

Intuitionistic Fuzzy n – Fold BCI - Positive Implicative Ideals in BCI-Algebras

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Abstract

We consider the notion of intuitionistic fuzzy n -fold positive implicative ideals in BCI-algebras. We analyse many properties of intuitionistic fuzzy n -fold positive implicative ideals. We also establish extension properties for intuitionistic fuzzy n -fold positive implicative ideals in BCI-algebras. This work generalizes the corresponding results in the crisp case.

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1. Introduction

Liu and Zhang [9], Wei and Jun [13] independently introduced BCI-positive implicative ideals and used these to completely describe positive implicative BCI-algebras (namely, weakly positive implicative BCI-algebras). Tebu et al. [11] introduced the notion of n -fold positive implicative ideals in BCI-algebra. Fuzzy ideals are useful tool to obtain results on BCI-algebras. The sets of provable formulas in the corresponding inference systems from the point of view of uncertain information can be described by fuzzy ideals of those algebraic semantics. The idea of “intuitionistic fuzzy set” was first published by Atanassov [1, 2] as a generalization of the notion of fuzzy sets. After that many researchers considered the intuitionistic

fuzzification of ideals and subalgebras in BCK/BCI-algebras. So, in this paper, we discuss the notion of intuitionistic fuzzy n -fold positive implicative ideals in BCI-algebras. We show that every intuitionistic fuzzy n -fold positive implicative ideal is an intuitionistic fuzzy ideal, and give a condition for an intuitionistic fuzzy ideal to be an intuitionistic fuzzy n -fold positive implicative ideal. Using the level set, we provide a characterization of intuitionistic fuzzy n -fold positive implicative ideals. Finally, we establish an extension property for intuitionistic fuzzy n -fold positive implicative ideal.

2. Preliminaries

Let us recall that an algebra $(X, *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

1. $((x * y) * (x * z)) * (z * y) = 0$,
2. $(x * (x * y)) * y = 0$,
3. $x * x = 0$,
4. $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y, z \in X$.

In a BCI-algebra, we can define a partial ordering " \leq " by $x \leq y$ if and only if $x * y = 0$. In a BCI-algebra X , the set $M = \{x \in X / 0 * x = 0\}$ is a subalgebra and is called the BCK-part of X . A BCI-algebra X is called proper if $X - M \neq \emptyset$. Otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:

5. $(x * y) * z = (x * z) * y$,
6. $x * 0 = 0$,
7. if $x \leq y$, then $x * z \leq y * z$ and $z * y \leq z * x$,
8. $0 * (x * y) = (0 * x) * (0 * y)$,
9. $x * (x * (x * y)) = x * y$,
10. $(x * z) * (y * z) \leq x * y$,
11. $0 * (x * y) = 0 * (0 * (y * x))$,
12. $0 * x = 0 * (y * (y * x))$,
13. $(0 * (x * y)) * (y * x) = 0$,
14. if $x \leq y$, then $0 * x = 0 * y$,
15. if $x \leq 0 * y$, then $x = 0 * y$.

Let n be a positive integer. Throughout this paper $X = (X, *, 0)$ denotes a BCI-algebra; $x * y^n = (\dots((x * y) * y) * \dots) * y$, in which y occurs n times; $x * y^0 = x$ and $x * \prod_{i=1}^n y_i$ denotes $(\dots((x * y_1) * y_2) * \dots) * y_n$, where $x, y, y_i \in X$.

An intuitionistic fuzzy set A in a non-empty set X is an object having the form

$$A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \right\},$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Such defined objects are studied by many authors (see for Example two journals:

1. Fuzzy Sets and 2. Notes on Intuitionistic Fuzzy Sets) and have many interesting applications not only in mathematics (See Chapter 5 in the book [2]).

For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \nu_A \rangle$ for the intuitionistic fuzzy set $A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \right\}$.

