Performance Analysis of Multiple Access 60 GHz System Using Frequency-shifted Gaussian Pulse and Non-carrier PSWF Pulse

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Abstract—In this paper, a kind of impulse radio (IR) 60 GHz pulse based on Prolate Spheroidal Wave Functions (PSWF) is proposed. The capacity and performance for multiple access 60 GHz communication system based on carrier pulse and impulse radio pulse are analyzed separately. Both frequency-shifted Gaussian pulse and Prolate Spheroidal Wave Functions (PSWF) pulse are considered and devised according to the federal communication commission (FCC) power constraints. Pulse position modulation (PPM) with time hopping spread spectrum (THSS) is employed in the multiple access 60GHz communication system. The channel capacity and error probability for 60GHz communication system with different pulse waveforms over additive white Gaussian noise (AWGN) channel are compared and analyzed. The simulation results showed that the PSWF pulse 60GHz system with has better channel capacity and error probability performance than frequency-shifted Gaussian pulse 60GHz system.

Index Terms—60 GHz, PSWF, frequency-shifted Gaussian pulse, multiple access, channel capacity, error probability

I. INTRODUCTION

In recent years, a massive amount of unlicensed spectra around 60 GHz has drawn a growing interest in academic and industrial institutions. Similar to the microwave ultra wideband (UWB) radio, the up to 7 GHz of bandwidth is very suitable for short-range wireless communication, but it suffers from less chance of intersystem interference than the UWB. With 7 GHz of bandwidth, 2-3 Gbit/s high definition media interface (HDMI) or wireless gigabit Ethernet could be achieved even using simple modulation methods [1], like PSK, PAM and QAM, etc. Furthermore, 60 GHz regulations allow a much higher transmit power (10W) compared to other existing wireless local area network (WLAN) and wireless personal area network (WPAN) systems. Higher transmit power is necessary to overcome the higher path loss at 60 GHz.

In 2001, the United States Federal Communications Commission (FCC) set aside 7 GHz of contiguous spectrum between 57 and 64 GHz for unlicensed use. The IEEE 802.15.3 Task Group 3c had introduced some wireless Medium Access Control (MAC) and Physical Layer (PHY) restrictions including Effective Isotropic Radiated Power (EIRP) limit, mmWave PHY channelization and transmit spectral mask [2].

Some researches on channel capacity and error probability for 60GHz systems have been undertaken. The capacity analysis for 60 GHz wireless communication system over AWGN channel, frequency selective fading channel, Ricean fading channel, Nakagami-m fading channel and IEEE 802.15.3c channel were respectively presented in [3-7]. The performance analysis for 60 GHz wireless communication system was discussion in [8-10]. However, all the researches were based on single-user systems with carrier frequency-shifted pulse. The performance of Non-carrier pulse 60GHz system and multiple access 60 GHz-IR system have not been studied. In this paper a new Non-carrier 60GHz pulse based on Prolate Spheroidal Wave Functions (PSWF) are designed. Multiple access 60 GHz system capacity and error probability performance using PSWF pulse and frequency-shifted Gaussian pulse are compared and analyzed.

The rest of this paper is organized as follows. Section II describes the design method and process for PSWF 60GHz pulse and frequency-shifted Gaussian 60GHz pulse. Meanwhile their power spectral densities (PSD) complying with the FCC frequency constraints are examined in this section. Multiple access 60 GHz system model is presented in Section III. The capacity and error probability performance of multiple access 60 GHz system are given and compared using these two types of waveforms in Section IV, these comparisons are based on pulse position modulation and under the AWGN channel. Finally, Section V concludes this paper.

II. WAVEFORM DESIGN METHOD
In this section, pulse waveforms for the 60GHz system are studied. The pulse waveforms are expected to use the 7 GHz frequency resources around 60 GHz as much as possible, so traditional Gaussian monocycles are not suitable. But a modulated Gaussian pulse can meet these requirements which can be written as [10]

\[ s_j(t) = g_j(t) \cos(2\pi f_c t) \]

Where \( f_c \) is modulating frequency which decides the center frequency of \( s_j(t) \), \( g_j(t) \) is the Gaussian pulse \((j=0)\) or its \( j \)th derivatives \((j>1)\).

\[ g_0(t) = Ae^{-\frac{2\pi t^2}{\alpha^2}}, \]
\[ g_j(t) = A\left(-\frac{4\pi t}{\alpha^2}\right)e^{-\frac{2\pi t^2}{\alpha^2}}, \]
\[ g_2(t) = A\frac{4\pi}{\alpha^2}e^{-\alpha^2 - 4\pi t^2}, \]

Where the amplitude \( A \) can be used to normalize the pulse energy, and \( \alpha \) is pulse shaping factor. In this paper, \( f_c=60.5\)GHz, \( \alpha =0.58\)ns.

