Role of Mutation Strategies of Differential Evolution Algorithm in Designing Hardware Efficient Multiplier-less Low-pass FIR Filter

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***Abstract*—In recent times, system designers are becoming very much apprehensive in reducing the structural complexity of digital systems with which they deal in practice. However, the uncontrolled minimization of any digital hardware always leads to significant deterioration of system performance making it incompatible for use in any practical system. As proper trade-off is inevitably essential between achievable performance and required hardware, researchers have sought a number of artificially intelligent optimization techniques to solve it out. Since such a technique generally involves variety of constructional alternatives, appropriate use of correct option demands justified attention. Numerous evolutionary computation techniques, being a branch of biologically inspired optimization process, are being increasingly used for a number of signal processing applications of late. This paper throws enough light to select the most suitable mutation strategy of Differential Evolution (DE) algorithm for efficient design of multiplier-less low-pass finite duration impulse response (FIR) filter. Computationally efficient mutation scheme has been identified by observing convergence behavior and error histogram plot for different alternatives. Performance of the designed filter has been compared in terms of its magnitude response and the requirement of various hardware blocks for four different lengths of the filter. Consequently the name of the most favorable mutation rule has been suggested upon analyzing all the factors. Finally the supremacy of our proposed design has been established by comparing its performance with that of other state-of-the-art multiplier-less low-pass FIR filters.**

***Index Terms*—convergence speed, Differential Evolution (DE), error histogram, finite duration impulse response (FIR) filter, sum of power of two, zero-valued filter coefficient (ZFC)**

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I. Introduction

Processing of digital data is accomplished by a wide variety of digital filters which can broadly be classified into two main categories; namely finite duration impulse response and infinite duration impulse response (IIR) filters [1]-[3]. These filters have found their proper relevance in a number of applications including voice/data communication, wireless communication, digital television and so on. Comparative analysis of these two types of filters has proved the superiority of one over other in different respects. Specifically, FIR filters are normally suitable because of its number of attractive characteristics such as linear-phase response, guaranteed stability etc [2]. However, for achieving equivalent magnitude response, FIR filters become computationally more expensive than its IIR counterparts in most of the cases.

A number of research articles have been published in recent times in order to get rid of the complicated structure of FIR filter. In [4], the coefficients of FIR filter have been represented by means of canonical signed digit (CSD), which has reduced the hardware complexity and thereby corresponding delay in a significant way. Another signed power of two representation of FIR filter coefficients have been demonstrated in [5]. The authors have focused to vary the number of signed power of two (SPT) terms for each coefficient resulting in a superior filter performance to those designed by mixed integer linear programming (MILP) and simulated annealing (SA) [6] without increasing the number of adders.

A novel non recursive signed common sub-expression elimination algorithm (NR-CSE) has been developed in [7] and consequently applied in the area of multiplier-less FIR filter design. The proposed NR-CSE algorithm requires a lower number of adders and logical depth along with optimal runtime operation due to its simplicity and thus proves its supremacy over other popular multiplier-less algorithms.

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Another method for designing discrete-coefficient linear-phase FIR filters with minimum normalized peak ripple magnitude has been reported in [8]. One efficient two-stage algorithm, consisting of designing a prototype filter using fast time-domain approximation followed by a Trellis search has been presented in [9]. Design examples have demonstrated the power of the algorithm in producing filters with a better frequency response than methods like [5], [10] and [11] while using fewer SPT terms. Minimization of number of adders to meet the given amplitude criterion has been executed by using one systematic algorithm in [12]. The proposed method has been compared with the results from MILP and Trellis search [9]. The efficiency of this common sub expression elimination based algorithm has been illustrated by means of several examples taken from the literature.

An explicit representation for the distribution of SPT terms of CSD numbers has been illustrated in [13]. Observing the distribution of SPT terms, the authors have proposed one algorithm for designing fixed-point linear-phase FIR filters with CSD coefficients. Design examples have proved the superiority of the proposed method to produce filters possessing a small amount of SPT terms. The design flow for a multiplier-less linear-phase FIR filter has been discussed in [14] where the supremacy of the proposed scheme has been firmly established than techniques like [10] and [11].

Efficient amalgamation of evolutionary computation mechanisms in the field of hardware friendly digital filter design has been studied extensively in literature. In this connection, FIR filter coefficients has been constructed by means of Genetic Algorithm (GA) where the coefficients of the filter has been constrained as sums of power of two [15]. With the aid of the proposed genetic technique, promising results have been achieved which is comparable or better than other state-of-the art techniques.

In recent times, quite a few papers have been published to demonstrate the supremacy of Differential Evolution (DE) algorithm over GA and least square method in connection to conventional digital filter design [16]-[17]. However, this evolutionary technique has not still been tested in designing hardware efficient multiplier-less FIR filter, as far as the research work in this particular domain is concerned. Thus in this paper, an attempt has been made to design a multiplier-free low-pass FIR filter with the help of DE technique. As the solution obtained through DE is greatly influenced by the choice of the mutation strategy of it, proper selection of appropriate mutation law plays an important role in searching the best result.

In this communication, we have extensively studied the impact of various mutation schemes of DE in designing hardware efficient multiplier-less low-pass FIR filter. Performance has been analyzed from different point of view such as computational complexity, magnitude response behavior and the resulting hardware cost. Effect of filter length on its performance has also been thoroughly examined in our work. To prove the superiority of the optimum filter response, quite a few such hardware friendly FIR filters from literature has been taken into consideration and the corresponding performances have been compared. The robustness of the designed filter has further been substantiated by implementing it on a Field Programmable Gate Array (FPGA) chip through XILINX ISE Design Suite 12.3 along with other models of interest.

