Complex Morlet Wavelet Amplitude and Phase Map Based Bearing Fault Diagnosis *

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Abstract –The continuous wavelet transform enables one to look at the evolution in the time scale joint representation plane. This advantage makes it very suitable for the detection of singularity generated by localized defects in mechanical system. The Fourier spectrum of complex Morlet wavelet is real, which the Fourier spectrum has no complex phase, the complex Morlet wavelet does not affect the phase of a signal in complex domain. This gives a desirable ability to detect the singularity characteristic of a signal precisely. In this study, the complex Morlet wavelet amplitude and phase map are used in conjunction to detect and diagnose the bearing fault. The complex Morlet wavelet amplitude and phase map are found to show distinctive signatures in the presence of bearing inner race or outer race damage. The experimental results show that the Morlet wavelet amplitude and phase map can extract the transients from strong noise signals and can effectively diagnose the faults of bearing.

Index Terms –Fault diagnosis, Bearing, Complex Morlet wavelet, Continuous wavelet transform.

I. INTRODUCTION

The rolling element bearing is an important component for power transmitting systems within the machine tool or gearbox drive train. Monitoring the condition of the bearing component provides advantages in the safety, operation and maintenance areas. Therefore, the predictive maintenance philosophy of using vibration information to lower operating costs and increase machinery availability has been the subject of intensive research throughout industry. Since most of the machinery in a predictive maintenance program contain rolling element bearings, it is imperative to establish a suitable condition monitoring procedure to prevent malfunction and breakage during operation.

The wavelet transform provides powerful multi-resolution analysis in both time and frequency domain and thereby becomes a favored tool to extract the transitory features of non-stationary vibration signals produced by the faulty bearing [1,2]. The wavelet analysis results in a series of wavelet coefficients, which indicate how close the signal is to the particular wavelet. In order to extract the fault feature of signals more effectively, an appropriate wavelet base function should be selected. Morlet wavelet is mostly applied to extract the rolling element bearing fault feature because of the large similarity with the impulse generated by the faulty bearing [3]. The impulse response wavelet is constructed and applied to extract the feature of fault vibration signal in [4]. A number of wavelet-based functions are proposed for mechanical fault detection with high sensitivity in [5], and the differences between single and double-sided Morlet wavelets are presented. An adaptive wavelet filter based on single-sided Morlet wavelet is introduced in [6]. An application of Morlet continuous wavelet transform for the detection of cracks in geared system is presented in [7]. The discrete and continuous wavelet transforms to detect abnormal transients generated by early gear damage are used in [8,9].

In this study, an alternative approach for detecting localized faults in the outer or the inner races of a rolling element bearing using the complex Morlet wavelet amplitude and phase map is investigated. The techniques are demonstrated by the experiments on a gearbox with a rolling element bearing under simulated crack on the inner race or the outer race. The characteristic periods related to the bearing defect can be effectively extracted. The experimental results show that this method based on the complex Morlet wavelet amplitude and phase map can effectively diagnose the faults of rolling element bearing.

To address the issues discussed above, this paper is organized as follows. Section I gives a brief introduction of bearing fault detection. Section II introduces the complex Morlet wavelet transform. Section III gives the applications of the complex Morlet wavelet amplitude and phase map to fault diagnosis of rolling element bearing. Finally, the main conclusions of this paper are given in Section IV.

II. COMPLEX MORLET WAVELET TRANSFORM

The complex Morlet wavelet is given as follows:
\[
\psi(t) = \pi^{-\frac{1}{4}} e^{-\frac{1}{2}t^2} \left( e^{-i\omega_0 t} - e^{-\frac{1}{2}(\omega - \omega_0)^2} \right)
\] (1)

The Fourier transform of Morlet wavelet is:
\[
\hat{\psi}(\omega) = \sqrt{2\pi} e^{-\frac{1}{2}(\omega - \omega_0)^2}
\] (2)

The Morlet wavelet, its real part, imaginary part, and its spectrum are displayed in Fig.1, respectively. From Fig.1, we can see that the real part or the imaginary part of Morlet wavelet has about 6 oscillations.

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In addition, we can extend the Morlet mother wavelet \( \psi(t) \) to its daughter wavelet in terms of the shift parameter \( b \) and the scale parameter \( a \),

\[
\psi_{ab}(t) = \frac{1}{a} \frac{1}{\sqrt{\pi}} e^{-\frac{t^2}{a^2}} e^{-i\omega_0 t} e^{-\frac{(t-b)^2}{2a^2}}
\]

(3)

\[
\hat{\psi}_{ab}(\omega) = \sqrt{2\pi} e^{-\frac{1}{2} (\omega-a_0)^2}
\]

(4)

Eq.(3) and Eq.(4) indicate that Morlet wavelet is complex in time domain and real in frequency domain.

