A Wideband MIMO Channel Model Derived From the Geometric Elliptical Scattering Model

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Abstract—In this paper, we present a reference model for a wideband multiple-input multiple-output (MIMO) channel based on the geometric elliptical scattering model. The model takes into account the exact relationship between the angle of departure (AOD) and the angle of arrival (AOA). Based on this relationship, the statistical properties of the reference model are studied. Analytical solutions are presented for the three-dimensional (3D) space-time cross-correlation function (CCF), the temporal autocorrelation function (ACF), the 2D space CCF, and finally the frequency correlation function (FCF). The correlation properties are studied and visualized under the assumption of isotropic as well as non-isotropic scattering conditions. The proposed reference model can be used as a starting point for the derivation of a frequency-selective space-time MIMO channel simulator enabling the performance evaluation of multi-antenna wideband communication systems. The reference model is also quite useful for studying the MIMO channel capacity under various propagation conditions imposed by the geometry of the underlying scattering model.

Keywords—Wireless propagation, channel characterization, MIMO channel modelling, wideband MIMO channels, space-time correlation, geometric elliptical scattering model.

I. INTRODUCTION

For the design and performance evaluation of MIMO wireless communication systems, it is of crucial importance to have accurate and realistic MIMO channel models [1]. Especially for the development of wideband wireless communication systems employing MIMO technologies, such as MIMO orthogonal frequency division multiplexing (OFDM) systems, channel models are required, which take into account the temporal, spatial, and frequency correlation properties. Such space-time-frequency MIMO channel models have been studied, for example, in [2].

In this paper, we derive a space-time-frequency MIMO channel model from the geometric elliptical scattering model. Together with the one-ring model [3]–[5] and the two-ring model [6]–[8], the elliptical model belongs to the most important geometrical models from which spatial channel models have been derived in the past. However, the one-ring model and the two-ring model have been used primarily to model narrowband MIMO channels with specific temporal and spatial correlation properties. This limits the usefulness of these channel models to performance studies of narrowband mobile communication systems. In contrast to the one-ring and the two-ring models, the elliptical model is predestinated for the modelling of wideband MIMO channels with characteristic temporal, spatial, and frequency correlation properties, as we will see in this paper. Especially, the frequency-selectivity feature makes the elliptical scattering model very attractive for developers of future mobile communication systems, since the interest has been turned recently to the design and performance investigation of high data rate wireless systems employing MIMO-OFDM techniques [9]. Originally, the geometric elliptical model was proposed in [10] for spatial channels in micro- and picocell environments, where the antenna heights are low, so that multipath scattering is just as likely near the base station (BS) as it is near the mobile station (MS). The geometrically based elliptical model was developed further, e.g., in [11], where a spatial channel model has been proposed for single-input multiple-output (SIMO) channels.

In the present paper, we extend our work in [11] with respect to multiple antennas at both the transmit side and the receive side. In our model, we take into account that the AOD and the AOA are not independent. The exact relationship between the AOD and the AOA was first derived in [11] and has recently been simplified in [12]. Using the relationship in [12], we derive general analytical solutions for the 3D space-time CCF, the temporal ACF, the 2D space CCF, and the FCF. The most important correlation properties will be discussed and visualized assuming isotropic and non-isotropic scattering conditions. Due to its infinite complexity, the proposed MIMO channel model is introduced here as a reference model. The reference model is an important framework for the derivation of an efficient MIMO channel simulator with given space-time-frequency correlation properties. Furthermore, the reference model is also quite useful for studying the capacity of MIMO channels under realistic propagation conditions imposed by the underlying geometrical scattering model.

Our paper is organized as follows. Section II reviews the geometric elliptical scattering model with local scatterers lying on an ellipse. Starting from the elliptical scattering model, we derive in Section III first the corresponding narrowband space-time MIMO channel model assuming an infinite number of random scatterers characterized by a given distribution function. The resulting model is called the reference model. Its statistical properties will also be analysed in Section III. The obtained theoretical results will then be illustrated in Section IV under the assumption of isotropic and non-isotropic scattering conditions. Section V presents two model extensions. The first one incorporates multi-cluster scenarios in...
whereas the AOA is described by \( \theta \). The angle between the
we will also profit from the reasonable assumption that the
inequalities (1)
channel with local scatterers
Fig. 1. Geometric elliptical scattering model for an
channel model is the geometric elliptical scattering model
draws the conclusion.

