Fuzzy Preference Relation Based Multi-Criteria Decision Making Approach for WiMAX License Award

Tsung-Han Chang, Tien-Chin Wang

Abstract—This paper develops a multi-criteria evaluation approach based on the preference relation to help the National Communication Commission (NCC) in Taiwan award a WiMAX license under fuzzy environment, where the vagueness and subjectivity are handled with linguistic variables parameterized by triangular fuzzy numbers. This study applies the fuzzy multi-criteria decision making (MCDM) method to determine the importance weights of evaluation criteria and synthesize the ratings of possible alternatives. Aggregated the evaluators’ attitude toward possible alternatives; then the non-dominated degree is employed to obtain a crisp overall performance value for each contender to make a final decision. This approach is demonstrated with a real case study involving seven evaluation criteria, eight mobile companies assessed by four evaluators from academia and telecommunication arena. 

Keyword: multiple criteria decision making; fuzzy sets theory; preference relation; WiMAX

I. INTRODUCTION

In the past several decades, the communication services in Taiwan were mostly owned and operated by government. Soon after Taiwan joined the World Trade Organization (WTO), the liberalization policy for telecommunication was implemented, and the National Communication Commission (NCC) announced the free license award in 2000. From then on, Taiwanese mobile telecommunication market formally marched toward the era of free competition. Since the mobile communication industry starts to be operated by private enterprises, they are subject to competitive environments rigorously [1]. For years, 2G and 3G mobile system have been successfully operated all over the world. With the extensive developments in wireless system technology, the powerful standards for various mobile applications have much attraction for markets. The expected demand for mobile data services providing high throughput, excellent quality of service and improved system capacity, has prompted managers to begin screening their options for the best choice in a 4G (WiMAX) mobile environment [2]. WiMAX (Worldwide Interoperability for Microwave Access, as shown in Fig. 1), enabling the mobile Internet to emerge as the latest mobile service, is a broadband application for wireless network. In the end of 2001, the Taiwan Directory General of Telecommunication awarded five 3G licenses.

In 2007, the NCC plans to award a WiMAX license among contenders. The award of WiMAX license has a significant influence not only on customers and 4G operators but also on the domestic mobile telecommunication industry. The challenge of selecting a best contender falls into satisfying multiple objectives, such as increase of governmental tax income, advancement of domestic communication service & technology, faster transmission rate and potential return on investment for 4G communication construction. This paper applied the multi-criteria decision making approach based on preference relation [3] to WiMAX license awarding under a fuzzy environment. This framework could serve as a blueprint for NCC regulators in establishing 4G license awarding mechanism and policies.

The remainder of this paper is organized as follows. Section II briefly introduces fuzzy sets theory as utilized in multi-criteria decision making process. A framework to evaluate a best contender using fuzzy MCDM approach based on preference relation is derived in Section III. Section IV presents an empirical case study, and conclusions are also proposed in Section V.

II. FUZZY SETS THEORY IN MULTI-CRITERIA DECISION MAKING

To express perception or judgement by natural language is always subjective, uncertain or vague. Such uncertainty and subjectivity have long been coped with traditional probability and statistics [4]. Since words are less accurate than numbers,
the concept of linguistic variable approximately characterizes phenomena that are too cumbersome or poorly defined to be described with conventional quantitative scales [5]. In order to resolve the vagueness, ambiguity and subjectivity of human judgment, fuzzy sets theory [6] was introduced to express the linguistic terms in decision-making (DM) process. Bellman and Zadeh [7] were the first researchers to survey the decision-making problem using fuzzy sets, and initiated the fuzzy multicriteria decision-making (FMCDM) methodology. FMCDM was developed to resolve the lack of precision in assigning importance weights of criteria and the ratings of alternatives regarding evaluation criteria [8]. This approach helps decision-makers solve complex decision-making problems in a systematic, consistent and productive way [9], and has been widely applied to tackle DM problems with multiple criteria and alternatives. Some studies have provided interesting results on multicriteria decision-making with the help of fuzzy sets theory [10]-[20]. Applications on solving MCDM problems by fuzzy sets theory have been published in professional journals of diversified disciplines, such as automotive industry [21], transfer strategy selection in biotechnology [22], electronic marketing strategies [23], enterprise intranet website evaluation [24], quality evaluation [25], tool steel material selection [26], election prediction [27], planning and design tender selection [28], nature resource management [29], broadband planning and design tender evaluation [30], students’ answerscripts evaluation [31]-[32], assessment of climate change [33], airline service quality evaluation [34], selecting strategic alliances partners for liner shipping [35] and others [36]-[39]. In the following, for the purpose of reference, some important definitions and notations of fuzzy sets theory from [30]-[39] will be reviewed.