The continuous wavelet transform (CWT) \( W_x(t,a) \) of the signal \( x(t) \) is defined as a convolution integral of \( x(t) \) with scaled and dilated versions of Morlet mother wavelet function \( \psi(t) \):

\[
W_x(t,a) = \frac{1}{a} \int x(\tau) \psi\left(\frac{t-\tau}{a}\right) d\tau = x(t) * \psi_{ab}(t)
\]

(5)

When the signal \( x(t) \) is periodic, Eq.(5) becomes a circular convolution, and the continuous wavelet transform (CWT) is better calculated in the frequency domain according to

\[
W_x(t,a) = \frac{1}{a} F^{-1}[X(\omega) \bullet aW(a\omega)]
\]

(6)

Where \( X(\omega) \) and \( aW(a\omega) \) are the Fourier transform of the signal \( x(t) \) and dilated wavelet \( \psi_{ab}(t) \), respectively. The operator \( F^{-1}[\bullet] \) denotes inverse Fourier transform. In our computer implementation, \( X(\omega) \) is calculated only once, and \( W(a\omega) \) is calculated for all values of the scale parameter \( a \).

Since the Morlet wavelet is complex-valued, the Morlet wavelet transform \( W_x(t,a) \) is also complex-valued. Thus, it gives the amplitude and phase modulation of the signal \( x(t) \) over the entire time-scale plane. The absolute value of \( W_x(t,a) \) indicates the transient energy of a signal \( x(t) \), which provides an amplitude plot, and its phase \( \theta_x(t,a) \) shows in Eq.(7) indicates the transient phase of a signal \( x(t) \), which provides an phase plot.

\[
\theta_x(t,a) = \tan^{-1}\left( \frac{\text{Im}[WT_x(t,a)]}{\text{Re}[WT_x(t,a)]} \right)
\]

(7)

where the operator \( \text{Re}(\bullet) \) and \( \text{Im}(\bullet) \) are the real part and imaginary part of \( W_x(t,a) \), respectively. The operator \( \tan^{-1}(\bullet) \) denotes inverse tangent.

III. BEARING FAULT DIAGNOSIS BASED ON MORLET WAVELET AMPLITUDE AND PHASE MAP

To demonstrate the performance of the proposed approach, this section presents the application examples for the detection and diagnosis of localized bearing defects. In all the example, the complex Morlet wavelet is as a wavelet transform mother function.

Bearings are installed in many kinds of machinery. A lot of problem of those machines may be caused by defects of the rolling bearing. Generally, local defects may occur on inner race, outer race or balls of bearing. A local fault may produce periodic impacts, the size and the repetition period which are determined by the shaft rotation speed, the type of fault and the geometry of the bearing. The successive impacts produce a series of impulse response, which maybe amplitude modulated because of the passage of fault through the load zone. The spectrum of such a signal would consist of a harmonics series of frequency components spaced at the component fault frequency with the highest amplitude around the resonance frequency. These frequency components are flanked by sidebands if there is an amplitude modulation due to the load zone. According to the period of the impulse, we can judge the location of the defect using characteristic frequency formulae. Because inner race defect has more transfer segments when transmitting the impulse to the outer surface of the case, usually the impulse components are rather weak in the vibration signal.

The tested bearing was used to study only one kind of surface failure: the bearing was damaged on the inner race or outer race. The rolling bearing tested has a groove on inner race or outer race. Localized defect was seed on the inner race or outer race by an electric-discharge machine to keep their size and depth under control. The size of the artificial defect was 1mm in depth and the width of the groove was 1.5mm. The input motion is produced by an AC motor. The speed of the spindle is 1500r/min, that is, the rotating frequency \( f \) is 25 Hz. The type of the ball bearing is 208. There are 10 balls in a bearing and the contact angle \( \alpha = 0^\circ \), ball diameter \( d = 55/3 \) mm, bearing pitch diameter \( D=97.5 \) mm. Then the characteristic frequency of the inner race defect and outer race defect can be calculated by the Eq.(8) and Eq.(9) respectively [10,11].
Therefore, according to Eq. (8) and Eq. (9), the characteristic frequency of the inner race defect and outer race defect are calculated to be at 148.5 Hz and 101.5 Hz, respectively.