Figure (1) describes pulse shapes of the modulated Gaussian pulse in time domain; figure (2) compares the PSD of the modulated Gaussian pulse and its derivatives with FCC transmit spectral mask. It shows that, with frequency shifting, the Gaussian pulse not only has no DC component, but also meets the transmitting spectral mask better than its derivatives. Since there is a gap in the center frequency, the PSD of Gaussian derivatives can not fully utilize the specified spectrum resources.

However, systems using frequency-shifted pulse require carrier modulation module and carrier recovery module. This increases the complexity of transceiver system structure and power consumption. In order to resolve these problems, a new pulse without carrier for 60 GHz systems is proposed. This pulse is based on PSWF. Compared with frequency-shifted pulse, this pulse simplifies the system structure, but this scheme demands high level narrow pulse circuits.

PSWF pulse has a good time-limited and frequency-limited characteristic, no matter in the time domain or frequency domain, and it also maintains orthogonal properties. PSWF expression is the following solution of differential equation [12].

\[ \int_{-\tau}^{\tau} \psi_k(x) \frac{\sin[W(t-x)]}{\pi(t-x)} dx = \lambda_k \psi_k(t) \]

Where \( \psi_k(t) \) is called the first \( k \) order PSWF waveform, \( \lambda_k \) is the corresponding energy concentration, \( \lambda_k \) is bigger, the energy concentration is better. \( W \) is bandwidth and pulse width is \( T \). \( \psi_i(t) \) is time limited signal and have the following form,

\[ \psi(t) = \begin{cases} p(t), & |t| < \frac{T_m}{2} \\ 0, & \text{elsewhere} \end{cases} \]

\( p(t) \) is the pulse waveform which meets the expected spectrum masks.

\[ h(t-\tau) = \frac{\sin[W(t-\tau)]}{\pi(t-\tau)} \]

Because \( \psi(t) \) is nonzero only in limited scope, then (3) could be rewritten as

\[ \lambda \psi(t) = \int_{-\tau}^{\tau} \psi(\tau) h(t-\tau) d\tau. \]

Although a closed-form solution, known as the prolate spheroidal function [12], [13], is difficult to find, our algorithm provides a numerical solution through the discretization of (6). By sampling at a rate of \( N \) samples per pulse period \( T_m \), (6) can be expressed as follows:

\[ \lambda \psi(n) = \sum_{m=-N/2}^{N/2} \psi(m) h(n-m), n = \frac{N}{2}, \ldots, \frac{N}{2} \]
Figure 3: The PSWF pulse with normalized amplitude for 60 GHz system.

Figure 4: The PSD of 60 GHz PSWF pulse.

The design philosophy of 60 GHz pulse based on PSWF can be simply summarized as: Select appropriate pulse width $T_m$, confirm the bandpass filter $h(t)$ according to the FCC frequency mask, sample $h(t)$ to $h(n)$, expand $h(n)$ to matrix form $H(n)$, seek the eigenvectors $\phi(n)$ and eigenvalues $\lambda$ of $H(n)$, $\phi(n)$ is the wanted pulse. But it is hard to get the exact pulse expression in time domain using this eigenvector method.

### III. 60 GHz Multiple Access System Model

A typical time-hopping M-ary PPM format for the output of the $k$th user in a IR 60 GHz system is given by [10]

$$s^{(k)}(t) = \sum_{j=0}^{\infty} A^{(k)} q(t - j T_f - c_j T_c - \delta_{j,n})$$

Where $A^{(k)}$ is the $k$ user’s signal amplitude, $q(t)$ represents the transmitted impulse waveform that nominally begins at time zero, and the quantities associated with $(k)$ are transmitter dependent. $T_f$ is the frame time, which is typically a hundred to a thousand times the...
impulse width resulting in a signal with very low duty cycle. Each frame is divided into \( N_s \) time slots with duration \( T_f \). The pulse shift pattern \( c_j^{(k)} \), \( 0 \leq c_j^{(k)} < Nh \), is pseudorandom, which provides an additional shift to avoid catastrophic collisions due to multiple access interference. The sequence \( d \) is the \( N \)-ary data stream generated by the \( k \)th source after channel coding, and \( \delta_{j(n)}^{(k)} \) is the additional modulation time shift utilized for PPM determined by the input data \( d \). If \( N_s>1 \), a repetition code is introduced, i.e., \( N_s \) pulses are used for the transmission of the same information symbol.

