Corner detection based on gradient correlation matrices of planar curves

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Abstract

An efficient and novel technique is developed for detecting and localizing corners of planar curves. This paper discusses the gradient feature distribution of planar curves and constructs gradient correlation matrices (GCMs) over the region of support (ROS) of these planar curves. It is shown that the eigen-structure and determinant of the GCMs encode the geometric features of these curves, such as curvature features and the dominant points. The determinant of the GCMs is shown to have a strong corner response, and is used as a “cornerness” measure of planar curves. A comprehensive performance evaluation of the proposed detector is performed, using the ACU and localization error criteria. Experimental results demonstrate that the GCM detector has a strong corner position response, along with a high detection rate and good localization performance.

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1. Introduction

Corners are important features in images, and are frequently used for scene analysis, stereo matching, robot navigation, stitching of panoramic photographs, and object tracking. There are many competing algorithms for detecting corners in images. Since the pioneering work of Förstner [1], and Harris and Stephens [2], the structure tensor of image gradients, denoted by \( P \), has become popular for corner detection. For example, Rohr [4,5] developed a rotationally invariant corner detector based solely on the determinant of \( P \). Tomasi and Kanade [6], and Shi and Tomasi [7] proposed corner detectors based on the smallest eigenvalue of \( P \). Recently, Kenney and Manjunath [8] defined a reciprocal value of the smallest eigenvalue of \( P \) as a “cornerness” measure according to condition number theory. Kenney, Zuliani, and Manjunath [3] discussed and reviewed five detectors, and presented an axiomatic approach to corner detection based on the structure tensor of image gradients. Structure tensors have been used rather successfully to find corners in images in these detectors, and applications in invariant region detection or shape detection have also been reported. It should be noted that these structure tensors are calculated from image intensities.

Corner detectors using boundary based methods [9–22] have been proposed as an alternative to intensity based methods. Tsai, Hou, and Su [9] proposed a boundary based corner detector using the eigenvalues of covariance matrices of contour coordinate points over the region of support (ROS). This approach requires that the radius of the ROS is large enough to suit the statistical characteristics of Tsai’s detector. If the radius is too large however, the responses of adjacent corners may interfere with each other causing the detector to miss small features; while if the radius is too small, the detector may suffer from sensitivity to noise. Rattarangsi and Chin [10], Mokhtarian and Suomela [11,12], and Mohanna and Mokhtarian [13] proposed a kind of curvature scale space (CSS) technique for corner detection. He and Yung [14] proposed an improved CSS corner detector with an adaptive curvature threshold and a dynamic region of support (ATCSS). Zhong and Liao [26] proposed a direct curvature scale space (DCSS) technique, and presented a corner detector combining CSS and DCSS. Zhang, Lei, and Yang [15] proposed a multi-scale curvature product (MSCP) technique for enhancing the response of corners, while suppressing the noise response. Yeh [20] proposed a curvature estimation method using wavelet transforms of eigenvectors of covariance matrices. These methods based on curvature estimation require calculating higher order derivatives of the smoothed versions of planar curves, and can suffer from sensitivity to noise. Awrangjeb and Lu [27] proposed a robust feature detector integrating multi-scale products, adaptive curvature thresholds, and chord-to-point distance accumulation (CPDA) into the discrete curvature estimation. The CPDA detector does not need to calculate derivatives of the planar curves, and is robust to geometric transformations. This approach may merge nearby features and miss some weak corners due to the larger radius of the ROS required however. Zhang and Wang [28] proposed a corner detector based on the scale evolution difference of contours and a difference of Gaussian filter. To the best of the
authors’ knowledge however, no methods using the structure tensor of planar curve gradients for boundary based corner detection have been presented in the literature.

In this paper a novel algorithm for corner detection based on the structure tensor of planar curve gradients is developed. Similar to the well-known Förstner [1], Harris [2] and Rohr [4,5] detectors, the proposed detector computes the structure tensor of the gradient and seeks corners at the maxima of its determinant. Instead of using the image gradient however, the proposed detector uses the contour’s gradient vectors. The gradient correlation matrix (GCM) formulated using Lagrange multipliers only requires calculation of the first derivative of the planar curves. This is advantageous as avoiding the higher order derivatives reduces the effect of noise. A small ROS radius may then be used to improve corner localization and to prevent nearby features from merging. Consequently, the proposed detector offers a high detection rate along with good localization performance. The main contributions and organization of this paper are as follows:

(1) Firstly, we introduce the motivation for using GCMs by analyzing the gradient feature distribution in the plane spanned by the gradients of planar curves, and formulate the GCM by least squares and the Lagrange multiplier method, where the GCM is viewed as the structure tensor of planar curve gradients.