Definition 2.1[8]: A non empty subset I of X is called an ideal of X if it satisfies:

$$(I_1) 0 \in I,$$

$$(I_2) x * y \in I \text{ and } y \in I \text{ imply } x \in I.$$

Definition 2.2 [8]: A non empty subset I of X is called an BCI-positive implicative of X if it satisfies (I_1) and

$$(I_3) (x * y^2) * (z * y) \in I \text{ and } z \in I \text{ imply } x * y \in I; \text{ for all } x, y, z \in X.$$

Definition 2.3: An IFS $A = \langle \mu_A, \nu_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:

$$(F1) \mu_A(0) \geq \mu_A(x) \ \& \ \nu_A(0) \leq \nu_A(x),$$

$$(F2) \mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \},$$

$$(F3) \nu_A(x) \leq \max \{ \nu_A(x * y), \nu_A(y) \}, \text{ for all } x, y \in X.$$

Definition 2.4: An IFS $A = \langle \mu_A, \nu_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy positive implicative ideal of X if it satisfies $(F1)$ and

$$(F4) \mu_A(x * y) \geq \min \{ \mu_A((x * y^2) * (z * y)), \mu_A(z) \},$$

$$(F5) \nu_A(x * y) \leq \max \{ \nu_A((x * y^2) * (z * y)), \nu_A(z) \}, \text{ for all } x, y, z \in X.$$

Definition 2.5: Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy set of a BCI-algebra X . For $t, s \in [0,1]$, the set $U(x;t) = \{x \in X / \mu_A(x) \geq t\}$ is called upper t-level of A and the set $L(x;s) = \{x \in X / \nu_A(x) \leq s\}$ is called lower s-level of A .

Definition 2.6: Every intuitionistic fuzzy ideal A , μ_A is order reversing and ν_A is order preserving, that is, if $x \leq y$ then $\mu_A(x) \geq \mu_A(y)$ & $\nu_A(x) \leq \nu_A(y)$.

Definition 2.7 [4]: Let X be a BCK-algebra and n be a positive integer. Then X is called an n -fold positive implicative BCK-algebra if $x * y^{n+1} = x * y^n$; for all $x, y \in X$.

Definition 2.8 [11]: Let X be a BCI-algebra and n be a positive integer. Then X is called an n -fold positive implicative BCI-algebra if $(x * y^{n+1}) * (0 * y) = x * y^n$; for all $x, y \in X$.

Definition 2.9 [11]: A nonempty set I of a BCI-algebra is called an n -fold BCI-positive implicative ideal of X if it satisfies (I_1) and

$$(I_4) (x * y^{n+1}) * (z * y) \in I \text{ and } z \in I \text{ imply } x * y^n \in I; \text{ for all } x, y, z \in X.$$

Definition 2.10: Let X be a BCK-algebra and n be a positive integer. Then the intuitionistic fuzzy set A of X is called an intuitionistic fuzzy n -fold positive implicative ideal in BCK-algebra if it satisfies $(F1)$ and

$$(F6) \mu_A(x * y^n) \geq \min \{ \mu_A((x * y^{n+1}) * z), \mu_A(z) \},$$

$$(F7) \nu_A(x * y^n) \leq \max \{ \nu_A((x * y^{n+1}) * z), \nu_A(z) \}, \text{ for all } x, y, z \in X.$$

3. Intuitionistic Fuzzy n – Fold Positive Implicative Ideals in BCI-Algebras

In this section, we introduce the notion of intuitionistic fuzzy n -fold positive implicative ideal in a BCI-algebra and study some important properties.

Definition 3.1: An intuitionistic fuzzy set A of a BCI-algebra X is said to be an intuitionistic fuzzy n -fold positive implicative ideal of X if it satisfies $(F1)$ and

$$(F8) \text{Min} \{ \mu_A((x * y^{n+1}) * (z * y)), \mu_A(z) \} \leq \mu_A(x * y^n),$$

$$(F9) \text{Max} \{ \nu_A((x * y^{n+1}) * (z * y)), \nu_A(z) \} \geq \nu_A(x * y^n) \quad \text{for all } x, y, z \in X.$$

Remark 3.2: (a) Notice that intuitionistic fuzzy 1-fold positive implicative ideal in BCI-algebra is an intuitionistic fuzzy positive implicative ideal in BCI-algebra.

(b) The notion of intuitionistic fuzzy n -fold positive implicative ideal in a BCI-algebra generalizes the notion of intuitionistic fuzzy n -fold positive implicative ideal in BCK-

algebras. This is because if X is a BCK-algebra, for every $x \in X, 0 * x = 0$.

(c) Every intuitionistic fuzzy n -fold positive implicative ideal in BCI-algebra is an intuitionistic fuzzy ideal.

The following example shows that the converse of Remark 3.2 (c) may also be true.