Let $X$ be the universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy set $\tilde{A}$ of $X$ is a set of ordered pairs \[\{(x_1, f_1(x_1)), (x_2, f_2(x_2)), \ldots, (x_n, f_n(x_n))\}\], where $f_i : X \rightarrow [0,1]$ is the membership function of $\tilde{A}$, and $f_i(x_i)$ stands for the membership degree of $x_i$ in $\tilde{A}$.

**Definition 1.** When $X$ is continuous rather than a countable or finite set, the fuzzy set $\tilde{A}$ is denoted as: $\tilde{A} = \int_{x \in X} f_i(x_i) \lambda(x_i) dx$, where $x \in X$.

**Definition 2.** When $X$ is a countable or finite set, the fuzzy set $\tilde{A}$ is represented as: $\tilde{A} = \sum_{i=1}^{n} f_i(x_i) \lambda(x_i)$, where $x_i \in X$.

**Definition 3.** A fuzzy number is a fuzzy subset in the universe of discourse $R$ that is not only convex but also normal.

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**Definition 5.** The $\alpha$-cut $\tilde{A}_\alpha$ and strong $\alpha$-cut $\tilde{A}_{\alpha s}$ of the fuzzy set $\tilde{A}$ in the universe of discourse $X$ is defined by:

\[ \tilde{A}_\alpha = \{x \in X | f_\alpha(x) \geq \alpha \} \]
\[ \tilde{A}_{\alpha s} = \{x \in X | f_\alpha(x) > \alpha \} \]

**Definition 6.** A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is convex if and only if every $\tilde{A}_\alpha$ is convex, that is $\tilde{A}_\alpha$ is a close interval of $\forall \alpha$. It can be written as:

\[ \tilde{A} = [\tilde{A}_1^\alpha, \tilde{A}_2^\alpha] \text{ where } \alpha \in [0,1]. \]

**Definition 7.** A triangular fuzzy number can be defined as a triplet $(a_1, a_2, a_3)$; the membership function of the fuzzy number $\tilde{A}$ is defined as:

\[ f_\alpha(x) = \begin{cases} 
(x - a_1)/a_2, & x \leq a_1, \\
(x - a_2)/a_2, & a_1 \leq x \leq a_2, \\
0, & x > a_2.
\end{cases} \]

Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers (TFN) parameterized by the triplet $(a_1, a_2, a_3)$ and $(b_1, b_2, b_3)$ respectively, then the operational laws of these two triangular fuzzy numbers are as follows.

\[ \tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \]
\[ \tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \]
\[ \tilde{A} \cdot \tilde{B} = (a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3) \]
\[ \tilde{A} \cdot \tilde{B} = (a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = \left( \frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \right) \]
\[ \tilde{A}^{-1} = \left( \frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \right) \]

### III. MULTI-CRITERIA DECISION MAKING BASED ON PREFERENCE RELATION FOR WIMAX LICENSING EVALUATION

Traditional evaluation methods always take the maximum benefits and minimum cost as single index of assessment, though these approaches are insufficient for dealing with the increasing complicated and diversified decision-making environment. This paper utilizes the fuzzy preference relation to evaluate the WiMAX licensing. The evaluation processes are divided into four stages.