(2) Secondly, we analyze the geometric properties of the GCM based on typical corner models [10], and find that the local maxima of the determinant of the GCMs correspond to the positions of dominant (corner) points. Numerical simulations based on the discrete END model are given to illustrate the behavior of the determinant of the GCMs, with experimental results used to validate this theoretical analysis. From this analysis, it follows that the determinant of the GCMs can be used as a “cornerness” measure of planar curves.

(3) Finally, we perform a comprehensive evaluation of the detection and localization performances of the proposed detector using the ACU [11] and the localization error (LE) [14,27] criteria. The proposed GCM corner detector has a strong corner position response along with a high detection rate and a good localization performance, in terms of the ACU and LE criteria.

2. Gradient correlation matrix

2.1. Motivation

Considering a regular planar curve

\[ C(t) = (x(t), y(t)) \quad (1) \]

parameterized by \( t \), where \( x(t), y(t) \) are coordinate functions. From (1), the gradient vector at any point on the curve can be expressed as

\[ C'(t) = (x'(t), y'(t)) = (dx, dy) \quad (2) \]

where \( dx, dy \) are the gradients of the planar curve \( C(t) \) in the \( x \) and \( y \) directions. In the following section we discuss an intuitive observation about the relationship between the dominant points of planar curves and the gradient distribution for a set of gradient vectors over the ROS.

If we think of any gradient vector in the ROS as a point in a plane described by axes \( dx \) and \( dy \), the distribution of gradient vectors for all points in the ROS will reflect the local feature structure. As an illustration, Fig. 1(a) shows a planar curve with three corners and two test points, \( P \) and \( Q \), on the curve, where \( P \) is a corner denoted by the “square”, while \( Q \) is a point on the straight line segment denoted by the “circle”. Fig. 1(b) shows the distribution of the gradient vectors over the ROS at the center of the points \( P \) and \( Q \), where the radius of ROS is equal to two. We see that the gradient distribution over the ROS of the point \( Q \) is highly concentrated, while in contrast the dispersion of the gradient distribution over the ROS of the point \( P \) is quite large. From Fig. 1(b), it follows that the gradient vectors tend to converge to a point if the ROS in the neighborhood of any center point is straight line segment, while the gradient distribution scatters around the origin if the ROS contains a corner. Furthermore, using linear least squares to find the line of best fit to a given set of the gradient vectors in the ROS of a point on a planar curve, the residual error will reflect the degree of dispersion of the gradient vectors in the gradient plane.

Based on the above intuitive observation, we regard the gradient vectors of planar curves as points in a two-dimensional plane. The corresponding gradient correlation matrices are constructed by fitting a line to a given set of the gradient vectors over the ROS of any point on the planar curves. These matrices describe the local geometric features of the planar curves, such as normal and gradient directions, and the dominant points. A simple and effective cornerness measure for planar curves is presented, using the eigen-structure and properties of the GCM.

Remark 1. When we regard any image gradient vector as a point in the plane spanned by image gradients, we obtain a similar geometric explanation. Hence, the structure tensor of image gradients can be derived by our approach.

2.2. Gradient correlation matrix calculation

We now consider the set of gradient vectors in the ROS of a planar curve, \( (dx_i, dy_i)^T, i = 1, \ldots, k \). Characterizing the line of best fit by its unit normal vector \( \mathbf{n} \), the perpendicular distance between each point \( (dx_i, dy_i) \) and the line of best fit is equal to the projection of the point onto \( \mathbf{n} \), \( d_i^2(\mathbf{n}) = (\mathbf{n}^T [dx_i; dy_i])^2 \). To find the line of best fit, we minimize the sum of the squared perpendicular distances:

\[ e^2 = \sum_{i=1}^{k} d_i^2(\mathbf{n}) = \mathbf{n}^T M \mathbf{n} \quad (3) \]

where

\[ M = \sum_{i=1}^{k} (dx_i, dy_i)^T (dx_i, dy_i) \]