Example 3.3: Consider a BCI-algebra $X = \{0, 1, 2, 3\}$ with Cayley table as follows:

*	0	1	2	3
0	0	0	3	2
1	1	0	3	2
2	2	2	0	3
3	3	3	2	0

Define an IFS $A = \langle \mu_A, \nu_A \rangle$ in X such that $\mu_A(0) = \mu_A(1) = 1, \mu_A(2) = \mu_A(3) = 0.5$ and $\nu_A(0) = \nu_A(1) = 0, \mu_A(2) = \mu_A(3) = 0.2$. Then A is an intuitionistic fuzzy ideal, and also an intuitionistic fuzzy 2-fold positive implicative ideal of X since

$$\min \{ \mu_A((1 * 2^3) * (3 * 2)), \mu_A(3) \} = 0.5 \leq 1 = \mu_A(1 * 2^2)$$

$$\text{and } \max \{ \nu_A((1 * 2^3) * (3 * 2)), \mu_A(3) \} = 0.2 \geq 0 = \nu_A(1 * 2^2).$$

Remark 3.4: In an n -fold positive implicative BCI-algebra, every intuitionistic fuzzy ideal is an intuitionistic fuzzy n -fold positive implicative.

Theorem 3.5: Let A be an intuitionistic fuzzy ideal of a BCI-algebra X . Then the following conditions are equivalent:

(i) $\mu_A((x * y^{n+1}) * (z * y)) \leq \mu_A((x * y^n) * z),$

$$\nu_A((x * y^{n+1}) * (z * y)) \geq \nu_A((x * y^n) * z), \text{ for all } x, y, z \in X,$$

(ii) A is an intuitionistic fuzzy n -fold positive implicative ideal,

(iii) $\mu_A((x * y^{n+1}) * (0 * y)) \leq \mu_A(x * y^n),$

$$\nu_A((x * y^{n+1}) * (0 * y)) \geq \nu_A(x * y^n), \text{ for all } x, y \in X,$$

Proof. (i) implies (ii). Let $x, y, z \in X$. We will prove that

$$\mu_A(x * y^n) \geq \min \{ \mu_A((x * y^{n+1}) * (z * y)), \mu_A(z) \}$$

$$\text{and } \nu_A(x * y^n) \leq \max \{ \nu_A((x * y^{n+1}) * (z * y)), \nu_A(z) \}.$$

Since A is an intuitionistic fuzzy ideal, we have

$$\mu_A(x * y^n) \geq \min \{ \mu_A((x * y^n) * z), \mu_A(z) \} \geq \min \{ \mu_A((x * y^{n+1}) * (z * y)), \mu_A(z) \}$$

$$\text{and } \nu_A(x * y^n) \leq \max \{ \nu_A((x * y^n) * z), \nu_A(z) \} \leq \max \{ \nu_A((x * y^{n+1}) * (z * y)), \nu_A(z) \}.$$

(ii) implies (iii). It is easy to obtain by setting $z=0$.

(iii) implies (i). Let $x, y, z \in X$ with $v = ((x * y^{n+1}) * (z * y))$ and $u = x * z$, it suffices to show that $\mu_A(u * y^{n+1}) \geq \mu_A(v)$ and $\nu_A(u * y^{n+1}) \leq \nu_A(v)$.

$$\begin{aligned} \mu_A(((u * y^{n+1}) * (0 * y)) * v) &= \mu_A((((x * y^{n+1}) * (0 * y)) * ((x * y^{n+1}) * (z * y))) * z) \\ &\geq \mu_A(((z * y) * (0 * y)) * z) \\ &\geq \mu_A((z * 0) * z) = \mu_A(0) \end{aligned}$$

and

$$\begin{aligned} \nu_A(((u * y^{n+1}) * (0 * y)) * v) &= \nu_A((((x * y^{n+1}) * (0 * y)) * ((x * y^{n+1}) * (z * y))) * z) \\ &\leq \nu_A(((z * y) * (0 * y)) * z) \\ &\leq \nu_A((z * 0) * z) = \nu_A(0). \end{aligned}$$

Thus, $\mu_A[(((u * y^{n+1}) * (0 * y)) * v)] = \mu_A(0)$ and $\nu_A[(((u * y^{n+1}) * (0 * y)) * v)] = \nu_A(0)$,

it follows that

$$\mu_A(u * y^n) = \min \{ \mu_A[(((u * y^{n+1}) * (0 * y)) * v)], \mu_A(v) \}$$

$$\text{and } \nu_A(u * y^n) = \max \{ \nu_A[(((u * y^{n+1}) * (0 * y)) * v)], \nu_A(v) \}.$$

Hence, $\mu_A((x * z) * y^n) \geq \mu_A(v)$ and $\nu_A((x * z) * y^n) \leq \nu_A(v)$.