II. THE GEOMETRIC ELLIPTICAL SCATTERING MODEL

Starting point for the derivation of the proposed MIMO
channel model is the geometric elliptical scattering model
shown in Fig. 1. This figure illustrates that all local scatterers
\( S^{(n)} \) \((n = 1, 2, \ldots, N)\) associated with a certain path length
are located on an ellipse, where the BS and the MS are located
at the focal points. The distance between the two focal points
is \( 2f \). The major axis half length and minor axis half length
are denoted by \( a \) and \( b \), respectively.

It is assumed that the BS is the transmitter, whereas the
MS plays the role of the receiver. We assume that both the
transmitter and the receiver are equipped with a uniform linear
antenna array consisting of \( M_T \) and \( M_R \) antenna elements,
respectively. The angle \( \alpha_T \) \((\alpha_R)\) denotes the tilt angle of the
transmit (receive) antenna array. The symbols \( \delta_T \) and \( \delta_R \)
are describing the antenna element spacings at the transmitter
and the receiver, respectively. Since the antenna dimensions are
generally small in comparison to the parameters \( a \) and \( f \),
we will also profit from the reasonable assumption that the
inequalities \((M_T - 1) \delta_T \ll a - f \) and \((M_R - 1) \delta_R \ll a - f \)
hold. The angle between the \( x \)-axis and the direction of motion
is denoted by \( \alpha_v \). Finally, the AOD is denoted by \( \phi_T^{(n)} \),
whereas the AOA is described by \( \phi_R^{(n)} \) \((n = 1, 2, \ldots, N)\).

![Geometric elliptical scattering model](image)

Fig. 1. Geometric elliptical scattering model for an \( M_T \times M_R \) MIMO
channel with local scatterers \( S^{(n)} \) lying on an ellipse.

III. THE REFERENCE MODEL

A. Derivation of the Reference Model

The starting point for the derivation of a reference model
for the MIMO channel is the geometrical scattering model
shown in Fig. 1. From this figure, we realize that the \( n \)th
homogeneous plane wave emitted from the \( l \)th transmit anten-
a element \( A_T^{(l)} \) \((l = 1, 2, \ldots, M_T)\) travels over the \( n \)th
local scatterer \( S^{(n)} \) \((n = 1, 2, \ldots, N)\) before impinging on the
\( k \)th receive antenna element \( A_R^{(k)} \) \((k = 1, 2, \ldots, M_R)\).
The reference model is based on the assumption that the
number \( N \) of local scatterers is infinite. As a consequence, the
diffuse component at the \( k \)th receive antenna \( A_R^{(k)} \) is composed
of an infinite number of homogeneous plane waves. Hence,
with reference to Fig. 1, the complex channel gain \( g_{lk}(\vec{r}_R) \)
describing the link from \( A_T^{(l)} \) \((l = 1, 2, \ldots, M_T)\) to \( A_R^{(k)} \)
\((k = 1, 2, \ldots, M_R)\) can be expressed as

\[
g_{lk}(\vec{r}_R) = \lim_{N \to \infty} \sum_{n=1}^{N} E_n e^{j(\theta_n - \vec{E}_R^{(n)} \cdot \vec{r}_R - k_0 D_n)} \tag{1}
\]

where \( E_n \) and \( \theta_n \) are the gain and phase shift, respectively,
caused by the interaction of the local scatterers \( S^{(n)} \), \( \vec{E}_R^{(n)} \)
denotes the wave vector pointing in the propagation direction of
the \( n \)th received plane wave, and \( \vec{r}_R \) is the spatial translation
vector of the receiver. Furthermore, \( k_0 \) is called the free-space
wave number, which is defined by \( k_0 = 2\pi/\lambda \) with \( \lambda \) being
the wavelength, and finally \( D_n \) denotes the length of the total
distance that a plane wave travels from \( A_T^{(l)} \) via \( S^{(n)} \) to \( A_R^{(k)} \).