(4)

Performing constrained minimization using a Lagrange multiplier, and minimizing (3) subject to \( \mathbf{n}^T \mathbf{n} = 1 \), we have \( 2Mn - 2\mathbf{n} = 0 \), i.e.,

\[ Mn = \mathbf{n} \quad (5) \]
In this paper $M$ denotes the gradient correlation matrix. From (5), it can be seen that the unit vector normal to the line of best fit is equivalent to the eigenvector of $M$ with the smallest eigenvalue. Moreover, substituting (5) into (3), we can show that the minimum eigenvalue of $M$ reflects the feature distribution—the degree of dispersion of the gradient vectors in the gradient plane. Therefore, $M$ characterizes the local geometric features of a planar curve.

**Remark 2.** The GCM is regarded as the standard outer product of the planar curve gradients, which is an extension of the structure tensor of image gradients. As such the GCM is built using the set of contour gradients in the ROS, $X_{GCM} = \{(dx_k, dy_k)T, \ldots, (dx_k, dy_k)T\}$, where $k$ is the radius of ROS. This is different from covariance matrix based approaches [9,20], which are built from the set of contour points in the ROS, $X_{COV} = \{(x_k, y_k)T, \ldots, (x_k, y_k)T\}$. While these forms become analogous when the expectation (mean) of the contour points is subtracted from $X_{COV}$, they remain quite distinct.

### 3. Behavior of the determinant of the GCM

To determine the relationship between the GCM and the dominant points of a planar curve, further investigation into the properties of $M$ is required. Two typical corner models presented by Rattarangsi [10] are examined, the $\Gamma$-corner model and the END model, which are treated as isolated segments to simplify the investigation. If the radius of the ROS is small enough, any complex corner model of a planar curve can be treated as a combination of the two models. Figs. 2(a) and (b) show the $\Gamma$-corner model and the END model respectively.

Assuming that $C(t)$ is a continuous differential curve, the GCM in (4) can be redefined as the continuous auto-correlation matrix between $dx$ and $dy$:

$$M(t) = \begin{bmatrix} \left\langle \frac{dx}{dy} \right\rangle & \langle dx \rangle \\ \langle dx \rangle & \langle d^2y \rangle \end{bmatrix}$$

where

$$\langle \ast \rangle = \int_{t-W}^{t+W} \ast ds$$

indicates an integral operation on the small interval $[t-W, t+W]$, centered at the point $t$, with $W$ the radius of ROS.

In the following sections, the relationship between the determinant of $M(t)$ and the dominant points of a planar curve is discussed based on $\Gamma$ and END models.

### 3.1. Behavior of the GCM determinant based on the $\Gamma$-corner model

Considering the $\Gamma$-corner model, in which two straight line segments intersect at a corner:

$$x(t) = \begin{cases} t \sin \theta, & t < 0 \\ t, & t \geq 0 \end{cases}$$

$$y(t) = \begin{cases} t \cos \theta, & t < 0 \\ 0, & t \geq 0 \end{cases}$$

(7)

where $\theta$ is the angle between one line segment and the vertical axis as shown in Fig. 2(a). A straight line is then a special case of the $\Gamma$-corner model where $\theta$ is equal to $90^\circ$.

**Property 1.** The determinant of $M(t)$, $\det(M(t))$, will reach a unique maximum at the intersection where $t$ equals 0, i.e., there exists a unique maximum at the corner.

From (6) and (7) we can express $M(t)$ as

$$M(t) = \begin{bmatrix} (W-t)\sin^2 \theta + t + W & (W-t)\sin \theta \cos \theta \\ (W-t)\sin \theta \cos \theta & (W-t)\cos^2 \theta \end{bmatrix}$$

else, $M(t) = 0$.

The determinant of $M(t)$ can then be rewritten as

$$\det(M(t)) = \begin{cases} (W^2 - t^2)\cos^2 \theta, & (t - W < 0) \text{ and } (t + W > 0) \\ 0, & \text{otherwise} \end{cases}$$

Clearly, $\det(M(t)) \leq W^2 \cos^2 \theta \forall t$. It is easy to verify that the unique maximum of $\det(M(t))$ is equal to $W^2 \cos^2 \theta$ if and only if $t = 0$, i.e., Property 1 holds.