The received signal can be modeled as the derivative of the transmitted pulse assuming propagation in free space [14], \( \rho(t) \) is the received pulse waveform.

\[
r(t) = \sum_{k=1}^{N_s} s^{(k)}(t - \tau_j) + n(t) \tag{14}
\]

To evaluate the average SNR, we make the following assumptions.

(a) \( s^{(k)}(t-\tau_j) \), for \( k=1,2, \ldots, N_s \), where \( N_s \) is the number of active users, and the noise \( n(t) \) is assumed to be independent.

(b) The time-hopping sequences \( c_j^{(k)} \) are assumed to be independent and identically distributed (i.i.d) random variables uniformly distributed over the time interval [0, \( Nh \)].

(c) All \( M \)-ary PPM signals are equally likely a priori.

(d) The time delay \( \tau_j \) is assumed to be i.i.d and uniformly distributed over [0, \( T_f \)].

(e) Perfect synchronization is assumed at the receiver, that is, \( \tau_j \) is known at the receiver.

Without loss generality, we assume the desired user corresponds to \( k=1 \). The single user optimal receiver is an \( M \)-ary pulse correlation receiver followed by a detector.

The \( M \)-ary cross-correlation receiver for user 1 consists of \( M \) filters matched to the basic functions defined as

\[
h_j^{(1)}(t) = \rho(t - \delta_j - \tau_j) \quad i = 1, \ldots, M. \tag{15}
\]

The output of each cross-correlation receiver at the sample period \([nN_j T_f \ ( (n+1)N_j -1) T_f ]\) is

\[
y_i = \sum_{j=1}^{N_j} \int_{i-1}^{i} r(t) h_j^{(1)}(t - j T_f - c_j^{(1)} (T_c - \delta_j)) \, dt, \tag{16}
\]

Assuming PPM signal is transmitted by user 1, (16) can be written as

\[
y_i = \begin{cases} N_s A^{(1)} \sqrt{\epsilon_r} + W_{MAI} + W, & i = n \text{ exist signal} \\ W_{MAI} + W & i \neq n \text{ no signal} \end{cases} \tag{17}
\]

where \( \epsilon_r \) is the average signal energy, WMAI is the MAI component given by

\[
W_{MAI} = \sum_{k=2}^{N_s} \sum_{j \neq n} \int_{(i-1)T_f}^{iT_f} \theta(t) \, dt \tag{18}
\]

where

\[
\theta(t) = A^{(i)} p(t - j T_f - c_j^{(k)} (T_c - \delta_{j(n)}) - \tau_k). \tag{19}
\]

and

\[
W = \sum_{j=1}^{N_j} \int_{i-1}^{i} n(t) (p(t - j T_f - c_j^{(1)} T_c - \delta_j - \tau_j)) \, dt. \tag{20}
\]

is the AWGN component. By defining the autocorrelation function of \( \rho(t) \) as

\[
\gamma(\Delta) = \int_0^{T_f} \rho(t) \rho(t - \Delta) \, dt \tag{21}
\]

(18) can be written as

\[
W_{MAI} = \sum_{k=2}^{N_s} \sum_{j \neq n} A^{(k)} \gamma(\Delta^{(k)}) \tag{22}
\]

where

\[
\Delta^{(k)} = (c_j^{(1)} - c_j^{(k)})(T_c - \delta_{j(n)}) - (\tau_k - \tau_j) \tag{23}
\]

is the time difference between user 1 and user \( k \). Under the assumptions listed above, \( \Delta \) can be modeled as a random variable uniformly distributed over \([-T_f, T_f]\). As in [14], [15], [16], the MAI is modeled as a Gaussian random process for the multi-user environment. Note that \( N_s \gg 1 \) justifies the Gaussian approximation even for a small number of users as illustrated in [16]. With the Gaussian approximation, we require the mean and variance of (17) to characterize the output of the cross-correlators. It is easy to show that the AWGN component has zero mean and variance \( N_s N_0 / 2 \). However, the mean and variance of the MAI component are determined by the specific of pulse waveform.

