DETERMINATION OF FUZZY LOGIC CONTROLLER MEMBERSHIP
FUNCTIONS USING TABU SEARCH ALGORITHM: AN APPLICATION TO
FIBER OPTIC SENSOR

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ABSTRACT
This paper presents a new approach for the optimum
determination of the membership functions for a fuzzy logic
controller based on the use of the tabu search algorithm. To
demonstrate the efficiency of the proposed approach,
estimating of the output power fraction of evanescent field
absorption fiber optic sensor is selected. The simulation
results showed that proposed approach can be employed as a
simple and effective optimization method for achieving
optimum determination of the membership functions.

I. INTRODUCTION

Fuzzy logic controllers (FLCs) are intelligent control
systems characterized by a set of linguistic statements
based on expert knowledge or experience [1-5].
Processing of uncertain information, modeling of physical
systems using common-sense rules and linguistic
statements are the basis for fuzzy logic control.

A simple FLC consists of four major elements: a fuzzifier,
rule base, inference engine and a defuzzifier (Fig.1) [3-5].
The fuzzifier converts real system variables into fuzzy
variables. The inference unit provides the necessary
connection between the controller input and output fuzzy
sets. The rule base expressed in the form of IF-THEN
rules is used by the inference unit. The defuzzifier takes
the results of fuzzy reasoning and produces a new real
control action.

The rule base of the FLC consists of the rules which
dictate control actions. Each rule has the potential to
suggest a control action. These rules are characterized by
a set of linguistic statements based on expert knowledge
or experience. The main advantage of this approach is that
is easy to implement the operator’s experiences. The rules
are often expressed using syntax of the form ‘IF
<antecedent>, THEN <consequent>’.

The antecedent of a rule gives a possible process state
while the consequent specifies a control action again in
terms of linguistic variables. The rule base is obtained
using the fuzzy sets that may be represented by a
mathematical formulation often known as the membership
function. This function gives a degree or grade
membership within a fuzzy set.

One of the main problems in designing any fuzzy system
is construction of the fuzzy membership functions
because the all changes in the membership functions will
affect the performance of the fuzzy control directly.
Therefore, optimum determination of the FLC
membership functions is an important factor for the
success of optimum process control.

In general, the membership functions and the rule base are
either acquired from a human expert using trial-and-error
method or from a referential data set with the
optimization algorithms such as genetic algorithm (GA)
[6-11], simulated annealing [12], tabu search [13,14].
GAs work with a set of solutions rather than a single
solution and consequently the computation time of the
GAs is too long. Moreover, evaluating the same solutions
several times is another drawback for GAs. By using a
standard simulated annealing algorithm, obtaining the
global optimum of the search space might became very
difficult and, there is no guarantee of global optimum. In
the methods employed the tabu search algorithm (TSA),
the search of solution is based on the basis of the
automatic learning of fuzzy rule table with the preselected
membership functions.

The main aim of this paper is to propose a new method
based on the TSA for determining the most appropriate
parameter values characterizing the fuzzy membership
functions. The tabu search-based optimization of the
membership functions performs a new approach that is
more systematic, powerful and faster than the other
heuristic methods.

I. TABU SEARCH ALGORITHM

Tabu search was first introduced by Glover as an
intelligent search technique to overcome local optimality
[15-17]. The flowchart of a standard TSA is given in
Fig.2.
One of the main ideas of TSA is to use of a flexible memory (tabu list) [4, 15-17]. The objective of tabu list is to exclude moves which would bring the algorithm back to where it was at some previous iteration and keep it trapped in a local minimum. The length of the tabu list is an important parameter that must be carefully defined for the effectiveness of the search. Too small a tabu list may cause cycling of the tabu search, while too large a tabu list may prohibit tabu search to reach certain good solution regions. Tabu list is initialized empty, constructed in consecutive iterations and updated circularly in later iterations.

To improve the quality of the solutions visited, the search moves from one solution to another using a neighbourhood structure [15-17]. The neighbourhood of a solution is the set of all formations that can be arrived at by a move.

In order to force the search, TSA uses three basic elements: frequency memory, recency memory and aspiration criteria [4]. Frequency (long-term) memory keeps the knowledge of how often the same choices have been made in the past. The recency (short-term) memory prevents cycles of length less than or equal to a predetermined number of iterations. Aspiration criteria is employed to avoid missing good solutions. As to this criteria, if a move on the tabu list leads to a solution with an objective function value strictly better than the best obtained so far, it is possible to allow this move. Frequency memory, recency memory and predetermined tabu conditions related with these factors are play an essential role for obtaining the tabu list.