As a special case of the $\Gamma$-corner model, we reach the following conclusion for a line segment model.

**Property 2.** For a line segment, $\det(M(t)) = 0 \forall t$.

### 3.2. Behavior of the GCM determinant based on the END model

Consider the END model, which contains two corners, $\theta_l$ and $\theta_r$, that are separated by a width of $\omega$, as shown in Fig. 3. This model is represented by

$$x(t) = \begin{cases} t \sin \theta_l, & 0 < t < \omega \\ t, & 0 < t < \omega \\ (t - \omega)\sin \theta_l + \omega, & \omega < t \end{cases}$$

$$y(t) = \begin{cases} t \cos \theta_l, & 0 < t < \omega \\ 0, & 0 < t < \omega \\ (t - \omega)\cos \theta_l + \omega, & \omega < t \end{cases}$$

(8)

If $2W \leq \omega$, we may regard the END model as two conjoint and independent $\Gamma$ models because any integral interval of (6) contains at most one corner. If $2W > \omega$, some integral interval of (6) must contain both corners, as shown in Fig. 3(b). Based on this, we only consider the situation where $2W > \omega$ for the END model.

Firstly, when $W + \omega \leq t$, or $t < -W$, we have

$$\det(M(t)) = 0$$

(9)
Secondly, for \(-W \leq t < \omega - W\), the interval only contains the corner \(\theta_i\) as shown in Fig. 3(a), and so

\[
M(t) = (W - t) \begin{bmatrix} \sin^2 \theta_i & -\sin \theta_i \cos \theta_i \\ -\sin \theta_i \cos \theta_i & \cos^2 \theta_i \end{bmatrix} + \begin{bmatrix} (t + W) & 0 \\ 0 & 0 \end{bmatrix}
\]

and

\[
det(M(t)) = (W^2 - t^2) \cos^2 \theta_i
\]

(10)

Thirdly, for \(\omega - W \leq t < W\), the interval contains both corners \(\theta_i\) and \(\theta_j\) as shown in Fig. 3(b). We then have

\[
M(t) = (W - t) \begin{bmatrix} \sin^2 \theta_i & -\sin \theta_i \cos \theta_i \\ -\sin \theta_i \cos \theta_i & \cos^2 \theta_i \end{bmatrix} + \begin{bmatrix} (t + W - \omega) & 0 \\ 0 & 0 \end{bmatrix}
\]

and

\[
det(M(t)) = \sin^2(\theta_i + \theta_j)(-t^2 + \omega t) + \omega \cos(2 \theta_i - \cos^2(\theta_i)) + \sin^2(\theta_i + \theta_j)W(W - \omega) + \omega W(\cos^2(\theta_i + \cos^2(\theta_i)) - \omega^2 \cos^2 \theta_i)
\]

(11)

Finally, for \(W \leq t < W + \omega\), the interval only contains the corner \(\theta_i\) as shown in Fig. 3(c), and so

\[
M(t) = (t + W - \omega) \begin{bmatrix} \sin^2 \theta_i & -\sin \theta_i \cos \theta_i \\ -\sin \theta_i \cos \theta_i & \cos^2 \theta_i \end{bmatrix} + \begin{bmatrix} (W - t + \omega) & 0 \\ 0 & 0 \end{bmatrix}
\]

and

\[
det(M(t)) = (W^2 - (t - \omega)^2) \cos^2 \theta_i
\]

(12)

**Property 3.** Using the END model, if \(\theta_i\) and \(\theta_j\) satisfy \(\sin(\theta_i + \theta_j) = 0\) or \(\theta_i + \theta_j = n \times 180^\circ\) (where \(n\) is any integer), the determinant of \(M(t)\) has two local maxima at \(t = 0\) and \(\omega\).

In this case, the first line segment is parallel to the final line segment, and hence \(dx\) and \(dy\) in the neighborhood of one corner are the same as \(dx\) and \(dy\) in the neighborhood of the other corner. Therefore, although the ROS changes from containing one corner to containing both corners, the dispersion of \(dx\) and \(dy\) in the space expanded by \(dx\) and \(dy\) will not change, and a transition region will occur in the interval \([\omega - W, W]\) of the curve.