II. MODELLING OF FIR FILTER

A. Design principle of conventional FIR filter

Finite duration Impulse Response (FIR) filters have gained their enormous applications in the field of communication systems due to ample favorable features. The structure of traditional FIR filters simply consists of a number of delay elements, adders and multipliers where the number of such elements is determined by the length of the filter [1]-[3], [18]. This conventional structure has restricted the use of FIR filter to several systems requiring high speed of operation with a higher degree of precision. This limitation actually arises due to the presence of multipliers in the filter structure and hence the search for multiplier-free structure is on high rise. However, the tap coefficients of an L-length FIR filter, which is nothing but the multiplication constants, can always be represented by means of a row-vector as:

$$h(n) = \{h_0 \; h_1 \; h_2 \; h_3 \; h_4 \; h_5 \; \dots\dots h_{L-1}\} \tag{1}$$

The output sequence $y(n)$of any digital filter is simply obtained by performing convolution between the input sequence $x(n)$ and the impulse response coefficients of the FIR filter [1]-[2]. This mathematical operation can be regarded as a combination of four basic steps like folding, shifting, multiplication and addition. These fundamental mathematical operations can be implemented very easily on an FIR architecture governed by the following equation [1]-[3]:

$$y(n) = \sum_{k=0}^{L-1} h_k . x(n-k) \tag{2}$$

Therefore, computation of a single element of output sequence $y\,(i), i \in Z^+$, thus calls for 'L' multiplications along with 'L-1' additions as observed from (2). The process of multiplication is a precise mathematical task when performed with digital hardware and thus necessitates the replacement of its hardware by some simplified equivalent blocks like delay elements and adders.

B. Design principle of multiplier-less FIR filter

Being motivated by the need of designing hardware friendly filter, digital filter designers have adopted several approaches to get rid of multipliers in the filter model. One such well recognized approach has been to replace the multipliers i.e. the filter coefficients by means of simple delay elements and adders without affecting the performance of the filter to a great extent. The resulting impulse response coefficient of the designed filter thus may have the form:

$$h_i = \sum_{j=0}^{\lambda-1} c_{ij} \,.\, 2^{-j}, \qquad i = 0, 1, 2, \ldots\ldots, L-1 \tag{3}$$

From (3), it can be easily observed that the individual coefficient can be treated as sum of power of two with the parameter 'λ' stands for the resolution of these terms and often known as word length (WL) of filter coefficients. The variable c_{ij} in the above equation holds the key in formulating the coefficients in the sense that they either allow or reject any power of two (PT) terms 2^{-j} to be included into h_i. As the term c_{ij} takes the binary decision of either inclusion or exclusion, it can assume values only from the binary set $\mathbb{B} = \{0, 1\}\ or\ \{0, -1\}$. As a matter of fact; the proper assignment of these coefficients for different values of the subscripts is a problem of optimization and has to be dealt with some powerful evolutionary optimization techniques of current interest.

III. Mathematical Background of Differential Evolution

Different types of non-linear and non-differentiable functions have found their wide application in a number of branches of engineering. In order to optimize these functions, the innovation of a new optimization technique was necessary. In the year 1995, one such new floating point encoded evolutionary computational technique for global optimization had been proposed by Storn and Price [19]-[24]. As this new optimization approach involves a new type of differential operator in it, the method has been termed as Differential Evolution (DE). It incorporates only a few control parameters with negligible parameter tuning and can be implemented very easily. This has made the algorithm very popular since its inception [23]. This particular optimization process can be utilized in an effective way in various engineering problems, such as design of digital filters [25]-[31] electromagnetic inverse problems [32]-[33], antennas [34], composite materials [35] and so on.

Being an evolutionary computation, it always starts with a number of D-dimensional search variable vectors. The total number of such vectors present at the very beginning of the algorithm is termed as the population size (P). The value of the parameter P is kept constant throughout the execution of the entire process. The objective of the algorithm is to optimize any function with D real parameters, subject to different constraints which may be contradictory to each other. The initial D-dimensional population vectors can be represented as [20]-[24]:

$$X_i(1) = [x_{i,1}(1), x_{i,2}\,(1), x_{i,3}(1), \ldots\ldots, x_{i,D}(1)]$$
$$i = 1,2,3, \ldots\ldots, P. \tag{4}$$

These vectors are referred to as the 'chromosomes' and the individual element of each vector is termed as 'gene'. Each of the initial search vectors in the population set undergoes four different operations namely initialization, mutation, recombination and selection respectively, as described elaborately in the following sub-sections.

A. Initialization

Better search results can be obtained from this global optimization mechanism if a suitable range of value is specified for the search variables supposed to be optimized by this technique. Thus if the j^{th} gene of each chromosome has its lower and upper bound as x_j^L and x_j^U respectively, then the corresponding gene of i^{th} chromosome can be initialized as [23]-[24]:

$$x_{i,j}(1) = x_j^L + rand\ (0,1).\,(x_j^U - x_j^L) \tag{5}$$

where $i = 1,2,3, \ldots\ldots, P$ and rand (0, 1) is a uniformly generated random number in [0, 1]. If any preliminary solution is available, the initial population is most often generated by adding normally distributed random deviations to the nominal solution [24].

B. Mutation

Initialization process only takes care of a particular portion of the entire search space. This search space has to be explored in order to find out the optimum solution of the given problem. The expansion of the search space is actually performed in the process of mutation in which various parameter vectors are combined in such a way to generate a new population vector, termed as mutant or donor vector. Depending upon the way the mutant vectors are generated from the population vector, there may exist different mutation strategies. Accordingly, the DE technique is termed differently for different mutation strategies.