At the last step of the algorithm, a stopping criterion terminates the tabu search procedure either after a specified total number of iterations have been performed in total or, currently best solution was found.

II. DETERMINATION OF MEMBERSHIP FUNCTIONS USING TSA

Fuzzy membership functions provide the characterization of fuzzy sets by establishing a connection between linguistic terms (such as cold, warm, hot) and precise numerical values of variables in a physical system.

The determination of fuzzy controller membership functions using a TSA takes place in three phases:

(i) creation of a primary membership functions which is incorrectly adjusted,
(ii) parameterization, and
(iii) adjustment of these membership functions

Triangular shaped membership functions are used in this work since it is one of the most common forms of membership functions. Parameterized fuzzy membership functions \((A_1, A_2, \ldots, A_k)\) are shown in Fig. 3.

![Figure 3. Parameterized fuzzy membership functions](image)

TSA is used to adjust the parameters of the membership functions in the fuzzy rule base. Optimum parameters of the membership functions are determined by minimizing an objective function.

When the same membership functions given in Fig. 3 are used for all variables of the system, for any physical system with two inputs and one output, a sample rule structure can be written as following:

**Linguistic Rule**

\[
\text{IF input variable (1) is } A_2, \text{ AND input variable (2) is } A_3, \text{ THEN output variable is } A_2. 
\]

**Parameterized Rule**

\[
\text{IF } (a_{21}, a_{22}, a_{23}), \text{ AND } (a_{31}, a_{32}, a_{33}), \text{ THEN } (a_{21}, a_{22}, a_{23}).
\]

As to this definition, the each rule consists of \([(\text{input number} + \text{output number}) \times 3]\) parameters. The number of the parameters required for defining a rule set is therefore \((\text{rule number} \times (\text{input number} + \text{output number}) \times 3)\). This value is also equal to the size of tabu list.

This definition of the membership functions has three main advantages.

1. Defining and understanding of the rules are easy.
2. It provides the research of the all possible solution points for the universe of discourse since the membership functions have not symetric structure,
3. Same control parameters can be obtained for different membership functions and hence, the number of membership functions can be reduced during the optimization.
The neighbourhood values of the parameters \((a_{11}, a_{12}, \ldots, a_{k3})\) used to represent the membership functions \((A_1, A_2, \ldots, A_k)\) are found using the neighbourhood structure given in Table 1. In this structure, for an element with \(N\) bits, there is a solution vector with \(N\) bits.

Table 1. The neighbourhood structure

| solution | 1 | 0 | 1 | 1 | 0 |
| neighbour (1) | 0 | 0 | 1 | 1 | 0 |
| neighbour (2) | 1 | 1 | 1 | 1 | 0 |
| neighbour (3) | 1 | 0 | 1 | 0 | 0 |
| neighbour (4) | 1 | 0 | 1 | 1 | 0 |
| neighbour (5) | 1 | 0 | 1 | 1 | 1 |

According to the problem considered, tabu conditions must be determined carefully. These conditions employed are based on the recency and frequency memory criteria. If an element of the solution vector does not satisfy tabu conditions, then it is accepted as tabu.

### III. APPLICATION

To demonstrate the efficiency of the proposed approach, estimating of the output power fraction of evanescent field absorption fiber optic sensor was selected.

An evanescent field (EF) is created in optical fibers when light undergoes to the total internal reflection between the core/cladding interface. The evanescent field is a fraction of the total input power of the fiber and travels in the cladding. If the fiber core is coated an absorbing material (or it has an absorptive cladding), the EF is attenuated and consequently the total power reached to the fiber end decreases. This is the principle of the evanescent field absorption fiber optic sensor (EFAFOS).

The calculation of the output power of the EFAFOS requires the solution of the eigenvalue equation of the multimode optical fiber. The eigenvalue equation under the weakly guiding approximation is given by [18],

\[
\eta_v = \frac{P_{\text{out}}}{P} = \frac{1}{V^2} \left[ (ha)^2 + (qa)^2 J_{\nu+1}(ha) J_{\nu-1}(ha) \right]
\]  

(4)

where \(n_1\) and \(n_2\) are the refractive indices of the core and the cladding, respectively. \(k_0\) is the wave number in vacuum and \(\beta\) is the propagation constant.