**Property 4.** Using the END model, if the width of the integral interval \(W\) satisfies \(2W > \omega\) and \(W < \omega\), the determinant of \(M(t)\) has three local maxima at \(t = 0, \omega\) and \(\omega/2\).

\[
\omega^2 \left[ \frac{1}{2} + \frac{(\cos^2 \theta_i - \cos^2 \theta_j)}{2 \sin^2(\theta_i + \theta_j)} \right]
\]

Here, as the support point of the ROS approaches \(\omega - W\), the ROS will change from containing one corner to containing both corners, and \(dx\) and \(dy\) in the neighborhood of one corner are not the same as \(dx\) and \(dy\) in the neighborhood of the other corner. This will result in the augmentation of the dispersions of \(dx\) and \(dy\). As the support point of the ROS lies in

\[
\omega^2 \left[ \frac{1}{2} + \frac{(\cos^2 \theta_i - \cos^2 \theta_j)}{2 \sin^2(\theta_i + \theta_j)} \right]
\]

the dispersion of \(dx\) and \(dy\) will become locally maximal. If \(\theta_i = \theta_j\), \(M(t)\) has a maximum at the center of the two corners.

**Property 5.** Using the END model, if the width of the integral interval \(W\) satisfies \(2W > \omega\) and \(W < \omega\), \(\theta_i = \theta_j\), the determinant of \(M(t)\) has three local maxima at \(t = 0, \omega\) and \(\omega/2\).

**Property 6.** If one of \(\theta_i\) or \(\theta_j\) is equal to \(90^\circ\), (11)–(13) become the result of \(I\)-Corner model. The determinant of \(M(t)\) has a unique maximum at \(t = 0\) or \(t = \omega\).

**Remark 3.** Property 3 can be derived from other corner models, such as the STAIR model [8], in a similar fashion. From properties 1–3, it follows that \(det(M(t))\) may be defined as a cornerness response function for the corners of planar curves.

### 3.3. Behavior of the GCM determinant based on the discrete END model

In this section the behavior of the GCM determinant is examined using the discrete END model and the discrete GCM, with experiments performed to test the effect that the angle parameters and radius of the ROS have on the determinant of the GCM.

Consider the continuous END model (8), with parametric values of the two corners set to \(t = 0\) and \(10\), in the region \(-10 \leq t \leq 30\). For the corresponding discrete END model, setting the sample step size to 0.2, the positions of the two corners are \(t_1 = 10/0.2 = 50\) and \(t_2 = 20/0.2 = 100\). The distance between the two corners is \(\omega = 10/0.2 = 50\). Following Eqs. (4) or (6), we have the following discrete GCM for the \(k\)th point:

\[
M(k) = \sum_{i = -W}^{W} \begin{bmatrix} \Delta x_{ki} & \Delta y_{ki} \\ \Delta x_{ki} & \Delta y_{ki} \end{bmatrix}
\]

(13)

where \(k = 1, \ldots, n\) denotes the index of the \(k\)th contour point, \(n\) is the number of the points on a contour, \(W\) is the radius of the ROS, and \((\Delta x_{ki}, \Delta y_{ki})\) is the \(k\)th difference over the ROS. Two experiments are presented in the following sections to test the effect that the angle parameters and the radius of the ROS have on the discrete GCM.

**Experiment 1.** In this experiment, the effect that the angle parameters of the END model have on the determinant of the GCM is examined, for two ROS radii. Several pairs of angle parameters \(\theta_i\) and \(\theta_j\) are chosen, with the reason for their selection given in the brackets. These angle pairs are: \(30^\circ\) and \(30^\circ\) (\(\theta_i = \theta_j\) – 30 and \(-40^\circ\) (\(\theta_i < \theta_j\)), 30 and \(-30^\circ\) (\(\theta_i + \theta_j = 0\)) 30 and \(150^\circ\) (\(\theta_i + \theta_j = 180\)), 30 and \(40^\circ\) (\(\theta_i \neq \theta_j\)), 30 and 90 (\(\theta_i = 90\)). The two different ROS radii used were: \(W_1 = 4/0.2 = 20\) and \(W_2 = 7/0.2 = 35\), satisfying \(2W_1 < \omega\) and \(2W_2 > \omega\).

