Decision Aiding

Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS

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Abstract

The multiple criteria decision making (MCDM) methods VIKOR and TOPSIS are based on an aggregating function representing “closeness to the ideal”, which originated in the compromise programming method. In VIKOR linear normalization and in TOPSIS vector normalization is used to eliminate the units of criterion functions. The VIKOR method of compromise ranking determines a compromise solution, providing a maximum “group utility” for the “majority” and a minimum of an individual regret for the “opponent”. The TOPSIS method determines a solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances. A comparative analysis of these two methods is illustrated with a numerical example, showing their similarity and some differences.

Keywords: Multiple criteria analysis; Compromise; VIKOR; TOPSIS; Comparison

1. Introduction

Many papers have proposed analytical models as aids in conflict management situations. Among the numerous approaches available for conflict management, one of the most prevalent is multicriteria decision making. Multicriteria decision making (MCDM) may be considered as a complex and dynamic process including one managerial level and one engineering level (Duckstein and Opricovic, 1980). The managerial level defines the goals, and chooses the final “optimal” alternative. The multicriteria nature of decisions is emphasized at this managerial level, at which public officials called “decision makers” have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are “off line” from the optimization procedure done at the engineering level. Very often, the preference structure is based on political rather than only technical criteria. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of noninferior...
The main steps of multicriteria decision making are the following:

(a) Establishing system evaluation criteria that relate system capabilities to goals;
(b) Developing alternative systems for attaining the goals (generating alternatives);
(c) Evaluating alternatives in terms of criteria (the values of the criterion functions);
(d) Applying a normative multicriteria analysis method;
(e) Accepting one alternative as “optimal” (preferred);
(g) If the final solution is not accepted, gather new information and go into the next iteration of multicriteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (d), a decision maker should express his/her preferences in terms of the relative importance of criteria, and one approach is to introduce criteria weights. These weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure). In this paper, we consider “importance weights” which represent the relative importance of criteria. Another approach is to introduce weights in a simple aggregating function (weighted sum), where weights reflect both criterion importance and measurement scale (“trade-offs” weights). Since criteria usually are expressed in different units (noncommensurable) it is difficult to determine the values of such weights. There are applications with “objective weights” determined from a performance matrix, and these have no relationship with preference of the decision maker (Deng et al., 2000).

In the engineering level, the main efforts are in generating and evaluating the alternatives (steps (b) and (c)); and these efforts are different for individual projects, since projects vary in the types of needs they meet or the problems they solve. The physical, environmental, and social settings in which planning takes place also differ from one location to another. Alternatives can be generated and their feasibility can be tested by mathematical models, physical models, and/or by experiments on the existing system or other similar systems. Constraints are seen as high-priority objectives, which must be satisfied in the process of generating alternatives. Generating alternatives can be a very complex process, since there is no general procedure or model, and no mathematical procedure could replace human creativity in generating and evaluating alternatives. However, after generating and evaluating the alternatives, an MCDM method (such as TOPSIS or VIKOR) could be applied to rank alternatives and to propose a solution to the decision maker.

Multicriteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several noncommensurable and conflicting (competing) criteria, and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of noninferior solutions, or a compromise solution according to the decision makers’ preferences.

Most multicriteria methods require definition of quantitative weights for the criteria, in order to assess the relative importance of the different criteria. The paper by Mareschal (1988) considers the stability of the ranking results during changes of the criteria weights. The procedure for sensitivity analysis defines stability intervals for the weights. The values of the weight of one criterion within the stability interval do not alter the results obtained with the initial set of weights, since all other weights have initial ratios. Wolters and Mareschal (1995) considered new types of stability analysis for additive MCDM methods, including additive utility function and outranking methods such as PROMETHEE (Brans et al., 1984; Olson, 2001). However, the compromise ranking method (called VIKOR) does not belong to this class of methods, but rather determines the weight stability intervals,
using the methodology presented in Opricovic (1998).