In DE/rand/1 scheme, donor vector of generation 'G+1' can be formed from the population vector of the generation 'G' as [19]-[22]:

$$V_i(G+1)_{rand/1} = X_{P1}(G) + F.\,[X_{p2}(G) - X_{p3}(G)] \tag{6}$$

In (6), F is called the weighting factor with $i, p_1, p_2, p_3 \in \mathbb{P}$, where $\mathbb{P}$ is the set of population members of the evolutionary computation. The parameter F is responsible for the amplification of the differential variation and generally lies within the range [0, 2].

DE/rand to best /1 scheme involves another control parameter other than the weighting factor. Apart from using two different population vectors, this mutation strategy also includes the current population vector and the best parameter vector of the current generation to generate a mutant vector of next generation as indicated in the following equation [23]-[24]:

$$V_i(G+1)_{randtobest/1} = X_i(G) + \lambda.\,[X_{best}(G) - X_i(G)] + F.\,[X_{p1}(G) - X_{p2}(G)] \tag{7}$$

The term λ in (7) assumes a value in the range [0, 2].

The donor vector of any particular generation can also be generated by incorporating the best vector of the current generation. The consequent DE scheme is referred to as DE/best/1 and DE/best/2 respectively. The only difference between those two mutation schemes is that one uses more parameter vector as compared to other. This can easily be observed from equations (8) and (9) respectively [23]-[24]:

$$V_i(G+1)_{best/1} = X_{best}(G) + F.[X_{p1}(G) - X_{p2}(G)] \quad (8)$$

$$V_i(G+1)_{best/2} = X_{best}(G) + F.[X_{p1}(G) + X_{p2}(G) - X_{p3}(G) - X_{p4}(G)] \quad (9)$$

DE/rand/2 mutation strategy involves five different parameter vectors and two different weighting factors for the generation of a mutant vector from the parameter vector of current iteration. The donor vector of this strategy can be mathematically represented as [23]-[24]:

$$V_i(G+1)_{rand/2} = X_{p1}(G) + F_1.[X_{p2}(G) - X_{p3}(G)] + F_2.[X_{p4}(G) - X_{p5}(G)] \quad (10)$$

where F_1 and F_2 are two weighting factors selected from the range [0, 1] and $i \neq p_1 \neq p_2 \neq p_3 \neq p_4 \neq p_5$

C. Recombination

Successful solutions from the previous generations are incorporated in the third step of DE, called cross-over or recombination. In order to enhance the potential diversity of the population, the process of recombination plays a very important role. It is nothing but a scheme of cross-over where donor vector exchanges its components, i.e., genes with the target vector. Two types of recombination exist in the literature, namely exponential and binomial cross-over. In exponential scheme, a random number n is generated from the interval [0, D-1] or [1, D], indicating the particular gene of the target vector from where the exchange of body parts or components with donor vector will actually start. Another integer is also chosen from the set [1, D] to signify the number of components contributed by the donor vector to the target vector. Selection of this integer is dependent on a uniformly generated random number within [0, 1] and a cross-over constant. As a result, another important parameter called cross-over probability (CR) plays a significant role along with the weighting factor (F) in the optimization process. The trial vector $U_i(G+1)$ in case of exponential cross-over has the form [20]-[24]:

$$U_i(G+1) = [u_{i,1}(G+1), u_{i,2}(G+1), \ldots\ldots, u_{i,D}(G+1)] \quad (11)$$

$$u_{i,j}(G+1) = \begin{cases} v_{i,j}(G+1); & j = mod(n-k+1, D) \\ x_{i,j}(G); & otherwise \end{cases} \quad (12)$$

where $k = 1,2,\ldots\ldots,L$ and L is another integer from the interval [1, D]. The parameter L signifies the number of components contributed by the donor vector to the target vector.

In case of binomial cross-over scheme, the cross-over is performed on each of the D genes whenever the uniformly generated random number is less than the cross-over or recombination probability (CR). The scheme has been depicted as [21]-[24]:

$$u_{i,j}(G+1) = \begin{cases} v_{i,j}(G+1); & rand(0,1) < CR \\ x_{i,j}(G); & elsewhere \end{cases} \quad (13)$$

D. Selection

There is a definite possibility that the trial vector of current generation may be included as a parameter vector of next generation just by replacing the corresponding parameter vector of current generation. This decision is taken at the last step of DE, called selection process. Depending upon the formulation of the given problem, a cost function is always associated with it. If the trial vector results in a lower cost function than parent parameter vector; the corresponding trial vector has been allowed to take part in the process of mutation and recombination in the next iteration. Otherwise, the present parameter vector passes through the next generation. This can well be described by means of the following equation [21]-[23]:

$$X_i(G+1) = \begin{cases} U_i(G+1); & \Im(U_i(G+1)) < \Im(X_i(G+1)) \\ X_i(G); & \Im(X_i(G)) < \Im(X_i(G+1)) \end{cases} \quad (14)$$

where $\Im(Y_i(G))$ identifies the cost functional value resulting from the i^{th} vector $Y_i(G)$ at iteration 'G'. It is noteworthy to mention here that the choice of the proper cost function is of paramount significance as far as the convergence behaviour of the optimization method is concerned.

The termination of the DE technique can be specified by means of a maximum allowable value of the averaged cost function. As long as the averaged cost function is more than this threshold value, the process of mutation, recombination and selection continues. When this cost function goes below the threshold; the algorithm stops and the parameter vector having the minimum cost function is chosen as the optimized solution to the given problem.