It will be shown that we assume that each information symbol only uses a single 60 GHz pulse, that is, \( N_s = 1 \) for simplicity. And Without loss of generality, make

\[
A^{(1)} = A, \quad \epsilon_r = 1.
\]

From (17), when the signal exist, the mean of \( y_i \) is

\[
y_i = E(N_s A^{(1)} \sqrt{\epsilon_r} + W_{MAI} + W) = A + E(W_{MAI})
\]

\[
= A + A(N_s - 1) \cdot E(\epsilon_r) \tag{24}
\]

The variance of \( y_i \) is


\[
\delta_1 = \text{Var}(N_s \cdot A^{(1)} \sqrt{E_p} + W_{\text{MAI}} + W) = \text{Var}(W_{\text{MAI}} + W)
\]

\[
= A^2 (N_s - 1) \cdot \text{Var}(\gamma(\Delta)) + \frac{N_s A^2}{2}
\]

(25)

When the signal does not exist, the mean and variance of \( y_i \) separately are

\[
u_2 = E(W_{\text{MAI}} + W) = E(W_{\text{MAI}}) = A(N_s - 1) \cdot E(\gamma(\Delta))
\]

(26)

\[
\delta_2 = \text{Var}(W_{\text{MAI}} + W) = A^2 (N_s - 1) \cdot \text{Var}(\gamma(\Delta)) + \frac{N_s A^2}{2} = \delta_1
\]

(27)

IV. MULTIPLE ACCESS 60GHZ SYSTEM CAPACITY AND PERFORMANCE ANALYSIS BASED ON PSWF PULSE AND FREQUENCY-SHIFTED GAUSSIAN PULSE

With more than one user active in the system, MAI is a factor limiting the capacity and performance, especially for a large number of users. As shown in [17], if the number of users is large [18] or a repetition code is used with \( N_s >> 1 \), MAI caused by the undesired users at the output of the desired user’s correlation receiver can be modeled as a zero-mean Gaussian random variable. In this section, multiple access capacity and error probability are investigated with PPM. Assuming that \( \delta \geq T_p \), that is, the \( M \)-ary PPM signal is an orthogonal signal with \( M \) dimensions.

A. Multiple Access Capacity

The theoretic capacity for a PPM system over an AWGN channel with a single user given in [19]

\[
C_{\text{M,PPM}} = \log_2 M - E_{\text{vh}} \left\{ \log_2 \sum_{i=1}^{M} \exp\left[ \sqrt{\gamma} (v_i - y) \right] \right\}
\]

(28)

bits/channel use

where \( \gamma \) is the channel SNR per symbol, \( V_i, i = 2,3 \cdots M \) and \( V_i \) are Gaussian random variables with distributions \( N(\sqrt{\gamma},1) \) and \( N(0,1) \) respectively. \( N(x,1) \) denotes a Gaussian distribution with mean \( x \) and variance 1. (28) can be extended to the multiple access case by substituting \( \gamma_{\text{MAI}} \) for \( \gamma \),

\[
\gamma_{\text{MAI}} = \frac{(A^{(1)})^2}{\text{Var}(W_{\text{MAI}} + W)} = \frac{A^2}{\delta_2}
\]

\[
= \frac{A^2 (N_s - 1) \cdot \text{Var}(\gamma(\Delta)) + \frac{N_s A^2}{2}}{A^2 (N_s - 1) \cdot \text{Var}(\gamma(\Delta)) + \frac{N_s A^2}{2}}
\]

(29)

\[
\rho = \frac{\rho}{(N_s - 1) \text{Var}(\gamma(\Delta)) \rho + 1}
\]

Where \( \rho = \frac{A^2}{N_s E_g} \), \( E_g \) is the average signal energy.

Figure 5 and 6 shows the channel capacity of 60 GHz system with different number of users and the users are
selected 5, 15, 30 respectively. It shows that the achievable channel capacity will decrease as the number of synchronous users increases due to the MAI.

Figure 7 compares the channel capacity of 60 GHz system using PSWF pulse and frequency-shifted Gaussian pulse. Systems using PSWF pulse has better capacity performance than systems using Gaussian pulse. The simulation results also illustrate that the superiority of PSWF pulse become more and more apparent as the users increase.