Theorem 3.6: Let X be a BCI-algebra and A be an intuitionistic fuzzy set in X . If A is an intuitionistic fuzzy n -fold positive implicative ideal of X , then

$$\mu_A((x * y^n) * (0 * (0 * y))) \leq \mu_A(x * y^{n+1}) \text{ and } \nu_A((x * y^n) * (0 * (0 * y))) \geq \nu_A(x * y^{n+1}).$$

Proof. Suppose that A is an intuitionistic fuzzy n -fold positive implicative ideal of X . Putting $s = x * y$, then

$$\mu_A((x * y^{n+1}) * (0 * y)) \leq \mu_A(x * y^n)$$

and $\nu_A((x * y^{n+1}) * (0 * y)) \geq \nu_A(x * y^n)$, for all $x, y \in X$.

$$\begin{aligned}
 & \mu_A((((s * y^{n+1}) * (0 * y)) * ((x * y^n) * (0 * (0 * y)))) \\
 &= \mu_A((((x * y^n) * (0 * y)) * y) * y) * ((x * y^n) * (0 * (0 * y))) \\
 &= \mu_A((((x * y^{n+1}) * (0 * y)) * ((x * y^n) * (0 * (0 * y)))) * y) * y) \\
 &\geq \mu_A((((0 * (0 * y)) * (0 * y)) * y) * y) \\
 &= \mu_A((((0 * (0 * y)) * y) * (0 * y)) * y) \\
 &= \mu_A((0 * (0 * y)) * y) \\
 &= \mu_A(0)
 \end{aligned}$$

and

$$\begin{aligned}
 & \nu_A((((s * y^{n+1}) * (0 * y)) * ((x * y^n) * (0 * (0 * y)))) \\
 &= \nu_A((((x * y^n) * (0 * y)) * y) * y) * ((x * y^n) * (0 * (0 * y))) \\
 &= \nu_A((((x * y^{n+1}) * (0 * y)) * ((x * y^n) * (0 * (0 * y)))) * y) * y) \\
 &\leq \nu_A((((0 * (0 * y)) * (0 * y)) * y) * y) \\
 &= \nu_A((((0 * (0 * y)) * y) * (0 * y)) * y) \\
 &= \nu_A((0 * (0 * y)) * y) \\
 &= \nu_A(0).
 \end{aligned}$$

We have $\mu_A(((x * y) * y^{n+1}) * (0 * y)) = \mu_A((s * y^{n+1}) * (0 * y)) = \mu_A(0)$

and $\nu_A(((x * y) * y^{n+1}) * (0 * y)) = \nu_A((s * y^{n+1}) * (0 * y)) = \nu_A(0)$,

since A is an intuitionistic fuzzy n -fold positive implicative ideal, we obtain

$$\mu_A(x * y^{n+1}) = \mu_A((x * y) * y^n) = \mu_A(u * y^n) \geq \mu_A((x * y^n) * (0 * (0 * y)))$$

and $\nu_A(x * y^{n+1}) = \nu_A((x * y) * y^n) = \nu_A(u * y^n) \leq \nu_A((x * y^n) * (0 * (0 * y)))$.

The proof is complete.

Now, we establish some transfer principle for intuitionistic fuzzy n -fold positive implicative ideals in BCI-algebras.

Proposition 3.7: Let A be an intuitionistic fuzzy ideal and I_A be an ideal of a BCI-algebra X . Then I_A is an n -fold positive implicative ideal if and only if its characteristic function $\langle \mu_{I_A}, \nu_{I_A} \rangle$ is an intuitionistic fuzzy n -fold positive implicative ideal.

Proof. Assume that I_A is an n-fold positive implicative ideal. Let $x, y \in X$. If $x * y^n \in I_A$, then

$$\mu_A(x * y^n) \geq \mu_A((x * y^{n+1}) * (0 * y))$$

and $\nu_A(x * y^n) \leq \nu_A((x * y^{n+1}) * (0 * y))$.