III. Mathematical Formulation of FIR Filter Design

The consequence of different mutation schemes of DE in designing multiplier-less low-pass FIR filter has been extensively studied in our work. Being an optimization algorithm, the efficiency of DE is always monitored by a mathematical expression called the cost function whose formulation is strongly dependent on the specific problem of concern. In many situations, for a fixed structure of error function, the cost functional value and thus the fitness value fluctuates appreciably depending upon the selection of structural strategy of the optimization technique. Thus the fitness value and cost functional value may be regarded as two different functions of identical items, as outlined in the following equations:

$$fitness\ value = \psi(\phi, \mathbb{M}, \mathbb{R}, \zeta) \quad (15)$$

$$cost\ functional\ value = \sigma(\phi, \mathbb{M}, \mathbb{R}, \zeta) \quad (16)$$

The parameter $'\phi'$ in (15) and (16) reflects the formulation of the error function and for the design of a low-pass filter with a cut-off frequency ϖ takes the form as shown in the following equation:

$$\varphi(k, p_i) = \begin{cases} \min_i[\max_k\{|1 - H_{p_i}(k)|\} & for\ k \le \frac{\varpi N}{2\pi} \\ \min_i[\max_k\{|H_{p_i}(k)|\} & for\ k > \frac{\varpi N}{2\pi} \end{cases} \quad (17)$$

; with $H(k) = \sum_{n=0}^{L-1} h(n) e^{\frac{-j2\pi kn}{L}}$ and $p_i \in \mathbb{P}$.

The other parameters within the parenthesis in (15) symbolize the set of mutation strategy, recombination strategy and control parameter and has been outlined as in (18), (19) and (20) respectively:

$$\mathbb{M} = \{rand/1, rand/2, best/1, best/2, rand\ to\ best/1\} \quad (18)$$

$$\mathbb{R} = \{binomial, exponential\} \quad (19)$$

$$\zeta = \{weighting\ factor(s), recombination\ probability\} \quad (20)$$

This paper investigates the most appropriate mutation rule for which the fitness value is the highest irrespective of the population size and iteration number of the evolutionary technique. Thus the optimized mutation scheme results in a fitness value which must satisfy the inequality, as proposed in the following equation:

$$fitness\ value\|_{\phi,\mathbb{M}_{opt},\mathbb{R}_{nom},\zeta_n} > fitness\ value\|_{\phi,\mathbb{M}_i,\mathbb{R}_{nom},\zeta_n} \quad (21)$$

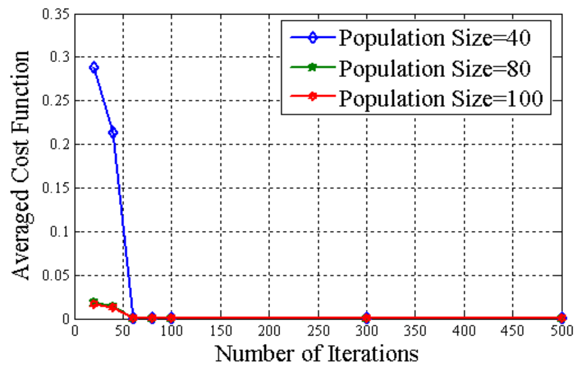
In this work, binomial recombination strategy has been chosen since it has been regarded as the conventional one for the most of the applications and it has been designated as $\mathbb{R}_{nom}$. Our main objective is to identify the suitable mutation rule $\mathbb{M}_{opt}$ for which the value of the fitness yields the highest score i.e. the lowest cost. However, in this approach, the control parameters of DE like weighting factor(s) and recombination probability have been maintained at its nominal values ζ_n. TABLE I below summarizes the choice of several parameters for our design problem of interest.

TABLE I
CHOICE OF DIFFERENT PARAMTERS FOR DESIGNING MULTIPLIER-LESS LOW-PASS FIR FILTER USING DIFFERENTIAL EVOLUTION ALGORITHM

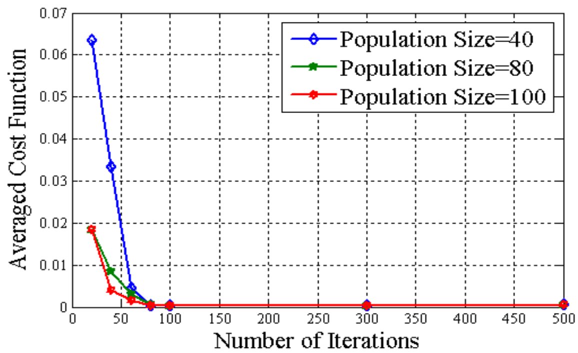
Parameter	Value
mutation rule	rand/1, rand/2, best/1, best/2, rand to best/1
weighting factor(s)	0.5
cross-over or recombination rule	binomial
cross-over or recombination probability	0.5
selection strategy	minimax
error threshold	10^{-4}
population size	40, 80, 100
maximum iteration number	500

IV. SIMULATION RESULTS & ITS CRITICAL ANALYSIS

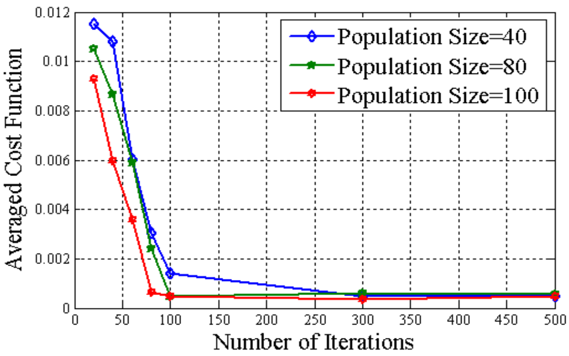
Computational efficiency of any optimization technique can be best evaluated in terms of its convergence curve, which shows the variation of averaged cost function with the number of iterations employed in the algorithm. Higher the computational efficiency, lesser the required number of iteration is. Convergence behavior of five different mutation strategies of DE, as discussed in this paper, has been depicted in Fig. 1 (a) to Fig. 1 (e) below.



