Lorentz Force Velocimetry

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We describe a noncontact technique for velocity measurement in electrically conducting fluids. The technique, which we term Lorentz force velocimetry (LFV), is based on exposing the fluid to a magnetic field and measuring the drag force acting upon the magnetic field lines. Two series of measurements are reported, one in which the force is determined through the angular velocity of a rotary magnet system and one in which the force on a fixed magnet system is measured directly. Both experiments confirm that the measured signal is a linear function of the flow velocity. We then derive the scaling law that relates the force on a localized distribution of magnetized material to the velocity of an electrically conducting fluid. This law shows that LFV, if properly designed, has a wide range of potential applications in metallurgy, semiconductor crystal growth, and glass manufacturing.

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In 1832 Faraday [1] attempted to determine the velocity of the Thames river near Waterloo bridge by measuring the electric potential difference induced by its flow across Earth's magnetic field lines. Today his invention, the electromagnetic flowmeter [2], enjoys broad success in the chemical and food industries but has fallen short of solving the grand challenge of flow measurement in high-temperature melts like steel, aluminum, or glass. Here we describe a technique which we term "Lorentz force velocimetry" (LFV) based on measuring the drag force on magnetic field lines which cross the melt flow. This noncontact technique is suited for high-temperature applications because it is free from the unavoidable electrode corrosion problem that has plagued Faraday's classical method.

When a liquid metal moves across magnetic field lines, as shown in Fig. 1, the interaction of the magnetic field with the induced eddy currents leads to a Lorentz force (with density $\mathbf{f} = \mathbf{j} \times \mathbf{B}$) which brakes the flow. The Lorentz force density is roughly

$$f \sim \sigma v B^2, \tag{1}$$

where σ is the electrical conductivity of the fluid, v its velocity, and B the magnitude of the magnetic field. This fact is well known [3–5] and has found a variety of applications for flow control in metallurgy and crystal growth [6]. Equally obvious but less widely recognized [2,7,8] is the fact that by virtue of Newton's law, an opposite force acts upon the magnetic-field-generating system and drags it along the flow direction as if the magnetic field lines were invisible obstacles. As we will show below, this force is proportional to the velocity and conductivity of the fluid, and its measurement is the key idea of LFV. Although the described phenomenon occurs no matter whether the magnetic field is generated by a (heavy) electromagnet or by a (lightweight) permanent magnet, it is thanks only to the recent advent of powerful rare earth permanent magnets [9] and design tools for permanent magnet systems [10] that a practical realization of this principle has now become possible. Here we describe two devices, one with a rotary and one with a fixed magnet system which are designed to measure a global quantity of the flow—its flow rate—and will therefore be referred to as Lorentz force flowmeters. Below we describe two series of measurements which characterize their performance and sensitivity.

In Fig. 2 we demonstrate for the first time that the flywheel principle which is well known from flywheel anemometers for air flow can be successfully embodied in a Lorenz force flowmeter. We arrange an array of permanent magnets on a rotatable disk such that the magnetic field lines cross the flow of a liquid metal. The velocimeter is mounted at a distance of 5 mm from a rectangular Plexiglas channel with a width of 10 mm which is filled with Ga-In-Sn eutectic alloy to a height of 80 mm. The flow rate of the liquid metal which is at room temperature is regulated using an electromagnetic pump situated sufficiently far away from the flowmeter not to disturb its operation. The mean velocity is determined by an ultra-

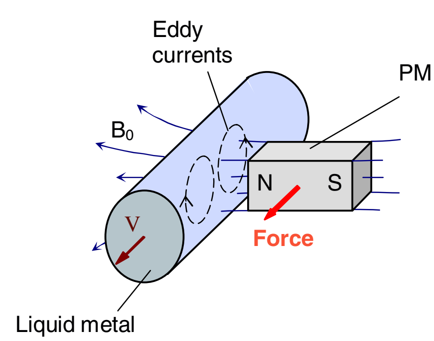

FIG. 1 (color online). Principle sketch of Lorentz force velocimetry showing the action of a permanent magnet (PM) upon the flow of an electrically conducting fluid. The magnetic-field-generating system can also consist of coils or a combination of coils, permanent magnets, and ferromagnetic material.

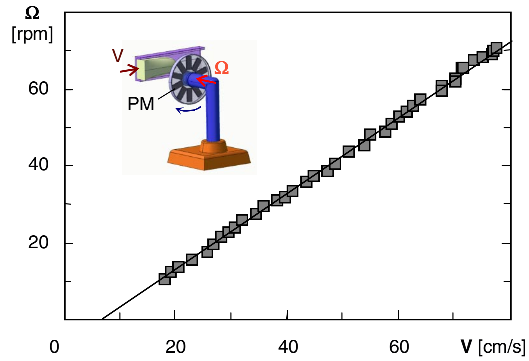

FIG. 2 (color online). Angular velocity of a rotary Lorentz force flowmeter as a function of the mean velocity of the liquid metal in a rectangular channel with insulating walls. Inset shows a simplified sketch of the device.

sonic Doppler velocimeter scanning the velocity profile at the inlet of the test section. Figure 2 shows that the magnetic flywheel is set into motion by the liquid metal flow and that its angular velocity increases linearly with the mean velocity. Some minimal velocity of the order of 10 cm/s is necessary to overcome friction within the ball bearings of the velocimeter.