Given the possible alternatives $A_i$ ($i = 1, 2, \ldots, m$) to be evaluated by experts $E_j$ ($g = 1, 2, \ldots, k$) with respect to criteria $C_j$ ($j = 1, 2, \ldots, n$). Then the fuzzy multi-criteria decision-making problem can be concisely expressed in matrix format as

\[ \tilde{D} = \begin{bmatrix}
\tilde{D}_{11} & \tilde{D}_{12} & \ldots & \tilde{D}_{1m} \\
\tilde{D}_{21} & \tilde{D}_{22} & \ldots & \tilde{D}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{D}_{n1} & \tilde{D}_{n2} & \ldots & \tilde{D}_{nm}
\end{bmatrix} \begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\vdots \\
\tilde{x}_m
\end{bmatrix} \]

where $\tilde{x}_i$ is the fuzzy rating of alternative $A_i$ with respect
A. Determine the fuzzy importance weight of criterion

Since the effects of criteria on evaluation are instinct with variance, we can not assume that they are of equal importance. There are several techniques adopted to determine the importance weights of criteria [45], such as weighted least square method, linear programming technique for multidimensions of analysis, eigenvector method and analytic hierarchy process. Here, this study provides the evaluators the linguistic variables parameterized with triangular fuzzy numbers to measure the importance weight of criterion. The decision matrix can be depicted as

\[ C_1 \quad C_2 \quad C_3 \quad \ldots \quad C_n \]

\[ W = E^{T} \vec{w}_1 \vec{w}_2 \vec{w}_3 \ldots \vec{w}_n \]

where \( \vec{w}_j \) indicates the fuzzy weight of \( j \) th criterion given by \( k \) th evaluator.

Because the evaluators’ perception toward the importance weight of criterion are diversified, this study therefore applies the method of average to integrate their judgments, that is

\[ \vec{w}_j = \frac{1}{k} \left( \vec{w}_{j1} \oplus \vec{w}_{j2} \oplus \vec{w}_{j3} \oplus \ldots \oplus \vec{w}_{jk} \right) \]

where “\( \oplus \)” denotes addition of triangular fuzzy numbers, \( \vec{w}_j \) is the aggregated fuzzy importance weight of criterion \( C_j \), and it can be denoted by triangular fuzzy numbers as \( \vec{w}_j = (w_{j1}, w_{j2}, w_{j3}) \).

B. Obtain the fuzzy rating and synthetic utility value of alternatives

The evaluators are asked to express their opinions for the rating of possible alternatives with respect to each criterion in linguistic terms. As stated aforementioned, to aggregate the different opinions, this study adopts the synthetic value notation to integrate the subjective judgments for \( k \) evaluators, given by

\[ \bar{x}_i = \frac{1}{k} \left[ \bar{x}_{i1} \oplus \bar{x}_{i2} \oplus \bar{x}_{i3} \oplus \ldots \oplus \bar{x}_{ik} \right] \]

where \( \bar{x}_i \) is the integrated fuzzy rating of \( i \) th possible alternative with respect to \( j \) th criterion, and \( \bar{x}_i \) can be expressed as \( \bar{x}_i = (a_{ij}, b_{ij}, c_{ij}) \).

In order to eliminate anomalies with different measurement units and preserve the property that the fuzzy numbers in the range of \([0,1]\), a linear transformation is used to obtain a normalized fuzzy decision matrix denoted by \( \bar{R} \) as

\[ \bar{R} = \left[ \bar{r}_{ij} \right]_{m \times n} \]

\[ \bar{r}_{ij} = \left( \frac{a_{ij} - b_{ij}}{c_{ij} - b_{ij}} \right), \quad j \in B, \quad c_j = \max \; e_j, \quad \text{if} \; j \in B \]

\[ \bar{r}_{ij} = \left( \frac{a_{ij} - b_{ij}}{c_{ij} - b_{ij}} \right), \quad j \in C, \quad a'_j = \min \; a_j, \quad \text{if} \; j \in C \]

where \( B \) and \( C \) are the sets of benefit criteria and cost criteria respectively.

Considering the different importance weight of each criterion, we can calculate the final synthetic utility value of each alternative, that is

\[ \bar{P} = \sum_{j=1}^{m} \bar{r}_{ij} \otimes \bar{w}_j, \quad i = 1,2,\ldots,m \]

where \( \bar{P} \) is the final synthetic utility value of alternative \( A_i \).