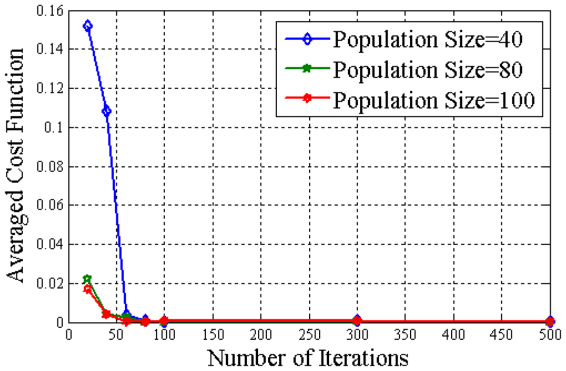
(a) rand/1



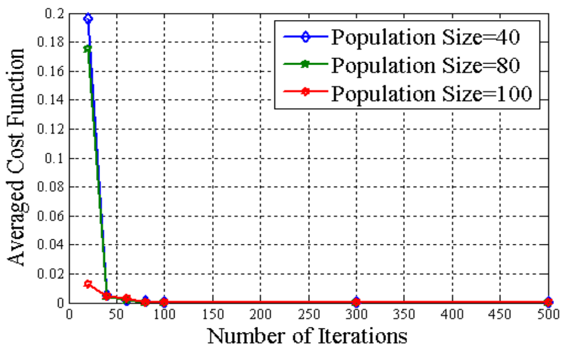
(b) rand/2



(c) best/1



(d) best/2

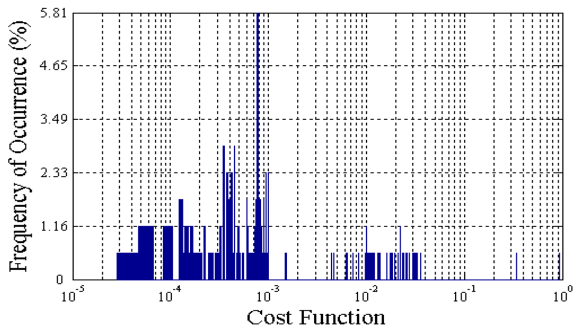


(e) rand to best/1

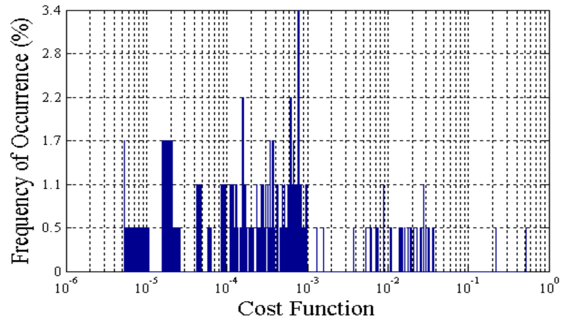
Figure 1 Convergence behavior of DE for the design of multiplier-less low-pass FIR filter

From Fig. 1, one useful observation can be made regarding the impact of mutation strategy on the size of the population of this evolutionary computational mechanism. As it can be seen from Fig. 1 (d), best/1 approach is the only mutation technique whose performance remains almost independent on the size of the population even at a lower number of iterations. For example, the cost function incurred with best/1 strategy is significantly less (1/5 to 1/10 times) than its counterparts for a population size of 40 with an iteration of 20. Moreover, for higher sizes of population, the best/1 technique also maintains its supremacy over its other competitors.

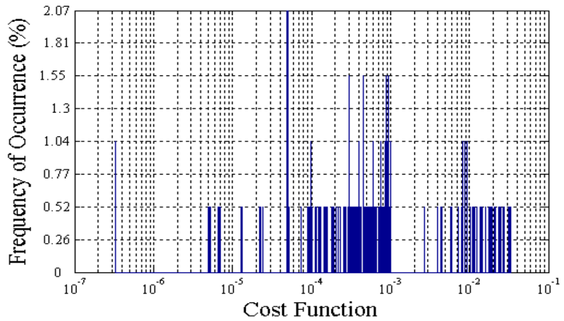
Fig. 1 only reflects the averaged value of the cost function with no information about the presence of individual cost function obtained through a number of trial runs. Occurrence of these individual values can be made presentable with the aid of a diagram called 'error histogram' which describes the variation of frequency of occurrence against each cost term. Peaks in the error histogram identify the error values with greater frequency. Therefore by observing only the peaks in the histogram, it would be very easy to comment on the computational efficiency of any mutation technique. Such a diagram has been demonstrated in Fig. 2 for five different mutation rules of Differential Evolution algorithm.



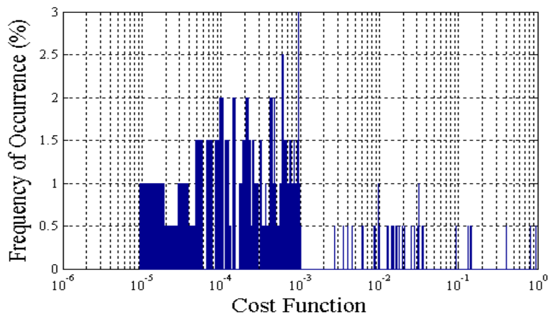
(a) rand/1



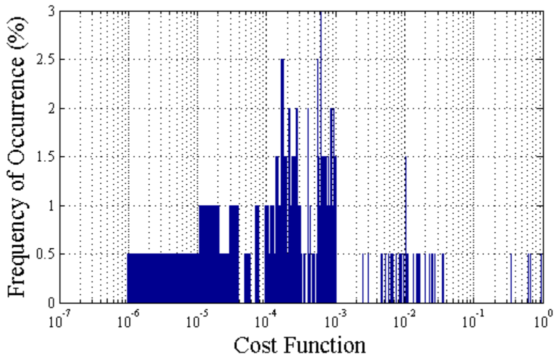
(b) rand/2



(c) best/1



(d) best/2



(e) rand to best/1

Figure 2 Error histogram of DE for five different mutation schemes

The effectiveness of best/1 strategy over the other existing techniques can well be understood from the plots of error histograms as indicated in Fig. 2. As for example, the maximum limit of cost functional value obtained with best/1 technique has been found to be 10^{-1} or 0.1; while the same limit is equal to 10^{0} or 1.0 for other mutation techniques. This fact has been substantiated by the presence of the discrete spikes between the range of cost functional values from 10^{0} (1.0) to 10^{-1} (0.1) for the other mutation strategies; where as, best/1 strategy does not provide so.