Figure 3 shows a second realization of the Lorentz force flowmeter in which the magnet system is at rest during the measurement. The magnet system is U shaped and contains two permanent magnets which are connected to each other with an iron yoke. The magnet system is connected to a counterweight and both are attached to a pendulum. We can either measure the angle of inclination of the pendulum under the influence of the flow or fix the pendulum and directly measure the force acting on the magnet system. The flowmeter is applied to measure the flow rate of a turbulent flow of Ga-In-Sn eutectic alloy at room temperature which flows in a circular pipe with inner diameter of 35 mm and stainless steel walls. As in the first measurement, the flow is set up by an electromagnetic pump and the flow rate is determined by a commercial Faraday-type electromagnetic flowmeter.

The result of the measurement is shown in Fig. 3. The force on the magnet system is a linearly increasing function of the flow rate, as was anticipated. The slope of the curve, which is a measure of sensitivity, decreases when the distance between the poles of the magnet system is increased. This linear behavior continues to even higher velocities as long as the magnetic Reynolds number $Rm = \mu_0 \sigma v D$ is smaller than unity. Here v denotes the mean velocity of the fluid and D the pipe diameter. A similar result (not shown) is obtained when we measure the angle of displacement of the magnet system as sketched in the inset of Fig. 3. It should be emphasized that LFV is the only noncontact electromagnetic flow measurement method that works in spite of the presence of electrically conducting walls. This is in contrast to noncontact methods [11,12] based on local magnetic field measurements.

It is desirable for a flowmeter that its output be independent of the shape of the velocity profile. "By a fortunate mathematical fluke" (p. 3 of Ref. [2]) this property holds for the electromagnetic flowmeter provided that the magnetic field is homogeneous and the flow is unidirectional and axisymmetric. In practice, however, the influence of the profile and of turbulence can be significant. In order to asses this effect in LFV, we have obstructed the cross section of the rectangular channel to 25%, 50%, and 75% by inserting an obstacle which blocks the flow while keeping the flow rate constant. Figure 4 shows that the force on a flowmeter of the type shown in Fig. 3 is only weakly affected by this change. Hence, LFV is well suited for the measurement of flow rate.

How can the results of our measurements be understood? To compute the force on the magnet systems shown

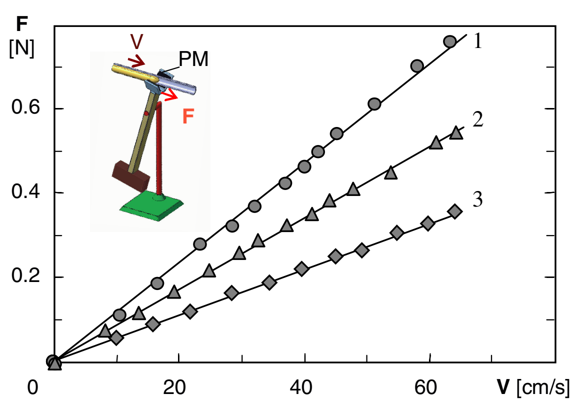

FIG. 3 (color online). Force on a Lorentz force flowmeter as a function of the mean velocity of the liquid metal in a circular pipe with electrically conducting walls. Inset shows a modified version of the flowmeter in which the magnet system is displaced under the action of the flow. The ratio between the distance of the magnets and the inner diameter of the pipe is 1.14, 1.28, and 1.43 for curves 1, 2, and 3, respectively.

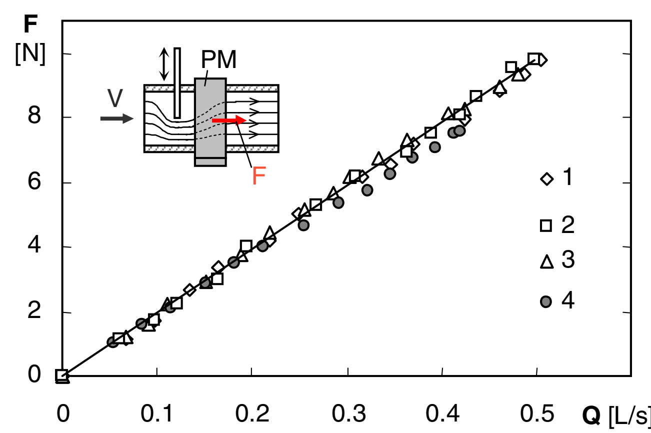

FIG. 4 (color online). Influence of the shape of velocity profile: Force on a Lorentz force flowmeter interacting with the liquid metal in a rectangular channel as a function of flow rate (in liters per second) for different levels (0% blocking for curve 1, 25% blocking for curve 2, 50% blocking for curve 3, and 75% blocking for curve 4) of obstruction.

