-point iterations for asymptotically nonexpansive mappings in Banach spaces*, J. Math. Anal. Appl. **267** (2002), no. 2, 444–453.

Somyot Plubtieng: Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand

E-mail address: somyotp@nu.ac.th

Rabian Wangkeeree: Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand

E-mail address: rabianw@nu.ac.th

Journal of Applied Mathematics and Decision Sciences

Special Issue on Intelligent Computational Methods for Financial Engineering

Call for Papers

As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning
- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

Authors should follow the Journal of Applied Mathematics and Decision Sciences manuscript format described at the journal site http://www.hindawi.com/journals/jamds/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/, according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Lean Yu, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; yulean@amss.ac.cn

Shouyang Wang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

K. K. Lai, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskklai@cityu.edu.hk