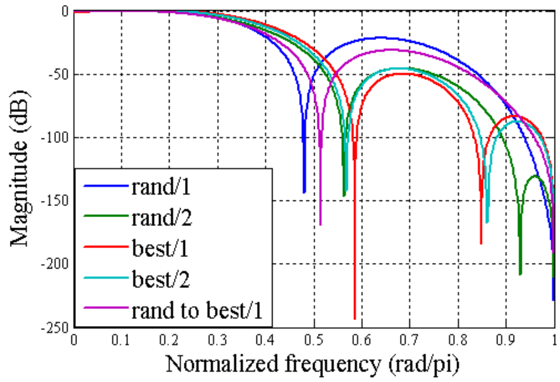
However, the majority of the error values are concentrated near the center of the plot (largely from 10^{-3} to10^{-5}) irrespective of the mutation technique employed. This dense concentration of error values actually accounts for the saturation region which spans through a wider number of iterations in connection to this specific design problem of FIR filter. Numerical characterization of cost function resulting from different mutation approaches and the corresponding simulation time for each of these procedures has been listed in TABLE II. The entire simulation has been carried out using MATLAB 7.0 software with Intel Pentium 4 CPU with 2 GB of RAM.

TABLE II
NUMERICAL VALUES OF COST FUNCTION (POPULATION SIZE=80)

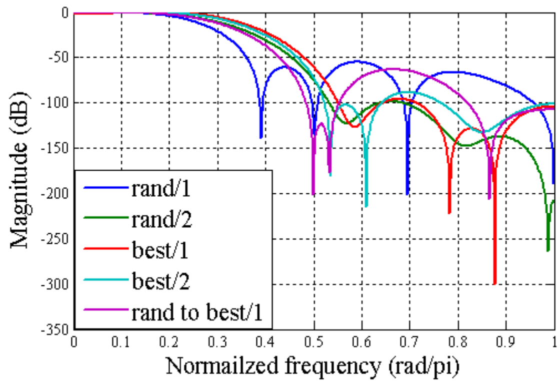
Rule	Cost functional value				Average simulation time
	Max	Min	Average	Std. Dev	
rand/1	0.0356	0.0000498	0.006115	0.009616	23.814 s
rand/2	0.0358	**0.0000019**	0.004959	0.009022	24.67 s
best/1	**0.0332**	0.00000515	**0.00399**	**0.00737**	**15.503 s**
best/2	0.148	0.000009	0.00463	0.0189	19.258 s
rand to best/1	0.6679	0.000021	0.028914	0.120897	20.897 s

From the numerical results in Table II, it can be observed that best/1 mutation technique requires the least average simulation time amongst all and thus proves to be the most efficient one as far as computational time is concerned. Furthermore, the error performance exhibited by this particular mutation scheme also looks fairly attractive since the average and standard deviation of cost function attains its minimum value with best/1 scheme.

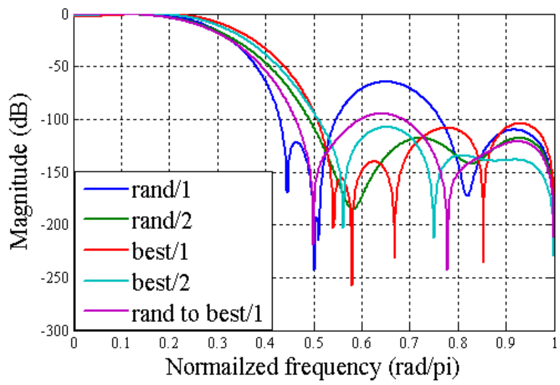
The impact of different mutation strategies associated with DE has been thoroughly studied in this paper for the design of hardware efficient multiplier-free low-pass FIR filter. From the computational point of view, the supremacy of best/1 scheme has firmly been established. But the effect of this superiority in terms of filter response is yet to be investigated. For this purpose, the magnitude response of the multiplier-free low-pass FIR filter for four distinct lengths, namely 8, 15, 22 and 29, has been plotted in Fig. 3 (a) through Fig. 3 (d) respectively for different mutation strategies.



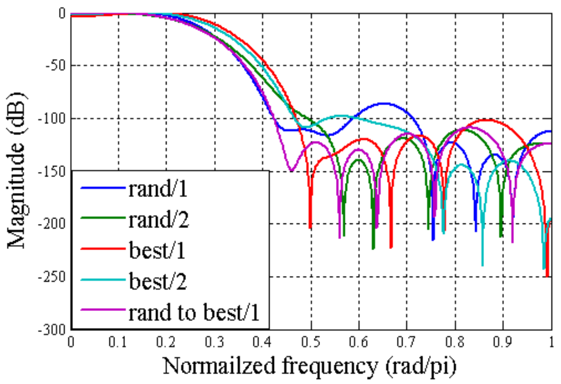
(a) 8-length



(b) 15-length



(c) 22-length



(d) 29-length

Figure 3 Magnitude response of the multiplier-less low-pass FIR filter designed with the aid of Differential Evolution algorithm

From the nature of the magnitude response, it can be clearly seen that irrespective of the order of the filter, all of these five mutation strategies behave identically in the pass-band region of interest. However, their performance differs significantly in the stop-band region and thus compels us to select the most appropriate mutation rule based on the information available from the value of the stop-band attenuation provided by the designed low-pass filter. In this connection, maximum and minimum stop-band attenuation can serve as a useful means to comment on the quality of the achievable solution. These two parameters have been listed in TABLE III for four different lengths of the FIR filter as considered in our study.