**Result 1.** The first column of Fig. 4 shows the corner model for the different angle pairs, while the second and the third columns show the determinant of the GCM at each sampled point. As shown in the second column of Figs. 4(a–f), when \(2W_1 < \omega\), local maxima occur only at the corner positions (the maxima position is invariant to the angle parameters). The third column of Fig. 4(c, d), shows that local maxima occur only at the corner positions, where \(2W_2 > \omega\) and \(\theta_i + \theta_j \neq 180\) (with \(n\) any integer). The third column of Figs. 4(a, b, e) however, show that an undesirable maximum appears between the two corners where \(2W_2 > \omega\) and \(\theta_i + \theta_j \neq 180\), with the undesirable maximum occurring at \(\omega/2\) when \(\theta_i = \theta_j\) as per Fig. 4(a).

The responses for the two corners when \(\theta_i \neq \theta_j\) and \(\theta_i + \theta_j \neq 180\) have unequal magnitude as shown in Figs. 5(b, e). Finally, the END model with \(\theta_i = 90^\circ\) reduces to the \(I\)-model and a unique maximum appears at \(t_i = 50\), as shown in Fig. 5(f).

**Experiment 2.** In this experiment, the effect that the radius of the ROS has on the determinant of the GCMs is examined, while the angles of the END model are kept constant. The following ROS radii are tested: 5, 15, 25, 35, 45, and 55, while \(\theta_i\) and \(\theta_j\) are set to the constant values of \(30^\circ\) and \(40^\circ\) respectively.
**Result 2.** When $W = 5, 15, 25$, i.e., $2W \leq \omega$, the responses of the two corners do not interact, and so local maxima only occur at the true corner locations. Figs. 5(a, b) show that the width of the local maxima corresponding to the corner locations increases as $W$ is increased, i.e. the corner resolution decreases as $W$ increases. When $W = 35, 45$, i.e., $2W > \omega$, an aliasing phenomenon appears in the corner response curves, and an undesirable maximum may occur between the two corners, as shown in Fig. 5(b). Furthermore, as $W$ approaches $\omega$, this undesired maximum may become the global maximum as the responses of the corners gradually diminish (relatively). As shown in the third column of Fig. 5(b), when $W = 55$, i.e., $W > \omega$, the responses at the corner positions are no longer locally maximal.

**Remark 4.** Based on the above results, we conclude that the behavior of the corner response is mainly effected by the radius of the ROS, and that the aliasing phenomenon (the undesirable maximum) will not occur in the corner responses when $2W \leq \omega$. Hence, for discrete planar curves the radius of the ROS is set to one for calculating the GCM in general. Thus, as the distance between any two corners must be greater than two pixels, no undesirable maximum will occur.

### 3.4. GCM corner detection algorithm

1. Utilize a “good” edge detector to extract edge contours from the original image.
(2) Extract the edge contours from the edge image and:
(a) Fill gaps in the edge contours.
(b) Locate the T-junctions and mark them as T-corners.
(3) Smooth the curves using a Gaussian kernel with standard derivation $\sigma$ to remove noise and trivial details.
(4) Calculate the corneress response function as described in Section 3.
(5) Determine the corners by comparing the local maxima of $\det(M(t))$ with a threshold value.

4. Performance evaluation and experiments

In this section, we present the results of three groups of experiments involving synthetic images, real images, and images disturbed by noise. Firstly, we summarize the experimental results of the parameter settings for the proposed GCM detector. Secondly, we compare the performance of the proposed corner detector with eight existing corner detectors: (a) CSS [11], (b) CPDA [27], (c) ATCSS [14], (d) MSCP [15], (e) Eigenvalue [9], (f) Eigenvector [20], (g) Wavelet [17], and (h) Harris [2].

4.1. Evaluation criteria

The performance of the detectors is evaluated using two metrics, accuracy (ACU) [13], and localization error (LE) [14,27]. Experiments were performed on both original images and their “degraded” (geometric transformed and noise disturbed) images.