C. Fuzzy preference relation

After the synthetic utility value of each alternative is obtained, the pairwise comparison of fuzzy preference relation between two adjoining alternatives can be made in this subsection. To define a preference relation of alternative \( A_i \) over \( A_j \), this study uses the membership function \( \bar{P} - \bar{P} \) to indicate the preference of \( A_i \) over \( A_j \). Using \( \bar{P} - \bar{P} \) can compare the difference between \( \bar{P} \) and \( \bar{P} \), and they are calculated as

\[ \bar{P}_i = \bar{P} - \bar{P}_j \]

where\( \bar{P}_i = \left[ p^i_1, p^i_2, \ldots, p^i_n \right] \), \( \bar{P}_j = \left[ p^j_1, p^j_2, \ldots, p^j_n \right] \), \( \bar{z}^i_0 = p^i_0 - p^j_0 \), \( \bar{z}^i_0 = p^i_0 - p^j_0 \), \( \bar{z}^i_0 > 0 \), \( \bar{z}^i_0 < 0 \).

If \( \bar{z}^i_0 > 0 \) then \( A_i \) is absolutely preferred to \( A_j \); \( \bar{z}^i_0 < 0 \) then \( A_i \) is not absolutely preferred to \( A_j \). If \( \bar{z}^i_0 < 0 \) and \( \bar{z}^i_0 > 0 \), we define \( e_{ij} \) as a fuzzy preference relation to present the degree of preference of alternative \( A_i \) over \( A_j \).

\[ e_{ij} = \frac{S_i}{S}, \quad S > 0 \]

where

\[ S_1 = \int_{x=a}^{b} \mu_{z^i_0}(x)dx, \quad S_2 = \int_{x=a}^{b} \mu_{z^i_0}(x)dx, \quad S = S_1 + S_2 \]
no difference between $A_i$ and $A_j$.

**D. Ranking alternatives**

By utilizing $e_i^j$, a fuzzy preference relation matrix $E$ can be established as

$$E = [e_i^j]_{mn}$$

According to the fuzzy preference relation matrix $E$, the fuzzy strict preference relation matrix is defined as

$$E' = [e_i^j]_{mn}$$

where

$$e_i^j = \begin{cases} e_i^j - e_j^i, & \text{when } e_i^j \geq e_j^i, \\ 0, & \text{otherwise} \end{cases}$$

The value of $e_i^j$ indicates a degree of strict dominance of alternative $A_i$ over $A_j$, and the non-dominated degree of each alternative can be determined by using the following preference relation matrix [46].

$$\mu^{\alpha}(A) = \min_{j \neq i} e_i^j = 1 - \max_{j \neq i} e_i^j$$

where $\mu^{\alpha}(A)$ is the non-dominated degree of each alternative $A_i$ and $\Omega$ is the set of alternatives. A larger value of $\mu^{\alpha}(A)$ indicates a higher non-dominated performance of alternative $A_i$. Thus this study use the $\mu^{\alpha}(A)$ to rank a set of possible alternatives.

**IV. EMPIRICAL CASE STUDY**

The National Communication Commission (NCC) in Taiwan intends to award a WiMAX license amongst eight contenders. The selection is made by a group of four evaluators $D_1$, $D_2$, $D_3$, and $D_4$. After preliminary screening, three candidates $A_1$, $A_2$, and $A_3$ are remained for further consideration with respect to seven criteria, namely: ratio of debt to total assets ($C_1$), current ratio ($C_2$), cash flow adequacy ratio ($C_3$), management ability ($C_4$), profitability of operation ($C_5$), human resource ($C_6$), and contribution to telecommunication industry ($C_7$). The benefit and cost sets are $B = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$ & $C = \{C_1\}$ respectively. The proposed framework is applied to cope with this MCDM problem.