TABLE III
FILTER PERFORMANCE IN TERMS OF STOP-BAND ATTENUATION

Length	Stop-band attenuation (dB)	rand/1	rand/2	best/1	best/2	rand to best/1
8	Max	210.2	208.4	**243.5**	167.6	191.1
	Min	21.76	45.54	**49.83**	45.75	31.27
15	Max	201.4	263.7	**300.7**	215.3	206.6
	Min	54.95	**98.5**	95.77	88.6	63.29
22	Max	242.3	207.3	**257.3**	212	243.2
	Min	64.61	**117.8**	104.1	107.5	94.77
29	Max	215.1	223.7	**250.2**	239.5	218
	Min	86.43	**111.1**	101.8	98.1	109

Looking at TABLE III, it can be well apprehended that in terms of maximum stop-band attenuation, best/1 strategy outperforms the other alternatives by a large margin as it always yields the highest maximum stop-band attenuation amongst five. On the other hand, in terms of minimum stop-band attenuation, the performance shown by best/1 rule is well acceptable by virtue of the fact that except 8-length filter, it always provides an attenuation value which is more than 90 dB.

As the design of hardware efficient FIR filter model is considered in this work, a number of performance parameter is therefore selected in order to compare the hardware cost of different architectures. These include total power of two (TPT) terms, multiplier delay flip-flop (MDF), multiplier adders (MA) and zero-valued filter coefficient (ZFC) in the impulse response. Such a comparative analysis in terms of these parameters has been carried out in this article whose outcomes have been elaborately illustrated in TABLE IV to TABLE VII.

TABLE IV
FILTER PERFORMANCE IN TERMS OF HARDWARE BLOCKS
(FILTER LENGTH=8)

Mutation strategy	TPT	MDF	MA	ZFC
rand/1	34	**124**	26	0
rand/2	**30**	136	**22**	0
best/1	42	172	34	0
best/2	32	130	24	0
rand to best/1	38	150	30	0

TABLE V
FILTER PERFORMANCE IN TERMS OF HARDWARE BLOCKS
(FILTER LENGTH=15)

Mutation strategy	TPT	MDF	MA	ZFC
rand/1	31	**135**	18	**2**
rand/2	33	151	20	**2**
best/1	**30**	140	**15**	0
best/2	32	146	17	0
rand to best/1	35	155	22	**2**

TABLE VI
FILTER PERFORMANCE IN TERMS OF HARDWARE BLOCKS
(FILTER LENGTH=22)

Mutation strategy	TPT	MDF	MA	ZFC
rand/1	**36**	**158**	**18**	4
rand/2	**36**	178	24	**6**
best/1	**36**	172	**18**	4
best/2	42	202	24	4
rand to best/1	40	190	22	4

TABLE VII
FILTER PERFORMANCE IN TERMS OF HARDWARE BLOCKS
(FILTER LENGTH=29)

Mutation strategy	TPT	MDF	MA	ZFC
rand/1	59	285	38	8
rand/2	44	219	25	10
best/1	46	228	27	10
best/2	**34**	**138**	**17**	**12**
rand to best/1	45	221	26	10

Careful observation of the numerical entries, as made in TABLE IV to TABLE VII, adds some essential flavors to our analysis. They unanimously suggest that there may not subsist a single mutation scheme which proves to yield minimum hardware cost in comparison to its competitors. For example, during the design of lower order FIR filter, the best performance is shown either by rand/1 or by rand/2 or by best/1. However, when the length of the FIR filter has been enhanced to 29; best/2 approach outperforms all by a considerable margin. This phenomenon is mostly due to the incorporation of maximum number of ZFC in the designed impulse response coefficient, as indicated in TABLE VII. Moreover, it includes minimum MA and moderate number of TPT and MDF amongst all architectures and proves to be more lucrative to be used as a hardware friendly model. In order to examine its proficiency to be employed as a low-pass filter, we need to refer to the numerical entries in TABLE III. It follows from TABLE III that the low-pass characteristic of the filter designed by best/2 technique is practically satisfactory since it provides a maximum and minimum stop-band attenuation of around 240 and 100 dB respectively.

In order to prove the robustness of Differential Evolution algorithm in designing multiplier-less low-pass FIR filter, a number of such multiplier-free FIR model has been selected from literature and consequently their performance has been compared in terms of various parameters and consequently listed in TABLE VIII. FIR filter of length 29, designed with the help of best/2 mutation rule of DE, is used as an optimum filter in this respect whose performance is compared with other state-of-the-art hardware friendly FIR structures.

TABLE VIII
COMPARATIVE ANALYSIS AMONGST VARIOUS STATE-OF-THE-ART MULTIPLIER-LESS FIR FILTERS IN TERMS OF HARDWARE BLOCKS

Algorithm	Length	WL	TPT	MDF	MA	ZFC
Lim [8]	35	10	58	378	23	0
Chen [9]	28	13	58	414	34	4
Saramaki [12]	29	11	49	326	26	6
Yao [13]	28	13	54	386	30	4
Jheng [14]	29	12	51	386	29	4
Proposed	29	**8**	**34**	**138**	**17**	**12**

TABLE VIII unambiguously suggests the proposed approach as a useful means to construct hardware friendly FIR model because of a number of reasons. Firstly, it has been possible to devise the multiplier-less FIR structures with a word length (WL) value of 8 using our approach which the researchers have failed to design previously. This will have the consequence of involving less hardware blocks in comparison with other structures. This statement can also be justified after looking at the other parameters in TABLE VIII. As can be noticed from the last row of the above table, proposed filter model is always in need of fewer TPT, MDF and MA than the design in [8], [9], [12], [13] and [14]. Amongst them, the filter designed by Saramaki [12] necessitates the use of minimum TPT and MDF. Percentage improvement resulting from our approach with respect to [12] is 30.61% and 57.67% in terms of TPT and MDF respectively. Impulse response coefficient of proposed FIR filter involves the use of 26.09% less MA than Lim's algorithm [8], which require the least MA amongst the other models considered in our work for the purpose of comparison. Moreover, using best/2 mutation scheme of DE, we became successful in obtaining more than 40% ZFC which will speed up the system response significantly.