If $x * y^n \notin I_A$, then $(x * y^{n+1}) * (0 * y) \notin I_A$ because I_A is an n-fold positive implicative ideal. So, we also have

$$\mu_A(x * y^n) \geq \mu_A((x * y^n) * (0 * y))$$

and $\nu_A(x * y^n) \leq \nu_A((x * y^n) * (0 * y))$.

Conversely, suppose that $\langle \mu_{I_A}, \nu_{I_A} \rangle$ is an intuitionistic fuzzy n-fold positive implicative ideal. Let $x, y \in X$ such that $(x * y^{n+1}) * (0 * y) \in I_A$. Then

$$\mu_{I_A}((x * y^{n+1}) * (0 * y)) = 1 \text{ and } \nu_{I_A}((x * y^{n+1}) * (0 * y)) = 0.$$

Since $\langle \mu_{I_A}, \nu_{I_A} \rangle$ is an intuitionistic fuzzy n-fold positive implicative ideal, $\mu_A(x * y^n) \geq \mu_{I_A}((x * y^{n+1}) * (0 * y)) = 1$ and $\nu_A(x * y^n) \leq \nu_{I_A}((x * y^{n+1}) * (0 * y)) = 0$.

We conclude that $x * y^n \in I_A$.

Theorem 3.8: Let A be an intuitionistic fuzzy ideal in a BCI-algebra X . Then A is an intuitionistic fuzzy n-fold positive implicative if and only if the level set

$A_{\langle t,s \rangle} = \{x \in X / \mu_A(x) \geq t, \nu_A(x) \leq s\}$ is either empty or is an n-fold positive implicative ideal.

Proof. Suppose that A is an intuitionistic fuzzy n-fold positive implicative ideal of X and $A_{\langle t,s \rangle} \neq \emptyset$, for all $t, s \in [0,1]$. We will show that $A_{\langle t,s \rangle}$ is an n-fold positive implicative ideal. Since $A_{\langle t,s \rangle}$ is nonempty, there exists $x_0 \in X$ such that $x_0 \in A_{\langle t,s \rangle}$, since $\mu_A(0) \geq \mu_A(x), \nu_A(0) \leq \nu_A(x)$, for all $x \in X$, we have $\mu_A(0) \geq \mu_A(x_0) \geq t$ and $\nu_A(0) \leq \nu_A(x_0) \leq s$, thus $0 \in A_{\langle t,s \rangle}$.

Let $x, y, z \in X$, such that $(x * y^{n+1}) * (0 * y) \in A_{\langle t,s \rangle}$.

Then $\mu_A(((x * y^{n+1}) * (0 * y))) \geq t$ and $\nu_A(((x * y^{n+1}) * (0 * y))) \leq s$, since A is an intuitionistic fuzzy n-fold positive implicative ideal, we have

$$\mu_A(x * y^n) \geq \mu_A(((x * y^{n+1}) * (0 * y))) \geq t$$

and $\nu_A(x * y^n) \leq \nu_A(((x * y^{n+1}) * (0 * y))) \leq s$.

Hence $x * y^n \in A_{\langle t,s \rangle}$ and $A_{\langle t,s \rangle}$ is an n -fold BCI-positive implicative ideal.

Conversely, assume that $A_{\langle t,s \rangle} \neq \phi$, for all $t, s \in [0,1]$ and $A_{\langle t,s \rangle}$ is an n -fold BCI-positive implicative ideal. We will show that A is an intuitionistic fuzzy n -fold positive implicative ideal.

It is easy to see that $\mu_A(0) \geq \mu_A(x)$, $\nu_A(0) \leq \nu_A(x)$, for all $x \in X$.