It is assumed that the gain \( E_n \) and the phase shift \( \theta_n \)
caused by a particular scatterer \( S^{(n)} \) are generally dependent
on the directions of the incoming and emerging waves seen
from \( S^{(n)} \). Since \((M_T - 1) \delta_T \ll a - f \), the waves emerging
from different transmit antennas arrive at a particular scatterer
\( S^{(n)} \) at approximately the same angle. Analogously, since
\((M_R - 1) \delta_R \ll a - f \) holds, we may state that waves
emerging from \( S^{(n)} \) arrive at different receive antennas at
approximately the same angle. This allows us to conclude that
the gain \( E_n \) and phase shift \( \theta_n \) caused by a particular scatterer
\( S^{(n)} \) are the same for waves arriving from (or travelling to)
different transmit (receive) antenna elements. Furthermore, it
is assumed that each scatterer \( S^{(n)} \) introduces a constant gain

\[
E_n = \frac{1}{\sqrt{N}} \tag{2}
\]

and a random phase shift \( \theta_n \). The phase shifts \( \theta_n \) are independent,
identically distributed (i.i.d.) random variables, each
having a uniform distribution over the interval \([0, 2\pi]\). Note
that (2) implies that all waves reaching the received antenna
array are equal in power.

The second phase component in (1), \( \vec{k}_R^{(n)} \cdot \vec{r}_R \), is due to the
movement of the receiver and can be written as

\[
\vec{k}_R^{(n)} \cdot \vec{r}_R = -2\pi f_{\text{max}} \cos \left( \phi_R^{(n)} - \alpha_v \right) t \tag{3}
\]

where \( f_{\text{max}} \) denotes the maximum Doppler frequency.
Furthermore, the third phase component in (1), $k_0 D_n$, is due to the total distance travelled and can be expressed as

$$k_0 D_n = \frac{2\pi}{\lambda} \left( D_T^{(l_n)} + D_R^{(n,k)} \right)$$  \hspace{1cm} (4)$$

where $D_T^{(l_n)}$ describes the distance from the $l$th transmit antenna $A_T^{(l)}$ to the scatterer $S^{(n)}$, and analogously $D_R^{(n,k)}$ denotes the distance from the scatterer $S^{(n)}$ to the $k$th receive antenna $A_R^{(k)}$. These two distances can be approximated by using $(M_T - 1) \delta_T \ll a - f$, $(M_R - 1) \delta_R \ll a - f$, and $\sqrt{1 + x} \approx 1 + x/2$ ($x \ll 1$) as

$$D_T^{(l_n)} \approx D_T^{(n)} = (M_T - 2l + 1) \frac{\delta_T}{2} \cos \left( \phi_T^{(n)} - \alpha_T \right)$$ \hspace{1cm} (5)$$

$$D_R^{(n,k)} \approx D_R^{(n)} = (M_R - 2k + 1) \frac{\delta_R}{2} \cos \left( \phi_R^{(n)} - \alpha_R \right)$$ \hspace{1cm} (6)$$

where $D_T^{(n)}$ and $D_R^{(n)}$ are the distances illustrated in Fig. 1.

Now, after substituting (2)-(4) in (1) and using the approximations in (5) and (6), the complex channel gain $g_{kl}(t)$ of the proposed reference model describing the link from the $l$th transmit antenna element $A_T^{(l)}$ ($l = 1, 2, \ldots, M_T$) to the $k$th receive antenna element $A_R^{(k)}$ ($k = 1, 2, \ldots, M_R$) can be expressed as

$$g_{kl}(t) = \lim_{N \to \infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_{tn} b_{kn} e^{j(2\pi f_n t + \theta_n + \theta_0)}$$ \hspace{1cm} (7)$$

where

$$a_{tn} = e^{j2\pi(M_T-2l+1)\delta_T/\lambda} \cos(\phi_T^{(n)} - \alpha_T)$$ \hspace{1cm} (8)$$