A compromise solution for a problem with conflicting criteria can help the decision makers to reach a final decision. The foundation for compromise solution was established by Yu (1973) and Zeleny (1982). The compromise solution is a feasible solution, which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. The VIKOR method was introduced as one applicable technique to implement within MCDM (Opricovic, 1998). The TOPSIS method determines a solution with the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Hwang and Yoon, 1981; Yoon, 1987).

In this paper two MCDM methods, VIKOR and TOPSIS are compared, focusing on modelling aggregating function and normalization, in order to reveal and to compare the procedural basis of these two MCDM methods. A comparative analysis is illustrated with a numerical example.

2. VIKOR method

The VIKOR method was developed for multicriteria optimization of complex systems. It determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multicriteria ranking index based on the particular measure of “closeness” to the “ideal” solution (Opricovic, 1998).

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The multicriteria measure for compromise ranking is developed from the \( L_p \)-metric used as an aggregating function in a compromise programming method (Yu, 1973; Zeleny, 1982). The various \( J \) alternatives are denoted as \( a_1, a_2, \ldots, a_J \). For alternative \( a_j \), the rating of the \( i \)th aspect is denoted by \( f_{ij} \), i.e. \( f_{ij} \) is the value of \( i \)th criterion function for the alternative \( a_j \); \( n \) is the number of criteria.

Development of the VIKOR method started with the following form of \( L_p \)-metric:

\[
L_{p,j} = \left\{ \frac{1}{n} \sum_{i=1}^{n} \left( w_i (f_{ij}^* - f_{ij}) / (f_{ij}^* - f_{ij}^-) \right)^p \right\}^{1/p},
\]

\( 1 \leq p \leq \infty; \ j = 1, 2, \ldots, J. \)

Within the VIKOR method \( L_{1,j} \) (as \( S_j \) in Eq. (1)) and \( L_{\infty,j} \) (as \( R_j \) in Eq. (2)) are used to formulate ranking measure. The solution obtained by \( \min S_j \) is with a maximum group utility (“majority” rule), and the solution obtained by \( \min R_j \) is with a minimum individual regret of the “opponent”.

The compromise solution \( F^* \) is a feasible solution that is the “closest” to the ideal \( F^* \), and compromise means an agreement established by mutual concessions, as is illustrated in Fig. 1 by \( \Delta f_1 = f_1^* - f_1^- \) and \( \Delta f_2 = f_2^* - f_2^- \).

The compromise ranking algorithm VIKOR has the following steps:

(a) Determine the best \( f_i^* \) and the worst \( f_i^- \) values of all criterion functions, \( i = 1, 2, \ldots, n. \) If the \( i \)th function represents a benefit then:

\[ f_i^* = \max_j f_{ij}, \quad f_i^- = \min_j f_{ij}. \]

(b) Compute the values \( S_j \) and \( R_j, j = 1, 2, \ldots, J, \) by the relations

\[ S_j = \sum_{i=1}^{n} w_i (f_{ij}^* - f_{ij}) / (f_{ij}^* - f_{ij}^-), \tag{1} \]

\[ R_j = \max_i \left[ w_i (f_{ij}^* - f_{ij}) / (f_{ij}^* - f_{ij}^-) \right], \tag{2} \]

![Fig. 1. Ideal and compromise solutions.](image-url)
where \( w_i \) are the weights of criteria, expressing their relative importance.

(c) Compute the values \( Q_j, j = 1, 2, \ldots, J \), by the relation

\[
Q_j = v(S_j - S^*)/(S^- - S^*) + (1 - v)(R_j - R^*)/(R^- - R^*)
\]

where

\[
S^* = \min_j S_j, \quad S^- = \max_j S_j,
\]

\[
R^* = \min_j R_j, \quad R^- = \max_j R_j,
\]

and \( v \) is introduced as weight of the strategy of “the majority of criteria” (or “the maximum group utility”), here \( v = 0.5 \).

(d) Rank the alternatives, sorting by the values \( S, R \) and \( Q \), in decreasing order. The results are three ranking lists.