Now assume that there exists $a, b \in X$ such that $\mu_A(a * b^n) < \mu_A((a * b^{n+1}) * (0 * b))$ and $\nu_A(a * b^n) > \nu_A((a * b^{n+1}) * (0 * b))$. Setting

$$t_0 = (1/2)[\mu_A(a * b^n) + \mu_A((a * b^{n+1}) * (0 * b))] \text{ and}$$

$$s_0 = (1/2)[\nu_A(a * b^n) + \nu_A((a * b^{n+1}) * (0 * b))], \text{ then}$$

$$\mu_A(a * b^n) < t_0 < \mu_A((a * b^{n+1}) * (0 * b)) \text{ and}$$

$\nu_A(a * b^n) > s_0 > \nu_A((a * b^{n+1}) * (0 * b))$, it follows that $(a * b^{n+1}) * (0 * b) \in A_{\langle t_0, s_0 \rangle}$, but $a * b^n \notin A_{\langle t_0, s_0 \rangle}$. This is a contradiction. Hence A is an intuitionistic fuzzy n -fold positive implicative ideal.

Theorem 3.9: If A is an n -fold intuitionistic fuzzy positive implicative ideal of a BCI-algebra X , then the set $X_{\langle \mu_A, \nu_A \rangle} = \{x \in X, \mu_A(x) = \mu_A(0), \nu_A(x) = \nu_A(0)\}$ is an n -fold positive implicative ideal of X .

Proof. Let A be an n -fold intuitionistic fuzzy positive implicative ideal of X . Clearly $0 \in X_{\langle \mu_A, \nu_A \rangle}$. Let $x, y, z \in X$ such that $(x * y^{n+1}) * (0 * y) \in X_{\langle \mu_A, \nu_A \rangle}$. Then

$$\mu_A(x * y^n) \geq \mu_A((x * y^{n+1}) * (0 * y)) = \mu_A(0)$$

$$\text{and } \nu_A(x * y^n) \leq \nu_A((x * y^{n+1}) * (0 * y)) = \nu_A(0).$$

It follows by $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$ for all $x \in X$, that $\mu_A(x * y^n) = \mu_A(0)$ and $\nu_A(x * y^n) = \nu_A(0)$ so that $x * y^n \in X_{\langle \mu_A, \nu_A \rangle}$. Hence $X_{\langle \mu_A, \nu_A \rangle}$ is an n -fold positive implicative ideal of X .

Theorem 3.10: (Extension property for intuitionistic fuzzy n -fold positive implicative ideals in BCI-algebra). Let A, B be two intuitionistic fuzzy ideals of a BCI-algebra X such that $\mu_A(0) = \mu_B(0), \nu_A(0) = \nu_B(0)$ and $A \subset B$, that is, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$. If A is an intuitionistic fuzzy n -fold positive implicative ideal then B is also an intuitionistic fuzzy n -fold positive implicative ideal.

Proof. Using Theorem 3.5, it is sufficient to show that B satisfies the inequality

$$\mu_B(x * y^n) \geq \mu_B((x * y^{n+1}) * (0 * y))$$

and $\nu_B(x * y^n) \leq \nu_B((x * y^{n+1}) * (0 * y))$ for all $x, y \in X$.

Let $x, y \in X$, setting $r = (x * y^{n+1}) * (0 * y)$, we have

$$\begin{aligned} \mu_B(0) &= \mu_A(0) = \mu_A(((x * y^{n+1}) * (0 * y)) * r) \\ &= \mu_A(((x * y^{n+1}) * r) * (0 * y)) \\ &= \mu_A(((x * r) * y^{n+1}) * (0 * y)) \\ &\leq \mu_A((x * y^n) * r) \\ &\leq \mu_B((x * y^n) * r) \end{aligned}$$

and

$$\begin{aligned} \nu_B(0) &= \nu_A(0) = \nu_A(((x * y^{n+1}) * (0 * y)) * r) \\ &= \nu_A(((x * y^{n+1}) * r) * (0 * y)) \\ &= \nu_A(((x * r) * y^{n+1}) * (0 * y)) \\ &\geq \nu_A((x * y^n) * r) \\ &\geq \nu_B((x * y^n) * r). \end{aligned}$$

Since B is an intuitionistic fuzzy ideal,

$$\mu_B(x * y^n) \geq \min\{\mu_B((x * r) * y^n), \mu_B(r)\} \geq \min\{\mu_B(0), \mu_B(r)\} = \mu_B(r)$$

and $\nu_B(x * y^n) \leq \max\{\nu_B((x * r) * y^n), \nu_B(r)\} \leq \max\{\nu_B(0), \nu_B(r)\} = \nu_B(r)$.

Hence by Theorem 3.5(iii), B is an intuitionistic fuzzy n -fold positive implicative ideal of X .

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