**Step 1.** The evaluators use the linguistic variables (shown in Table I) to assess the importance weights of criteria and Eq.(13) is used to integrate the fuzzy weights of criteria. The fuzzy importance weight of each criterion is listed as follows.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>54.25</td>
<td>32.8</td>
<td>40.62</td>
</tr>
<tr>
<td>$C_2$</td>
<td>491.31</td>
<td>452.12</td>
<td>432.98</td>
</tr>
<tr>
<td>$C_3$</td>
<td>135.62</td>
<td>152.1</td>
<td>132.6</td>
</tr>
<tr>
<td>$C_4$</td>
<td>126.44</td>
<td>116.37</td>
<td>120.23</td>
</tr>
<tr>
<td>$C_5$</td>
<td>54.25</td>
<td>32.8</td>
<td>40.62</td>
</tr>
<tr>
<td>$C_6$</td>
<td>135.62</td>
<td>152.1</td>
<td>132.6</td>
</tr>
<tr>
<td>$C_7$</td>
<td>126.44</td>
<td>116.37</td>
<td>120.23</td>
</tr>
</tbody>
</table>

Step 2. The evaluators utilized the linguistic terms (shown in Table II) to evaluate the rating of contenders with respect to each criterion and the fuzzy linguistic matrix is listed in Table III. Then Eq. (14) is used to average the evaluators’ opinions into a fuzzy decision matrix, and the fuzzy decision matrix is shown in Table IV.

**Step 3.** Use Eq. (15)-(17) to construct the fuzzy normalized decision matrix as shown in Table V.

**Step 4.** The final fuzzy evaluations of three contenders are computed by utilizing Eq. (18) and are shown as below.

$P_1 = (1.456, 2.828, 4.362); P_2 = (1.393, 2.766, 4.386); P_3 = (1.563, 2.951, 4.483)$

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Candidate</th>
<th>Decision Makers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>A1</td>
<td>54.25</td>
</tr>
<tr>
<td>$C_2$</td>
<td>A2</td>
<td>13.23</td>
</tr>
<tr>
<td>$C_3$</td>
<td>A3</td>
<td>14.27</td>
</tr>
<tr>
<td>$C_4$</td>
<td>A1</td>
<td>491.31</td>
</tr>
<tr>
<td>$C_5$</td>
<td>A2</td>
<td>157.65</td>
</tr>
<tr>
<td>$C_6$</td>
<td>A3</td>
<td>202.83</td>
</tr>
<tr>
<td>$C_7$</td>
<td>A1</td>
<td>135.62</td>
</tr>
<tr>
<td>$C_8$</td>
<td>A2</td>
<td>238</td>
</tr>
<tr>
<td>$C_9$</td>
<td>A3</td>
<td>126.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Candidate</th>
<th>Decision Makers</th>
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<tbody>
<tr>
<td>$C_1$</td>
<td>A1</td>
<td>54.25</td>
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<td>$C_2$</td>
<td>A2</td>
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<td>A1</td>
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</tr>
<tr>
<td>$C_8$</td>
<td>A2</td>
<td>238</td>
</tr>
<tr>
<td>$C_9$</td>
<td>A3</td>
<td>126.44</td>
</tr>
</tbody>
</table>
TABLE IV  
FUZZY DECISION MATRIX  

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>41.86</td>
<td>17.50</td>
<td>13.96</td>
</tr>
<tr>
<td>C2</td>
<td>448.04</td>
<td>183.98</td>
<td>219.22</td>
</tr>
<tr>
<td>C3</td>
<td>(2.75, 5.625)</td>
<td>(2.5, 5.65)</td>
<td>(6, 8.75)</td>
</tr>
<tr>
<td>C4</td>
<td>(3, 5.5, 7.25)</td>
<td>(3.5, 5)</td>
<td>(3.75, 5.25, 6.75)</td>
</tr>
<tr>
<td>C5</td>
<td>(4.5, 6.5)</td>
<td>(4.5, 6.5, 8.25)</td>
<td>(3, 5)</td>
</tr>
<tr>
<td>C6</td>
<td>(4.8, 6)</td>
<td>(4.6, 7.75)</td>
<td>(7, 8.75, 9.75)</td>
</tr>
</tbody>
</table>

**Step 5.** The difference between two evaluation values is calculated by Eqs. (6), (19) and (20), and the results are listed as follows.