(e) Propose as a compromise solution the alternative \( a' \) which is ranked the best by the measure \( Q \) (minimum) if the following two conditions are satisfied:

**C1**. “Acceptable advantage”:

\[
Q(a'') - Q(a') \geq DQ
\]

where \( a'' \) is the alternative with second position in the ranking list by \( Q; DQ = 1/(J-1) \);

\( J \) is the number of alternatives.

**C2**. “Acceptable stability in decision making”: Alternative \( a' \) must also be the best ranked by \( S \) of \( R \). This compromise solution is stable within a decision making process, which could be: “voting by majority rule” (when \( v > 0.5 \) is needed), or “by consensus” \( v \approx 0.5 \), or “with veto” \( v < 0.5 \). Here, \( v \) is the weight of the decision making strategy “the majority of criteria” (or “the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives \( a' \) and \( a'' \) if only condition C2 is not satisfied, or
- Alternatives \( a', a'', \ldots, a^{(M)} \) if condition C1 is not satisfied; and \( a^{(M)} \) is determined by the relation \( Q(a^{(M)}) - Q(a') < DQ \) for maximum \( M \) (the positions of these alternatives are “in closeness”).

The best alternative, ranked by \( Q \), is the one with the minimum value of \( Q \). The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the “advantage rate”.

Ranking by VIKOR may be performed with different values of criteria weights, analyzing the impact of criteria weights on proposed compromise solution. The VIKOR method determines the weight stability intervals, using the methodology presented in Opricovic (1998). The compromise solution obtained with initial weights \( (w_i, i = 1, \ldots, n) \), will be replaced if the value of a weight is not within the stability interval. The analysis of weight stability intervals for a single criterion is performed for all criterion functions, with the same (given) initial values of weights. In this way, the preference stability of an obtained compromise solution may be analyzed using the VIKOR program.

VIKOR is a helpful tool in multicriteria decision making, particularly in a situation where the decision maker is not able, or does not know how to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum “group utility” (represented by \( \min S \), Eq. (1)) of the “majority”, and a minimum of the individual regret (represented by \( \min R \)) of the “opponent”. The compromise solutions could be the basis for negotiations, involving the decision makers’ preference by criteria weights.

### 3. TOPSIS method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang (1992), with reference to Hwang and Yoon (1981). The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution.

The TOPSIS procedure consists of the following steps:

1. Calculate the normalized decision matrix. The normalized value \( r_{ij} \) is calculated as
\[ r_{ij} = f_{ij} \sqrt{\sum_{j=1}^{J} f_{ij}^2}, \]
\[ j = 1, \ldots, J; \ i = 1, \ldots, n. \]

2. Calculate the weighted normalized decision matrix. The weighted normalized value \( v_{ij} \) is calculated as
\[ v_{ij} = w_i r_{ij}, \quad j = 1, \ldots, J; \ i = 1, \ldots, n, \]
where \( w_i \) is the weight of the \( i \)th attribute or criterion, and \( \sum_{i=1}^{n} w_i = 1. \)

3. Determine the ideal and negative-ideal solution.
\[ A^* = \{ v^*_1, \ldots, v^*_n \} = \{ (\max_j v_{ij} | i \in I'), (\min_j v_{ij} | i \in I'') \}, \]
\[ A^- = \{ v^-_1, \ldots, v^-_n \} = \{ (\min_j v_{ij} | i \in I'), (\max_j v_{ij} | i \in I'') \}, \]
where \( I' \) is associated with benefit criteria, and \( I'' \) is associated with cost criteria.