$$b_{kn} = e^{j2\pi(M_R-2k+1)\delta_R/\lambda} \cos(\phi_R^{(n)} - \alpha_R)$$ \hspace{1cm} (9)$$

$$f_n = f_{\max} \cos(\phi_T^{(n)} - \alpha_T)$$ \hspace{1cm} (10)$$

$$\theta_0 = -4\pi a/\lambda.$$ \hspace{1cm} (11)$$

Without loss of generality, the constant phase shift $\theta_0$ can be set to zero in (7), since this parameter has no influence on the statistics of the reference model. Analyzing the statistical properties of the complex channel gain $g_{kl}(t)$ in (7) reveals that the mean value and the variance of $g_{kl}(t)$ are equal to 0 and 1, respectively. Thus, from the central limit theorem [13], it follows that $g_{kl}(t)$ is a complex Gaussian process with zero mean and unit variance. Therefore, we can conclude that the distribution of the envelope $|g_{kl}(t)|$ equals the Rayleigh distribution.

In our reference model, we take into account the exact relationship between the AOD $\phi_T^{(n)}$ and the AOA $\phi_R^{(n)}$. By using the results in [11] and [12], we can express the AOD $\phi_T^{(n)}$ in terms of the AOA $\phi_R^{(n)}$ as follows

$$\phi_T^{(n)} = \begin{cases} 
\frac{\pi}{2} \sin \phi_R^{(n)} & \text{if } 0 < \phi_R^{(n)} \leq \phi_0 \\
\frac{\pi}{2} + \phi_T^{(n)} & \text{if } \phi_0 < \phi_R^{(n)} \leq 2\pi - \phi_0 \\
\phi_T^{(n)} + \pi & \text{if } 2\pi - \phi_0 < \phi_R^{(n)} \leq 2\pi
\end{cases}$$ \hspace{1cm} (12)$$

where

$$f(\phi_R^{(n)}) = \arctan \left[ \frac{(\kappa_0^2 - 1) \sin(\phi_R^{(n)})}{2\kappa_0 + (\kappa_0^2 + 1) \cos(\phi_R^{(n)})} \right]$$ \hspace{1cm} (13)$$

and

$$\phi_0 = \pi - \arctan \left( \frac{\kappa_0^2 - 1}{2\kappa_0} \right).$$ \hspace{1cm} (14)$$

The parameter $\kappa_0$ in (13) and (14) equals the reciprocal value of the eccentricity $e$ of the ellipse, i.e., $\kappa_0 = 1/e = a/f$.

B. Correlation Functions of the Reference Model

The 3D space-time CCF of the links $A_T^{(l)} - A_R^{(k)}$ and $A_T^{(l')} - A_R^{(k')}$ is defined as the correlation between the channel gains $g_{kl}(t)$ and $g_{kl'}(t)$ according to

$$\rho_{kl,kl'}(\delta_T, \delta_R, \tau) := E\{g_{kl}(t) g_{kl'}^*(t + \tau)\}$$ \hspace{1cm} (15)$$

where $E\{\cdot\}$ denotes the expectation operator. Note that the expectation operator has to be applied on all random variables $(\theta_n$ and $\phi_R^{(n)})$, where we recall that the AOD $\phi_T^{(n)}$ is a function of the AOA $\phi_R^{(n)}$ according to (12). The solution of (15) can be achieved by substituting (7) in (15) and averaging in the first step over the random phases $\theta_n$. This allows us to express the 3D space-time CCF as

$$\rho_{kl,kl'}(\delta_T, \delta_R, \tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} E \left\{ c_{il}^{(n)} d_{kk'}^{(n)} e^{-j2\pi f_n \tau} \right\}$$ \hspace{1cm} (16)$$

where

$$c_{il}^{(n)} = e^{-j2\pi(l-l')(\delta_T/\lambda) \cos(\phi_T^{(n)} - \alpha_T)}$$ \hspace{1cm} (17)$$

$$d_{kk'}^{(n)} = e^{-j2\pi(k-k')(\delta_R/\lambda) \cos(\phi_R^{(n)} - \alpha_R)}.$$ \hspace{1cm} (18)$$