\[ \bar{P}_1 - \bar{P}_2 = (1.456 - 4.386, 2.828 - 2.766, 4.362 - 1.393) \]
\[ = (-2.93, 0.062, 2.969); \]
\[ \bar{P}_1 - \bar{P}_2 = (1.456 - 4.483, 2.828 - 2.951, 4.362 - 1.563) \]
\[ = (-3.027, -0.123, 2.799); \]
\[ \bar{P}_2 - \bar{P}_1 = (1.393 - 4.483, 2.766 - 2.951, 4.386 - 1.563) \]
\[ = (-3.09, -0.185, 2.823); \]

**TABLE V**  
FUZZY NORMALIZED DECISION MATRIX  

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.333</td>
<td>0.797</td>
<td>1.000</td>
</tr>
<tr>
<td>C2</td>
<td>1.000</td>
<td>0.411</td>
<td>0.489</td>
</tr>
<tr>
<td>C3</td>
<td>0.584</td>
<td>1.000</td>
<td>0.543</td>
</tr>
<tr>
<td>C4</td>
<td>(0.29, 0.47, 0.66)</td>
<td>(0.26, 0.47, 0.68)</td>
<td>(0.63, 0.84, 1)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.48, 0.76, 1)</td>
<td>(0.41, 0.69, 0.97)</td>
<td>(0.52, 0.72, 0.93)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.59, 0.82, 1)</td>
<td>(0.53, 0.76, 0.97)</td>
<td>(0.35, 0.59, 0.82)</td>
</tr>
<tr>
<td>C7</td>
<td>(0.41, 0.62, 0.82)</td>
<td>(0.41, 0.62, 0.79)</td>
<td>(0.72, 0.9, 1)</td>
</tr>
</tbody>
</table>

**Step 6.** Using Eqs. (21)-(22) to obtain the value of \( e_{12} \), \( e_{13} \) and \( e_{23} \) for constructing the fuzzy preference relation matrix \( E \). Taking \( e_{13} \) as an example, the area of triangular \( S \) is calculated as.

\[ \bar{P}_1 - \bar{P}_2 = (-3.027, -0.123, 2.799); \]
\[ S = \frac{1}{2} [(3.027 + 2.799) \odot 1] = 2.913 \]
\[ S_1 = \left\{ \frac{1}{2} (2.799 \odot x) \right\} \]
\[ 0.123 \quad \frac{1}{2} - x \quad 2.799 \]
\[ x = 0.96 \]
\[ e_{13} = \frac{1.34352}{2.913} = 0.46; \quad e_{23} = 1 - 0.46 = 0.54 \]

Likewise, \( e_{12} \) and \( e_{23} \) can be computed by the same procedures mentioned above.

**Step 7.** Eqs. (24)-(25) are adopted to construct the fuzzy strict preference relation matrix \( E' \).

\[ E' = \left[ \begin{array}{ccc}
0.5 & 0.51 & 0.46 \\
0.49 & 0.5 & 0.45 \\
0.54 & 0.55 & 0.5
\end{array} \right] \]

**Step 8.** Consequently, using Eq. (26) for computing the non-dominated degree of each alternative

\[ \mu_{10}(A_1) = 1 - 0.08 = 0.92 \]
\[ \mu_{10}(A_2) = 1 - 0.1 = 0.90 \]
\[ \mu_{10}(A_3) = 1 - 0 = 1.0 \]

Therefore, the ranking order of these three contenders is \( A_1 > A_2 > A_3 \). This study suggests that the NCC award the contender A3 a WiMAX license.

**V. CONCLUSIONS**

Although MCDM plays an important role in real life problems, it is hard to accept a MCDM approach as being a multidimensional methodology for dealing with various multi-criteria decision making problems all the time. This study presents a systematic and objective MCDM method based on preference relation to award a WiMAX license with respect to qualitative and quantitative criteria simultaneously under fuzzy environment. It can be regarded as one of the contributions of this paper. This study not only allows the evaluators to determine the ranking order of contenders, but express their preference of each neighboring contenders. The evaluation framework could be a reference for regulators and officers of NCC in establishing 4G license awarding mechanism and policies.

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