4. Calculate the separation measures, using the \( n \)-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as
\[ D^+_j = \sqrt{\sum_{i=1}^{n} (v_{ij} - v^*_i)^2}, \quad j = 1, \ldots, J. \] (4)
Similarly, the separation from the negative-ideal solution is given as
\[ D^-_j = \sqrt{\sum_{i=1}^{n} (v_{ij} - v^-_i)^2}, \quad j = 1, \ldots, J. \] (5)

5. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative \( a_j \) with respect to \( A^* \) is defined as
\[ C^*_j = D^-_j / (D^+_j + D^-_j), \quad j = 1, \ldots, J. \] (6)

6. Rank the preference order.

Eq. (6) represents the “basic principle” in the TOPSIS method (Chen and Hwang, 1992). In step 5, the sentences “Calculate the relative closeness to the ideal solution. The relative closeness of alternative \( a_j \) with respect to \( A^* \) is defined as:” which are taken from the book by Chen and Hwang (1992, p. 39), although simplified seem incorrect. In Triantaphyllou (2000, p. 21) states that “The best (optimal) alternative can now be decided according to the preference rank order of \( C^*_j \). Therefore, the best alternative is the one that has the shortest distance to the ideal solution. The previous definition can also be used to demonstrate that any alternative which has the shortest distance from the ideal solution is also guaranteed to have the longest distance from the negative-ideal solution”.

This could be correct, but not always (see Section 4.1, condition 2 in Eq. (7)). In fact, the chosen alternative has the maximum value of \( C^*_j \), defined in Eq. (6), with the intention to minimize the distance from the ideal solution and to maximize the distance from the negative-ideal solution. The previous authors did not consider the relative importance of distances \( D^+_j \) (Eq. (4)) and \( D^-_j \) (Eq. (5)) within Eq. (6), although this could be a major concern in decision making. They simply summed \( D^+_j \) and \( D^-_j \) in Eq. (6), without using any parameter that could represent the relative importance of these two distances. This issue was considered by Lai et al. (1994) introducing the “satisfactory level” for both criteria of the shortest distance from the ideal and the farthest distance from the negative ideal, and concluding “The compromise solution will exist at the point where the satisfactory levels of both criteria are the same. In future studies, applying compensatory operators should be emphasized”. Thus, the relative importance remained an open question. Deng et al. (2000) presented a modified TOPSIS method, for which they used the ranking index formulated in Eq. (6).

4. Comparing VIKOR and TOPSIS

The MCDM methods VIKOR and TOPSIS are based on an aggregating function representing closeness to the reference point(s). Our comparative analysis points out that these two methods introduce different forms of aggregating function (\( L_p \)-metric) for ranking. The VIKOR method introduces \( Q_j \) function of \( L_1 \) and \( L_\infty \), whereas the TOPSIS method introduces \( C^*_j \) function of \( L_2 \).
These two MCDM methods use different kinds of normalization to eliminate the units of criterion functions: the VIKOR method uses linear normalization, and the TOPSIS method uses vector normalization.

4.1. Aggregating function

The VIKOR method is based on following aggregating function (derived from \( L_p \)-metric):

\[
L_{p,j} = \left\{ \frac{\sum_{i=1}^{n} \left[ w_i (f^*_i - f_{ij})/(f^*_i - f^{-}_{ij}) \right]^p}{k} \right\}^{1/p},
\]

\[1 \leq p \leq \infty; \quad j = 1, \ldots, J.\]

The measure \( L_{p,j} \) represents the distance of the alternative \( a_j \) to the ideal solution, as introduced by Duckstein and Opricovic (1980).

In the VIKOR method \( S_j = L_{p,j} \) (Eq. (1)) and \( R_j = L_{\infty,j} \) (Eq. (2)), \( j = 1, \ldots, J \), are introduced (as "boundary measures"). The solution obtained by \( \min_j S_j \) is with a maximum "group utility" ("majority" rule). The solution obtained by \( \min_j R_j \) is with a minimum individual regret of an "opponent". According to Eqs. (1) and (2), the VIKOR result stands only for the given set of alternatives. Inclusion (or exclusion) of an alternative could affect the VIKOR ranking of new set of alternatives. This seems logical if VIKOR ranking is considered as a competition. By fixing the best \( f_i^* \) and the worst \( f_i^- \) values, this effect could be avoided, but that would mean that the decision maker could define a fixed ideal solution.