The next step for the calculation of the output power of the sensor is to determine the power fraction of each mode. By substituting \(ha\) and \(qa\) pair obtained from the eigenvalue equation into the equation given by [18],

\[
\eta_v = \frac{P_{\text{out}}}{P} = \frac{1}{V^2} \left[ (ha)^2 + (qa)^2 J_{\nu+1}(ha) J_{\nu-1}(ha) \right]
\]  

(4)

modal power fraction of \(v\)th mode is determined. \(V\) is the normalized frequency of the fiber and is given by [19],

\[
V = k_0 a \left( n_1^2 - n_2^2 \right)^{\frac{1}{2}}
\]  

(5)

By assuming, all fiber modes are excited with equal powers, the output power of step-index multimode fiber with a lossless core is given by [20],

\[
P_{\text{out}} = \frac{P_{\text{in}}}{N} \sum_{\nu=1}^{N} \exp \left( -\alpha \eta_v L \right)
\]  

(6)

where \(\alpha\) and \(L\) are the bulk absorption coefficient and the length of the absorptive cladding, respectively. \(N\) is the total number of the guided modes and given by [19],

\[
N = \frac{V^2}{2}
\]  

(7)

Fig. 4 shows the output power fractions \((P_{\text{out}}/P_{\text{in}})\) calculated for different \(\alpha L\) values against \(V\).
In this work, $(\alpha L)$ and $V$ are the input variables of the fuzzy model with the output of $(P_{out}/P_{in})$ value. In order to obtain the modeling of this system, a fuzzy rule base with six rules represented by 54 parameters has been selected intuitively. During the evaluation of the different fuzzy rule sets, Mamdani’s minimum operation rule has been used as a fuzzy implication function. The fuzzification strategy was selected center of gravity method.

For $i^{th}$ element of the solution vector, the following tabu conditions are used by the search algorithm:

1. $\text{recency}(i) \leq \text{rec.n}$
2. $\text{frequency}(i) \geq \text{freq.avgfreq}$

Here, $\text{rec}$ and $\text{freq}$ are recency and frequency factors, $n$ is the number of element in the binary sequence and $\text{avgfreq}$ is the average chance of bits. In this study, $\text{rec}$, $\text{freq}$ and $n$ are selected as 0.2, 2 and 5, respectively. The number of element in the tabu list is determined as 54. The objective function employed is defined by,

$$E = \left\{ \frac{1}{t} \sum_{k=1}^{t} \left[ y_{d}(k) - y_{f}(k) \right]^2 \right\}^{1/2}$$

where $y_{d}$ and $y_{f}$ are the desired and the actual outputs for modeled system, $t$ is the total number of the outputs. The membership functions optimized by TSA and the fuzzy rule base are given in Figure 5 and Table 2, respectively. The simulation results obtained using the fuzzy rule base in Table 2 are given in Figure 6.

![Figure 5. The membership functions optimized by TSA for (a) $\alpha L$, (b) $V$ and, (c) $(P_{out}/P_{in})$](image)

<table>
<thead>
<tr>
<th>$\alpha L$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>c6</td>
<td>--</td>
</tr>
<tr>
<td>$a_2$</td>
<td>--</td>
<td>--</td>
<td>c5</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$a_3$</td>
<td>c2</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$a_4$</td>
<td>--</td>
<td>c1</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$a_5$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>c3</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$a_6$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>c4</td>
<td>--</td>
</tr>
</tbody>
</table>

![Table 2. Optimum fuzzy rule base](image)

![Figure 6. Simulation results](image)

According to the Figure 6, performance of the fuzzy model with the membership functions optimized by TSA has a very good agreement with the desired results. These results are not based on any mathematical equation. Moreover, a significant reduction for the computation time can be obtained using the fuzzy rule structure in Table 2.
IV. CONCLUSION

In this paper, we present a new method for optimum determination of the membership functions in a fuzzy rule set based on the use of the tabu search algorithm. The simulated responses of the system with the optimum fuzzy rule base indicated that proposed optimization method can be efficiently used for obtained the optimum membership functions of a fuzzy logic controller.

V. REFERENCES

[8]. M.A.Lee & H.Takagi, Dynamic Control of Genetic Algorithms using Fuzzy Logic Techniques, Proc. of 5th Int. Conf. on Genetic Algorithms (ICGA’93), California, USA, pp.76-83, 1993