in Figs. 2 and 3 one would have to solve the equations of magnetohydrodynamics including the nonlocal backreaction of the velocity field upon the complex-shaped magnetic system. This task is beyond the scope of the present Letter. However, the general scaling law that relates the measured force to the unknown velocity can be derived with reference to the simplified situation shown in Fig. 5. Here a small permanent magnet with dipole moment m is located at a distance L above a semi-infinite fluid moving with uniform velocity v parallel to its free surface. The magnetic field of the dipole which we refer to as the primary field is of the order $B \sim \mu_0 m L^{-3}$ at the surface of the fluid [13]. By virtue of the fluid's motion, eddy currents with amplitude $J \sim \sigma v B \sim \mu_0 \sigma v m L^{-3}$ are induced which are horizontal and concentrated below the surface as shown in Fig. 5(a). The eddy currents interact with the primary field to produce the Lorentz force which brakes the flow. More important for our understanding of LFV, however, is the fact that the eddy currents surround themselves with a magnetic field b, which we call the secondary field and which is sketched in Fig. 5(b). This magnetic field extends to the location of the dipole, and its magnitude there is $b \sim \mu_0 J L \sim \mu_0^2 \sigma v m L^{-2}$. As known from classical electrodynamics, a magnetic dipole experiences a force of the order $F \sim m b L^{-1}$ when subjected to a magnetic field gradient b/L. This provides us with the estimate

$$F \sim \mu_0^2 \sigma v m^2 L^{-3} \tag{2}$$

for the force acting of the magnet.

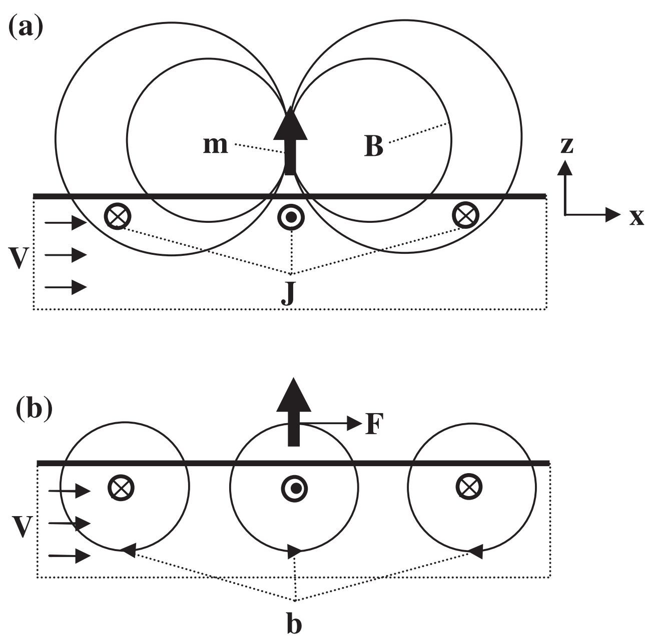

FIG. 5. Spatial distribution of magnetic fields in Lorentz force velocimetry: (a) primary magnetic field **B** and eddy currents **J** produced by a magnetic dipole interacting with a uniformly moving electrically conducting fluid; (b) secondary magnetic field **b** due to the horizontal eddy currents **J**.

We can draw a number of useful conclusions from this formula. First, the force is proportional to the product of velocity and electrical conductivity. This is in contrast to electromagnetic flow meters [2] where the induced voltage does not depend on the conductivity. Second, F grows with the second power of the magnetization (or with the second power of the electric current if the magnetic field were produced by an electromagnet). This is a consequence of the fact that the magnet acts simultaneously as a source of the primary and a sensor of the secondary magnetic field. In contrast to other contactless methods [11,12] the sensitivity of LFV can thus be increased by increasing the strength of the magnetic field. This makes LFV potentially suitable even for low conductivity melts like glass which are inaccessible to any other noncontact electromagnetic measurement method.