The TOPSIS method introduces an aggregating function for ranking in Eq. (6). According to the formulation of \( C_j^* \) (ranking index), alternative \( a_i \) is better than \( a_k \) if \( C_j^* > C_k^* \) or \( D_j^*/(D_j^* + D_k^-) > D_k^*/(D_k^* + D_k^-) \), which will hold if

1. \( D_j^* < D_k^* \) and \( D_j^- > D_k^- \); or
2. \( D_j^* > D_k^* \) and \( D_j^- > D_k^- \), but

\[
D_j^* < D_k^* D_j^- /D_k^-.
\]

Condition 1 shows the "regular" situation, when alternative \( a_i \) is better than \( a_k \) because it is closer to the ideal and farther from the negative-ideal. On the contrary, condition 2 in Eq. (7) shows that an alternative \( a_i \) could be better than \( a_k \) even though \( a_j \) is farther from ideal then \( a_k \). Let \( a_k \) be the alternative with \( D_k^* = D_k^- \) and \( C_k^* = 0.5 \). In this case, all alternatives \( a_j \) with \( D_j^* > D_k^* \) and \( D_j^- > D_k^- \) are better ranked than \( a_k \), although \( a_k \) is closer to the ideal \( A^* \). The distances considered by VIKOR and TOPSIS are illustrated in Fig. 2. An alternative \( a_j \) is better then \( a_k \) as a TOPSIS result, but \( a_k \) is better then \( a_j \) ranked by VIKOR because \( a_k \) is closer to the ideal solution. The relative importance of distances \( D_j^* \) (Eq. (4)) and \( D_j^- \) (Eq. (5)) was not considered within Eq. (6), although it could be a major concern in decision making.

4.2. Normalization effects

Normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless. The same criterion function could be evaluated in different convertible units, for example: length \( f_i \, [\text{m}] \) or \( f_i \, [\text{km}] \), and temperature \( f_i \, [\text{C}^0] \) or \( f_i \, [\text{F}^0] \). These "convertible" units are related as follows:

\[
\phi_i = \alpha f_i + \beta, \quad \alpha > 0.
\]

Does evaluation of the \( i \)th criterion function as \( f_i \) or \( \phi_i \) affect the result of MCDM method? The answer should be NO, although there are normalization procedures with effects on the final MCDM result (Pavlicic, 2001).

The VIKOR method uses a normalized value as follows:

\[
d_{ij}(f) = (f_i^* - f_{ij})/(f_i^* - f_i^-).
\]
The normalized value $d_{ij}(\phi)$ is the following:

$$d_{ij}(\phi) = (\phi^*_{ij} - \phi^-_{ij})/(\phi^*_{ij} - \phi^-_{ij}).$$

Since in Eq. (8), $\alpha > 0$ and $\beta = \text{const.}$, we have:

$$\phi^*_{ij} = \alpha f^*_{ij} + \beta, \quad \text{and} \quad \phi^-_{ij} = \alpha f^-_{ij} + \beta$$

and the following equality holds:

$$d_{ij}(\phi) = d_{ij}(f).$$

The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function.

The normalized value $r_{ij}$ in the TOPSIS method is calculated as

$$r_{ij}(f) = f_{ij} / \sqrt{\sum_{j=1}^{J} f_{ij}^2},$$

$$r_{ij}(\phi) = \phi_{ij} / \sqrt{\sum_{j=1}^{J} \phi_{ij}^2}$$

$$= (\alpha f_{ij} + \beta) / \sqrt{\sum_{j=1}^{J} (\alpha f_{ij} + \beta)^2}.$$ 

The normalized value in the TOPSIS method could depend on the evaluation unit if $\phi_{ij} = \alpha f_{ij} + \beta$, and the vector normalization leads to $r_{ij}(\phi) \neq r_{ij}(f)$. The equality $r_{ij}(\phi) = r_{ij}(f)$ holds only if $\phi_{ij} = \alpha f_{ij}$.