In the second step, we compute the statistical expectation with respect to the random variable $\phi_R^{(n)}$. We notice that if the number of local scatterers $N$ approaches infinity, then the discrete random variables $\phi_R^{(n)}$ and $\phi_T^{(n)}$ become continuous random variables denoted by $\phi_R$ and $\phi_T$, respectively, while we still take into account that $\phi_T$ is a function of $\phi_R$ according to (12). The infinitesimal power of the diffuse component corresponding to the differential angles $\delta_R$ is proportional to $p_{\phi_R}(\phi_R) d\phi_R$, where $p_{\phi_R}(\phi_R)$ denotes the distribution of $\phi_R$. As $N \to \infty$, this infinitesimal contribution must be equal to 1/$N$, i.e., $1/N = p_{\phi_R}(\phi_R) d\phi_R$. From this fact, it follows from (16) that the 3D space-time CCF of the reference model can be written as

$$\rho_{kl,kl'}(\delta_T, \delta_R, \tau) = \int_{-\pi}^{\pi} c_{il}^{(\delta_T, \phi_T)} d_{kk'}^{(\delta_R, \phi_R)} p_{\phi_R}(\phi_R) d\phi_R$$ \hspace{1cm} (19)$$

where

$$c_{il}^{(\delta_T, \phi_T)} = e^{-j2\pi(l-l')(\delta_T/\lambda) \cos(\phi_T - \alpha_T)}$$ \hspace{1cm} (20)$$

$$d_{kk'}^{(\delta_R, \phi_R)} = e^{-j2\pi(k-k')(\delta_R/\lambda) \cos(\phi_R - \alpha_R)}$$ \hspace{1cm} (21)$$

$$f(\phi_R) = f_{\max} \cos(\phi_R - \alpha_R).$$ \hspace{1cm} (22)$$

It is interesting to note that the 3D space-time CCF $\rho_{kl,kl'}(\delta_T, \delta_R, \tau)$ is independent of the parameters $a, b$ and $f$ describing the ellipse.
The temporal ACF $r_{gkl}(\tau)$ of the complex channel gain $g_{kl}(t)$ is defined as $r_{gkl}(\tau) := E\{g_{kl}(t)g_{kl}^*(t + \tau)\}$. Alternatively, the temporal ACF $r_{gkl}(\tau)$ can be obtained from the 3D space-time CCF $\rho_{kl,k'l'}(\delta_T, \delta_R, \tau)$ by setting the antenna element spacings $\delta_T$ and $\delta_R$ to zero, i.e., $r_{gkl}(\tau) = \rho_{kl,k'l'}(0, 0, \tau)$. In both cases, we obtain

$$
r_{gkl}(\tau) = \int_{-\pi}^{\pi} e^{-j2\pi f_{\text{max}} \cos(\phi_R - \phi_k)} p_{\phi_R}(\phi_R) d\phi_R \tag{23}
$$

for all $k = 1, 2, \ldots, M_R$ and $l = 1, 2, \ldots, M_T$. Notice that the temporal ACFs $r_{gkl}(\tau)$ of the complex channel gains $g_{kl}(t)$ are identical for all links from $A_{kl}^{(j)}$ ($l = 1, 2, \ldots, M_T$) to $A_{kl}^{(j)}$ ($k = 1, 2, \ldots, M_R$). In case of isotropic scattering, characterized by $p_{\phi_R}(\phi_R) = 1/(2\pi)$, the above integral can be solved analytically leading to $r_{gkl}(\tau) = J_0(2\pi f_{\text{max}} \tau)$, where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind.