The monitoring and diagnostic system is composed of four accelerometers, amplifiers, B&K 3560 spectrum analyzer and a computer. The sampling frequency is 32768 Hz.

A. Application of Morlet wavelet amplitude and phase map to fault detection of inner race

The original vibration signal of inner race defect is displayed in Fig. 2. It is clear that there are periodic impacts in the vibration signal. There are significant fluctuations in the peak amplitude of the signal, and there are also considerable variations of frequency content. From Fig. 2, we can hardly find the characteristic period of the inner race defect.

Fig. 3 shows the FFT of the vibration signal with inner race fault. There is no characteristic defect frequencies component around $f_{inner}$ and its harmonics. Therefore, classical Fourier analysis has some limitation such as being unable to extract the characteristic defect frequency of inner effectively.

To the data of Fig. 2, the Morlet wavelet transform is applied, resulting in wavelet amplitude and phase map. The Morlet wavelet amplitude and phase map are shown in Fig. 4(a) and Fig. 4(b), respectively. In Fig. 4, we can clearly see the distinctive signature for the vibration signal with bearing inner race fault. The wavelet amplitude and phase map both show about 36 ‘jumps’ which correspond to the bearing characteristic defect cycle of the inner race ($T_{inner} = 0.006734s$, $f_{inner} = 1 / T_{inner}$). The presence of inner race fault results in a sudden increase of vibration energy. For the defective bearing, transient vibrations caused by the rolling elements-defect interactions are clearly seen through the characteristic defect frequency and its high order harmonic frequencies. In addition, these transient vibrations have shown a repetitive pattern with $T_{inner}$ interval, which corresponds to a repetitive characteristic defect cycle of the inner race, resulting from the structural defect on the inner race. Such repetitive cycle reflects degradation of the inner race health condition as the defect propagated through the bearing inner race. Physically, impacts generated by the rolling ball-defect interactions excite intrinsic modes of the bearing system, giving rise to a train of transient vibrations at the mode-related resonant frequencies.

The phase wavelet map showed in Fig. 4(b) is clearer when compared with amplitude wavelet map shown in Fig. 4(a). Therefore, the phase wavelet map is often much more sensitive than amplitude wavelet map. The phase wavelet map for scales below 80 does not show a clear pattern. This is expected for the low scales where many noise sources contaminate the signal.

The simplicity of the Morlet wavelet amplitude and phase representation can be put down to the ability of this signal processing method to eliminate undesirable modulation effects. Therefore, the Morlet wavelet amplitude and phase map have shown to provide an effective tool for bearing inner fault diagnosis.
B. Application of Morlet wavelet amplitude and phase map to fault detection of outer race

The original vibration signal of the outer race defect is displayed in Fig.5. To the data of Fig.5, the Morlet wavelet transform is applied, resulting in wavelet amplitude and phase map. The Morlet wavelet amplitude and phase map are shown in Fig.6(a) and Fig.6(b), respectively. In Fig.6, we can clearly see the distinctive signature for the vibration signal with bearing outer race fault. The wavelet amplitude and phase map both show about 25 ‘jumps’ which correspond to the bearing characteristic defect cycle of the outer race (\(T_{\text{outer}} = 0.009852\text{s}, \ f_{\text{outer}} = 1/T_{\text{outer}}\)). In addition, these transient vibrations have shown a repetitive pattern with interval, resulting from the structural defect on the outer race. Such repetitive cycle reflects degradation of the outer race health condition as the defect propagated through the bearing outer race.

![Fig.5 Original vibration signal with bearing outer race fault](image)

![Fig.6 The Morlet wavelet amplitude and phase map of vibration signal with outer race fault](image)

IV. CONCLUSIONS

The complex Morlet wavelet is a complex valued wavelet. Since the Fourier spectrum of Morlet wavelet is real, which the Fourier spectrum has no complex phase, the Morlet wavelet does not affect the phase of a signal in complex domain. This gives a desirable ability to detect the singularity characteristic of a signal precisely. This research shows that the Morlet amplitude and phase map are very effective in detection and diagnosis bearing fault. The technique exceeds in extracting transients, which are often the indicators of incipient defect in bearing system. Therefore, we can recognize the vibration modes that coexist in the system, and to have a better understanding of the nature of the fault information contained in the vibration signal. The experimental results have been shown that the Morlet amplitude and phase map can effectively diagnose the bearing fault.

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