Linear normalization, such as that in Eq. (9), was subsequently introduced into the TOPSIS method by Lai and Hwang (1994, p. 72), as the following:

$$r_{ij} = f_{ij}/(f^*_{ij} - f^-_{ij}), \quad i \in I' \text{ (benefits)};$$

and

$$r_{ij} = f_{ij}/(f^-_{ij} - f^*_{ij}), \quad i \in I'' \text{ (costs).}$$

The normalized value in Eq. (10) does not depend on the evaluation unit of a criterion function.

**4.3. Essentials of VIKOR and TOPSIS**

The main features of VIKOR and TOPSIS are summarized here in order to clarify the differences between these two methods.

**Procedural basis.** Both methods assume that there exists a performance matrix $\|f\|_{\alpha,\beta}$ obtained by the evaluation of all the alternatives in terms of each criterion. Normalization is used to eliminate the units of criterion values. An aggregating function is formulated and it is used as a ranking index. In addition to ranking, the VIKOR method proposes a compromise solution with an advantage rate.

**Normalization.** The difference appears in the normalization used within these two methods. The VIKOR method uses linear normalization in Eq. (9), and the normalized value does not depend on the evaluation unit of a criterion. The TOPSIS method uses vector normalization, and the normalized value could be different for different evaluation unit of a particular criterion. A later version of the TOPSIS method uses linear normalization in Eq. (10).

**Aggregation.** The main difference appears in the aggregation approaches. The VIKOR method introduces an aggregating function representing the distance from the ideal solution. This ranking index is an aggregation of all criteria, the relative importance of the criteria, and a balance between total and individual satisfaction. The TOPSIS method introduces the ranking index in Eq. (6), including the distances from the ideal point and from the negative-ideal (nadir) point. These distances in TOPSIS are simply summed in Eq. (6), without considering their relative importance. However, the reference point could be a major concern in decision making, and to be as close as possible to the ideal is the rationale of human choice. Being far away from a nadir point could be a goal only in a particular situation, and the relative importance remains an open question (see Section 3). The TOPSIS method uses $n$-dimensional Euclidean distance that by itself could represent some balance between total and individual satisfaction, but uses it in a different way than VIKOR, where weight $v$ is introduced in Eq. (3).

**Solution.** Both methods provide a ranking list. The highest ranked alternative by VIKOR is the closest to the ideal solution. However, the highest ranked alternative by TOPSIS is the best in terms of the ranking index, which does not mean that it is always the closest to the ideal solution. In
addition to ranking, the VIKOR method proposes a compromise solution with an advantage rate.

5. Numerical example

A mountain climber (beginner) must choose an alternative from a set of three alternatives, i.e. destinations \{A_1,A_2,A_3\}. The alternatives are evaluated as presented in Tables 1 and 2. Let us suppose that both criteria are equally important, i.e. the weights of criteria are \(w_i = 1/2\), \(i = 1,2\).

The relationships between values of criteria \(f\) and \(\phi\) are as follows:

\[
\phi_{ij} = f_{ij} + 5 \quad \text{and} \quad \phi_{2j} = f_{2j}/1000 - 1.
\]

The results obtained by the VIKOR and the TOPSIS methods are presented in Table 3.

The compromise solution obtained by VIKOR is \(A_2\). The same solution is obtained for problem \(f\) and problem \(\phi\), illustrating that the "convertible" results with vector normalization depend on the convertible units related as \(\phi_{ij} = \alpha f_{ij} + \beta\) do not affect the result of the VIKOR method.