The 2D space CCF $\rho_{kl,k'l'}(\delta_T, \delta_R)$, which is defined as $\rho_{kl,k'l'}(\delta_T, \delta_R) := E\{g_{kl}(t)g_{kl'}^*(t')\}$, is equal to the 3D space-time CCF $\rho_{kl,k'l'}(\delta_T, \delta_R, \tau)$ at $\tau = 0$, i.e., $\rho_{kl,k'l'}(\delta_T, \delta_R) = \rho_{kl,k'l'}(\delta_R, \delta_R, 0)$. Hence,

$$
\rho_{kl,k'l'}(\delta_T, \delta_R) = \int_{-\pi}^{\pi} c_{kl'}(\delta_T, \delta_R) d\phi_R \rho_{\phi_R}(\phi_R) d\phi_R. \tag{24}
$$

The above result shows that the 2D space CCF $\rho_{kl,k'l'}(\delta_T, \delta_R)$ does generally not allow a Kronecker representation, meaning a representation of $\rho_{kl,k'l'}(\delta_T, \delta_R)$ as a product consisting of two terms—the first being a function of $\delta_T$ and the second of $\delta_R$ only. This statement is in agreement with the MIMO channel model based on the geometrical one-ring scattering model [5], but in sharp contrast to the two-ring scattering model [7], where the AOD $\phi_T$ and AOA $\phi_R$ are independent.

IV. ILLUSTRATIVE EXAMPLES AND NUMERICAL RESULTS

In this section, we will present some illustrative examples for the temporal ACF $r_{gkl}(\tau)$ and the 2D space CCF $\rho_{kl,k'l'}(\delta_T, \delta_R)$ in case of isotropic and non-isotropic scattering. For this purpose, we employ the von Mises density [14] to characterize the distribution of the AOA $\phi_R$. The von Mises density is given by

$$
p_{\phi_R}(\phi_R) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\phi_R - \mu)}, \quad \phi_R \in (-\pi, \pi] \tag{25}
$$

where $I_0(\cdot)$ denotes the zeroth-order modified Bessel function, $\mu \in (-\pi, \pi]$ accounts for the mean value of the AOA $\phi_R$, and $\kappa \geq 0$ is a real-valued parameter that controls the angular spread of $\phi_R$. For large values of $\kappa$, the angular spread of $\phi_R$ is approximately equal to $2/\sqrt{\kappa}$ [14]. Isotropic scattering is obtained if $\kappa = 0$. In this case, the von Mises distribution in (25) reduces to the uniform distribution $p_{\phi_R}(\phi_R) = 1/(2\pi)$.

Some numerical results obtained for the temporal ACF $r_{gkl}(\tau)$ and the 2D space CCF $\rho_{11,22}(\delta_T, \delta_R)$ are presented in Figs. 2 and 3, respectively. These results are valid under the assumption of isotropic scattering ($\kappa = 0$). The Figs. 4 and 5 illustrate the behavior of $r_{gkl}(\tau)$ and $\rho_{11,22}(\delta_T, \delta_R)$ under non-isotropic scattering conditions assuming that the AOA $\phi_R$ follows the von Mises distribution with parameters $\kappa = 10$ and $\mu = 0$.

V. MODEL EXTENSIONS

In this section, we will show how the proposed MIMO channel model can be extended to multiple clusters of scatterers as well as to frequency selectivity.

A. Extension to Multiple Clusters of Scatterers

Let us consider a propagation scenario consisting of $C$ clusters of scatterers located on a single ellipse as shown in Fig. 6. To distinguish between individual clusters, we add the subscript $(\cdot)_c$ ($c = 1, 2, \ldots, C$) to all affected symbols, e.g., we write $g_{kl,c}(t)$, $\kappa_c$, $\mu_c$ etc. For different values of $c$,
Fig. 4. The temporal ACF $r_{h_{k,l}}(\tau)$ of the reference model (non-isotropic scattering, von Mises density with parameters $\kappa = 10$ and $\mu = 0$).

Fig. 5. The 2D space CCF $\rho_{1,2}(\delta_r, \delta_R)$ of the reference model (non-isotropic scattering, von Mises density with parameters $\kappa = 10$ and $\mu = 0$).