As a second method of comparing the hardware costs between other state-of-the-art multiplier-less FIR filters and the proposed design, all of them have been realized on an FPGA kit using XILINX ISE Design Suite 12.3. The outcome of the experiment has been serially tabulated in TABLE IX below.

TABLE IX
COMPARATIVE ANALYSIS AMONGST VARIOUS STATE-OF-THE-ART MULTIPLIER-LESS FIR FILTERS AFTER REALIZING ON FPGA KIT

Algorithm	Length	I/O Buffer	Full Adder	Subtractor	Delay
Lim [8]	35	3	17	1	39
Chen [9]	28	3	18	1	38
Saramaki [12]	29	3	14	1	39
Yao [13]	28	3	18	1	38
Jheng [14]	29	3	17	1	39
Proposed	29	3	**13**	1	**30**

The implementation results reveal the fact that all of these architectures are requiring identical number of I/O buffers and subtractors. However, in terms of full-adder and delay requirement, different algorithms behave quite differently and this can help in selecting the most favorable design alternative in this respect. As can be observed from the numerical entries in TABLE IX, proposed scheme requires minimum number of full-adder and delay flip-flops amongst all of the multiplier-free architectures considered in this article.

Accumulation of all these results, as described in this paper, will provide us with a number of interesting dimensions. It has already been proved that best/1 mutation rule is the fastest amongst all and requires less number of iterations to converge. This has not certainly indicated a pre-mature solution of the optimization mechanism since resulting filter performance from best/1 rule is very much promising. When the attention is focused towards the associated hardware cost, it has been inferred that best/2 strategy outperforms best/1 and even other popularly known multiplier-less FIR architectures when the length of the filter is selected as 29. However, the convergence speed and magnitude response characteristic of best/2 strategy is highly satisfactory and therefore leads to an optimum filter response in this regard.

V. CONCLUSIONS

Influence of bio-diversity in the field of evolutionary computation has led to a number of evolutionary computational techniques which are popularly employed in the vast area of signal processing. Such a robust computational technique called Differential Evolution algorithm has been efficiently employed in this paper in connection to the design of multiplier-less low-pass FIR filter. Since the performance of evolutionary approaches is largely governed by the way evolution takes place, the impact of various mutation strategy of DE has been critically analyzed in this paper. The detailed analytical study has established the fact that best/1 mutation strategy can be regarded as the best approach for this specific design problem as far as the convergence speed and the low-pass nature of the designed filter is concerned. It has also been concluded that for the design of lower order filter, no single mutation rule comes up with reduced hardware complexity. However when the filter length is enhanced to 29, best/2 mutation rule outperforms the other alternatives in terms of associated hardware cost. Moreover, the superiority of best/2 mutation scheme over other state-of-the-art hardware efficient filter models has also been substantiated by listing numerous experimental outcomes. It has been appreciated that the choice of the most favorable mutation rule is significantly determined by the requirement as set by the system designer. Specifically, it depends on maximum allowable deviation from the ideal filter characteristic, length of the designed filter and constraint on the hardware cost. However, the contribution made by different cross-over or recombination rules has yet to be studied and can be included as future extension of this work. Other existing computational techniques may also be applied in this domain of signal processing and finally a comparative study may be carried out for the same design problem. Additionally, the application of the designed multiplier-less low-pass FIR filter in present and future generation wireless system can also be investigated in future research work too.

APPENDIX A TAP COEFFICIENTS OF THE DESIGNED MULTIPLIER-LESS LOW-PASS FIR FILTER
(MUTATION STRATEGY: BEST/1, LENGTH=29)

Tap no.		Tap coefficients
0	28	0
1	27	0
2	26	0
3	25	0
4	24	0
5	23	-2^{-7}
6	22	$2^{-6}+2^{-7}$
7	21	2^{-7}
8	20	$-2^{-5}-2^{-6}-2^{-7}$
9	19	$-2^{-3}-2^{-5}$
10	18	$-2^{-3}-2^{-5}-2^{-6}-2^{-7}$
11	17	2^{-7}
12	16	$2^{-2}+2^{-3}+2^{-5}$
13	15	$2^{-1}+2^{-2}+2^{-4}+2^{-6}+2^{-7}$
14		$2^{0}+2^{-6}$

APPENDIX B TAP COEFFICIENTS OF THE DESIGNED MULTIPLIER-LESS LOW-PASS FIR FILTER
(MUTATION STRATEGY: BEST/2, LENGTH=29)

Tap no.		Tap coefficients
0	28	0
1	27	0
2	26	0
3	25	0
4	24	0
5	23	2^{-7}
6	22	2^{-6}
7	21	0
8	20	-2^{-4}
9	19	$-2^{-3}-2^{-6}$
10	18	-2^{-3}
11	17	$2^{-4}+2^{-6}$
12	16	$2^{-2}+2^{-3}+2^{-4}+2^{-6}$
13	15	$2^{-1}+2^{-2}+2^{-4}+2^{-5}$
14		$2^{0}+2^{-6}$

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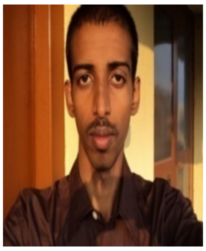

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