The final rankings obtained by the TOPSIS criterion values, \(C_j^f\) (\(j = 1,2,3\)), are different for two normatively equivalent problems. Thus, although \(A_1\) is proclaimed as the best solution for problem \(f\), alternative \(A_2\) is suggested as the best for the problem \(\phi\) (see Table 3, “TOPSIS vector normalization”). This shows that the TOPSIS results with vector normalization depend on the convertible units related as \(\phi_{ij} = \alpha f_{ij} + \beta\). According to the TOPSIS results, alternative \(A_2\) is the closest to the ideal \(A^*\) for both problems (\(f\) and \(\phi\)), but different alternatives are denoted as the “farthest” from the “negative ideal”. In problem \(f\), the TOPSIS method considers \(A_1\) as the “farthest” from the “negative ideal”, while in problem \(\phi\), alternative \(A_3\) is proclaimed the “farthest”. The VIKOR method considers both \(A_1\) and \(A_3\) as the “farthest” from the “negative ideal”.

Analyzing the results of TOPSIS, we find that the results from TOPSIS with vector normalization for problem \(f\) are interesting due to the following. According to \(C_1^f\), the best solution is \(A_1\), i.e. \(C_1^f = 0.762\), \(C_2^f = 0.722\); and it is the best according to \(D_1^f\), i.e. \(D_1^f = 0.365\), \(D_2^f = 0.280\). However, \(A_1\) is not the closest to the ideal, i.e. \(D_1^f = 0.114\), \(D_2^f = 0.108\). According to relation (7), in this case, \(A_1\) is ranked best by TOPSIS, although it is not the closest to the ideal, because \(D_1^f < D_2^f D_1^f / D_2^f\), i.e. \(0.114 < 0.141\). This is an example of the case discussed in Section 4.1 in Eq. (7) and presented in Fig. 2.

Ranking results by TOPSIS with vector normalization by Eq. (10) are the same as the results by the VIKOR method (see Table 3, “TOPSIS linear normalization”).

Figs. 3–6 illustrate the positions of the alternatives and their distances to the ideal \(A^*\) and negative-ideal \(A^-\). Weight stability intervals for a single criterion obtained by VIKOR are as follows:

\[
0.415 \leq w_1 \leq 0.630 \quad \text{(input } w_1 = 0.5),
\]
\[
0.370 \leq w_2 \leq 0.585 \quad \text{(input } w_2 = 0.5).
\]

The alternative \(A_2\) is a single compromise solution with the weights within these intervals. It is ranked as first in the set of compromise solution \{\(A_2,A_1\}\) with the weights outside the above intervals, but within the following intervals:

\[
0.366 \leq w_1 \leq 0.746 \quad \text{(input } w_1 = 0.5),
\]
\[
0.254 \leq w_2 \leq 0.634 \quad \text{(input } w_2 = 0.5).
\]
In this example, the alternative $A_2$ is a real compromise, as “something between extremes”. The TOPSIS method with vector normalization selects $A_1$ as a compromise solution for problem $f$, although it is difficult to accept $A_1$ as a compromise being better than $A_2$ (see Fig. 3).

These results illustrate the difference between MCDM methods VIKOR and TOPSIS. The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function, whereas the normalized values by vector normalization in the TOPSIS method may depend on the evaluation unit (see $r(f) \neq r(\phi)$ in Table 3).
6. Conclusions

The MCDM methods VIKOR and TOPSIS are both based on an aggregating function representing “closeness to the ideal”. The VIKOR method introduces the ranking index based on the particular measure of “closeness” to the ideal solution. In contrast, the basic principle of the TOPSIS method is that the chosen alternative should have the “shortest distance” from the ideal solution and the “farthest distance” from the “negative-ideal” solution. The TOPSIS method introduces two “reference” points, but it does not consider the relative importance of the distances from these points.

These two MCDM methods use different kinds of normalization to eliminate the units of criterion functions, whereas the VIKOR method uses linear normalisation, the TOPSIS method uses vector normalization. The normalized value in the VIKOR method does not depend on the evaluation unit of a criterion function, whereas the normalized values by vector normalization in the TOPSIS method may depend on the evaluation unit.

A comparative analysis shows that these two methods use different normalizations and that they introduce different aggregating functions for ranking.

The paper does not consider the trade-offs involved by normalization in obtaining the aggregating function ($Q$ or $C^*$), and this topic remains for further research.

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