B. Multiple Access Error Probability

The theoretic error probability for a PPM system over an AWGN channel with a single user given in [19]

\[ P_e = \int_{-\infty}^{+\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}x^2} dx \right)^{M-1} p(r_i) dr_i \]  

where

\[ p(r_i) = \frac{1}{\sqrt{\pi}N_0} \exp\left( -\frac{(r_i - E_g)^2}{N_0} \right) \]  

the probability of a symbol error for an \( M \)-ary PPM is

\[ P_M = 1 - P_e \]  

The error probability of multiple access PPM 60 GHz systems over AWGN channel can be obtained from (30) (31) by substituting \( \delta_2 \) for \( N_0/2 \),

\[ P_e = \int_{-\infty}^{+\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}x^2} dx \right)^{M-1} p(r_i) dr_i \]

\[ = \int_{-\infty}^{+\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(X_i - \gamma(A_i))^2} e^{-x^2} dx \right)^{M-1} p(r_i) dr_i \]

\[ = \int_{-\infty}^{+\infty} \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left( -\frac{(r_i - E_g)^2}{N_0} \right) dr_i \right) \]

\[ \times \frac{1}{\sqrt{2\pi\delta_2}} \exp\left( -\frac{(r_i - E_g)^2}{2\delta_2^2} \right) \]

\[ = \int_{-\infty}^{+\infty} \exp\left( -\frac{(r_i - E_g)^2}{2\delta_2^2} \right) \]

\[ \times \frac{1}{\sqrt{2\pi\delta_2}} \exp\left( -\frac{(r_i - E_g)^2}{2\delta_2^2} \right) \]

Figure 8 and 9 compare the error probability of 60 GHz system with different number of users and the users are selected 5, 15, 30 respectively. When \( E_s/N_0 \) is small, the error probability mainly depends on thermal noise, so we can improve system performance through increasing each transmission pulse energy or increasing the transmission power. Once the transmission power increasing, the \( E_s/N_0 \) will also increase in receiving terminal, and the SER will decrease. When \( E_s/N_0 \) is large, the error probability mainly depends on MAI, and due to the MAI, the error probability increases as the number of synchronous users increases.
Figure 10 presents the error probability comparison between PSWF pulse and frequency-shifted Gaussian pulse 60 GHz system. In this paper, the numbers of users are selected 10, 30 respectively. PSWF pulse has better error probability performance than frequency-shifted Gaussian pulse as the same $E_b/N_0$. And the simulation result also illustrates that PSWF pulse has superior performance than frequency-shifted Gaussian pulse.

From (28), (29), (33), (34), pulse waveforms impact the channel capacity and error probability of multiple access 60 GHz systems. The main reason for the diversity of channel capacity and error probability is different $\text{Var}(\gamma(\Delta))$ in (29), (33), (34). The pulse autocorrelation variance $\text{Var}(\gamma(\Delta))$ is smaller, the channel capacity is larger and the performance is better. In this paper, based on the calculation, the values of $\text{Var}(\gamma(\Delta))$ for PSWF pulse and frequency-shifted Gaussian pulse are 2.3646e-3 and 2.9554e-3 separately when the $T_m=1\text{ns}$. Thus systems using the PSWF pulse have better capacity and error probability than systems using Gaussian pulse.

V. CONCLUSION

This paper provides an alternative IR 60GHz system other than carrier 60GHz system, and this subject is one rarely studied project in this field. This IR 60 GHz pulse design method is based on Prolate Spheroidal Wave Functions (PSWF). Meanwhile a carrier pulse design method using frequency-shifted Gaussian pulse is also presented. Multiple access 60 GHz wireless communication system model using time-hopping spread spectrum based on impulse radio (IR) is proposed in this paper. Researching on capacity and error probability of multiple access 60 GHz system is significant for 60 GHz system multiuser access and anti-interference, so capacity and error probability of 60 GHz multiple access systems were analyzed and compared in this paper. The numerical results showed that pulse waveforms impact the channel capacity and error probability of multiple access 60 GHz systems, the multiple access channel capacity and SER performance of $M$-ary PPM systems using PSWF pulse are superior to that of systems using frequency-shifted Gaussian pulse.

REFERENCE


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