The analysis leading to the scaling relation (2) can be made quantitative by assuming that the magnet is a point dipole with dipole moment $\mathbf{m} = m\mathbf{e}_z$ whose magnetic field is given by

$$\mathbf{B}(\mathbf{R}) = \frac{\mu_0}{4\pi}\left\{3\frac{(\mathbf{m}\cdot\mathbf{R})\mathbf{R}}{R^5} - \frac{\mathbf{m}}{R^3}\right\} \tag{3}$$

[13], where $\mathbf{R} = \mathbf{r} - L\mathbf{e}_z$ and $R = |\mathbf{R}|$. Assuming a velocity field $\mathbf{v} = v\mathbf{e}_x$ for $z < 0$, the eddy currents can be computed from Ohm's law for a moving electrically conducting fluid [3,4]

$$\mathbf{J} = \sigma(-\nabla\phi + \mathbf{v}\times\mathbf{B}) \tag{4}$$

subject to the boundary conditions $J_z = 0$ at $z = 0$ and $J_z \to 0$ as $z \to -\infty$. First, the scalar electric potential is obtained as

$$\phi(\mathbf{r}) = -\frac{\mu_0 v m}{4\pi}\frac{x}{R^3}, \tag{5}$$

from which the electric current density is readily calculated [14]. They are indeed horizontal. Once they are known, the Biot-Savart law [13] can be used to compute the secondary magnetic field $\mathbf{b}(\mathbf{r})$. Finally, the force is given by

$$\mathbf{F} = (\mathbf{m}\cdot\nabla)\mathbf{b}, \tag{6}$$

where the gradient of $\mathbf{b}$ has to be evaluated at the location of the dipole. For the problem at hand all these steps can be carried out analytically without any approximation leading to the result

$$\mathbf{F} = \frac{\mu_0^2 \sigma v m^2}{128\pi L^3}\mathbf{e}_x. \tag{7}$$

Let us use this expression to interpret our experimental results. The dynamics of the rotary velocimeter is approximately described by the equation of motion $I(\dot{\Omega} + \gamma\Omega) = FR$, where $I \approx 10^{-2}$ kg m^2 is its moment of inertia, $\gamma \approx 10^{-1}$ s^{-1} its (experimentally determined) frictional decay time, and FR the torque due to the force F acting on the rotating permanent magnets located $R = 0.05$ m away from the axis of rotation. The angular velocity at steady state ($\dot{\Omega} = 0$) is therefore given by $\Omega = FR/\gamma I$. To compute F assume that the dominant contribution comes from the permanent magnet which is closest (i.e., $L \sim 0.01$ m) to the liquid metal. The magnet has a volume of $V \sim 3$ cm^3 and is made of a material with magnetization density $M \sim 10^6$ A/m; hence its magnetization is $m = MV \sim 3$ Am^2. With $v \sim 1$ m/s and $\sigma \sim 10^6\ \Omega^{-1}\,\mathrm{m}^{-1}$ we can use Eq. (7) to estimate $F \sim 0.024$ N and obtain an angular velocity $\Omega \sim 10$ rpm, which is in good qualitative agreement with our observations shown in Fig. 2. Notice that the real magnet system contains ferromagnetic material and has a complex geometry, so a quantitative agreement cannot be expected based on such simple theory.

A similar estimate can be made to compute the force shown in Fig. 3. Here the volume $V \sim 10$ cm^3 of the magnets is larger than in the previous case, thus $m = 10$ Am^2. Keeping the values for v, L, and σ we obtain $F \sim 0.25$ N, which is again in good qualitative agreement with the experiment.

In summary, we have described a noncontact measurement method for the velocity of electrically conducting fluids and demonstrated its feasibility for measuring one scalar quantity. Much more information on the flow field can be obtained [15] by combining a Lorentz force flowmeter with a more sophisticated force measurement technique that has been originally developed for the testing of airplane models in wind tunnels, namely, wind tunnel balances. By simultaneously measuring all six components of force and torque acting on the magnet system one can identify additional parameters such as the width of the channel, which could be of interest for fundamental turbulence research in liquid metals [16–18] as well as for industrial applications.

Let us conclude our work with a brief historical speculation. If Faraday were aware of the principle of LFV, he might have tried another experiment in 1832, namely, placing a horseshoe magnet above the water and measuring the magnetic force acting upon it. Would he have succeeded? The electrical conductivity of Thames river back in 1832 is difficult to estimate but was probably of the order $\sigma \sim 0.1\ \Omega^{-1}\,\mathrm{m}^{-1}$. With a flow velocity of $v \sim 1$ m/s and the magnetic field of a horseshoe magnet $B \sim 0.1$ T the Lorentz force density acting on the water according to Eq. (1) would be $f \sim 10^{-3}$ N/m^3. Assuming that the field affects a volume of $V = 100$ cm^3 of water, the force on the water and thereby the counterforce on the magnet would amount to only $F \sim 10^{-7}$ N, which is about a factor 10^{-8} less than the weight of the magnet. This would have required a force measurement accuracy comparable to that in Cavendish's 1798 experiment for the determination of the gravitational constant. Whether Faraday could have performed such accurate measurements is therefore an interesting open question for the history of science but outside the scope of the present Letter.

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