Each complex channel gain $g_{k,l,c}(t)$ might be characterized by different parameters $\kappa_c$ and $\mu_c$. The resulting complex channel gain $z_{k,l}(t)$ describing the link from $A_{k,0}^{(l)} (l = 1, 2, \ldots, M_T)$ to $A_{k,0}^{(l)} (k = 1, 2, \ldots, M_R)$ in a multi-cluster $M_T \times M_R$ MIMO channel is then given by the superposition of the received scattered components of all $L$ clusters, i.e.,

$$z_{k,l}(t) = \sum_{c=1}^{C} w_c g_{k,l,c}(t)$$

where $w_c$ is a real constant representing the weighting factor of the $c$th cluster. We impose the boundary condition $\sum_{c=1}^{C} w_c^2 = 1$ on the weighting factors $w_c$ to normalize the mean power of $z_{k,l}(t)$ to unity.

B. Extension to Frequency Selectivity

The underlying geometric elliptical scattering model for the proposed frequency-selective MIMO channel is presented in Fig. 7. The complex channel gain associated with the $l$th discrete propagation path $\tau'_l$ will be denoted by $z_{k,l}(t)$. This notation allows us to express the impulse response of the proposed reference model as

$$h_{k,l}(\tau', t) = \sum_{c=1}^{C} a_c z_{k,l,c}(t) \delta(\tau' - \tau'_c)$$ (27)

where $\delta(\cdot)$ is the delta function, $L$ denotes the number of discrete propagation paths with different propagation delays $\tau'_c$, and $a_c$ represents the gain of the $l$th path.

The Fourier transform of the impulse response $h_{k,l}(\tau', t)$ with respect to $\tau'$ is known as the time-variant transfer function $H_{k,l}(f', t)$. Hence, from (27) it follows

$$H_{k,l}(f', t) = \sum_{c=1}^{C} a_c z_{k,l,c}(t) e^{-j2\pi f' \tau'_c}.$$ (28)

The FCF of the reference model, denoted as $r_{\tau'}(v')$ and defined by $r_{\tau'}(v') := E\{H_{k,l}(f', t) H_{k,l}^{*}(f' + v', t)\}$, can be expressed as

$$r_{\tau'}(v') = \sum_{c=1}^{C} a_c^2 e^{-j2\pi v' \tau'_c}.$$ (29)

The above result shows that the FCF $r_{\tau'}(v')$ is completely determined by the number of propagation paths $L$, the path gains $a_c$, and the propagation delays $\tau'_c$. Proper values for these parameters can be found in many specifications for channel models, such as the COST 207 channel models [15], the SUI channel models [16], and the HIPERLAN/2 channel models [17]. This is an important observation, since it allows us to fit the FCF $r_{\tau'}(v')$ of the proposed channel model to any given specified (or measured) FCF characterized by the sets $\{a_c\}_{c=1}^{C-1}$ and $\{\tau'_c\}_{c=1}^{C-1}$. The same statement holds of course for any given (specified or measured) discrete power delay profile (PDP) due to the Fourier transform relationship between the FCF and the PDP.

An example illustrating the behaviour of the absolute value of the FCF, $|r_{\tau'}(v')|$, is shown in Fig. 8. The results have been obtained by using the 18-path HIPERLAN/2 channel model C specified in [17].
A reference model has been derived for wideband space-time MIMO channels. The starting point of the procedure was the geometric elliptical scattering model with a single cluster of scatterers lying on an ellipse. The fundamental procedure has then been extended to enable the modelling of more realistic multi-cluster scenarios. A further extension has been made with respect to frequency-selectivity by introducing a geometric elliptical model consisting of several ellipses.

The statistical properties of the derived MIMO channel model have been analysed. Especially, general formulas have been derived for the 3D space-time CCF, the temporal ACF, the 2D space CCF, and the FCF. Moreover, illustrative examples have been presented for the most important correlation functions in case of isotropic and non-isotropic scattering conditions.

The new wideband space-time MIMO channel model includes the SIMO channel model introduced in [11] as special case. Our proposed MIMO channel model is not only useful for the design, test, and analysis of future wideband mobile communication systems using MIMO-OFDM techniques, but is also of central importance for studying the channel capacity of frequency-selective MIMO channels under realistic propagation conditions. Last not least, the proposed reference model provides an important framework for the design of stochastic and deterministic MIMO channel simulators [12].

REFERENCES