The ACU accuracy criterion was proposed by Mohanna and Mokhtarian [13], and takes into account the number of corners in...
Letting \( N_0 \) denote the number of corners detected in the original image (\( N_0 \neq 0 \)), \( N_r \) the number of the true corners, and \( N_a \) the number of the correctly matched corners. The ACU accuracy criterion is

\[
\text{ACU} = \frac{(N_0 - N_a + N_a / N_r)}{2} \times 100\%
\]

The LE criterion [14,27] for evaluating the localization performance of corner detectors is defined as the root mean square (RMS) error of the corner locations in the test images, and the corresponding corners in the reference images. This can be expressed as

\[
\text{LE} = \sqrt{\frac{1}{N_r} \sum_{i=1}^{N_r} ((x_{gi} - x_{ti})^2 + (y_{gi} - y_{ti})^2)}
\]

where \((x_{gi}, y_{gi})\) and \((x_{ti}, y_{ti})\) are the positions of the \( i \)th matched corner in the reference and test images respectively, and \( N_r \) is the number of corners matched between the test image and reference images.

Ground truth corner sets were created for the original images by manually selecting the corner positions. Zooming in on each corner in the image allowed for sub pixel accuracy during this process. This process was performed twice, with the ground truth corner position the average of the two hand labeled corner positions. The number of true detections, the number of true corners that were missed (false non-detections), and the number of false detections were all collected. For geometric transformations, the ground truth corners were transformed using the transformation parameters prior to matching. Corners detected in a 3\( \times \)3 neighborhood of the true corners were considered to be matches.

**Remark 5.** The ACU is similar to the repeatability measure presented by Awrangjeb and Lu [27]. Unlike the repeatability measure presented in [27] however, the ACU measure requires not only high repeatability, but also that many true corners are detected. The LE measure also requires that many true corners are detected.

### 4.2. Image database

A total of 20 different original gray-level images including 7 real images and 13 synthetic images were used for experimentation. Many of these original images were collected from standard databases [9–21], and included: Block, House, Lab, Pentagon, Airplane, Flower, Leaf, Gear, Key, Fish, Shark, and some simple planar curves. Selected images are shown in Fig. 10. Five types of “degradations” were applied to each of the 20 original images giving a total data base size of 1960 images. These five types of degradations were:

- **Rotation:** The original image was rotated, with rotation angles chosen by sampling the interval \([-80\degree, +80\degree]\) at a 10\degree resolution.
- **Uniform scaling:** The original image was scaled with scale factors chosen by sampling the interval \([0.5, 2] \) at a resolution of 0.1.
- **Non-uniform scaling:** The \(x\) and \(y\) scale parameters were independently chosen by sampling the interval \([0.5, 1.8] \) at a resolution of 0.1.
- **Planar affine transforms:** Planar affine transforms were applied using: rotation angles in the interval \([-50\degree, +80\degree]\) at a 10\degree resolution, and non-uniform scaling with \(x\) and \(y\) scale parameters chosen in the interval \([0.5, 1.8] \) at a resolution of 0.1.

![Fig. 6.](image)

**Table 1** Parameters of the comparative detectors.

<table>
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<tr>
<th></th>
<th>GCM</th>
<th>CSS</th>
<th>CPDA</th>
<th>ATCSS</th>
<th>MSCP</th>
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<th>Eigenvector</th>
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<tr>
<td>Threshold for real-image</td>
<td>0.003</td>
<td>0.04</td>
<td>0.05</td>
<td>1.8</td>
<td>0.005</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
<td>2000</td>
</tr>
<tr>
<td>Threshold for noise images</td>
<td>0.0045</td>
<td>0.05</td>
<td>0.04</td>
<td>1.75</td>
<td>0.006</td>
<td>0.5</td>
<td>0.2</td>
<td>0.25</td>
<td>4000</td>
</tr>
<tr>
<td>Radius of ROS</td>
<td>1</td>
<td>–</td>
<td>R1: 10</td>
<td>–</td>
<td>–</td>
<td>10</td>
<td>10</td>
<td>–</td>
<td>10 (radius of region considered in non-maximal suppression)</td>
</tr>
<tr>
<td>Sigma</td>
<td>3</td>
<td>Low: 2</td>
<td>3</td>
<td>High: 3</td>
<td>Low: 2</td>
<td>High: 3</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Scale factor</td>
<td>–</td>
<td>–</td>
<td>R2: 20</td>
<td>R3: 30</td>
<td>–</td>
<td>–</td>
<td>2</td>
<td>2</td>
<td>k=0.04 (the constant k in the Harris cornerness measure)</td>
</tr>
</tbody>
</table>
Gaussian noise: Zero mean Gaussian white noise was added to the original image. Variances were chosen by sampling the interval [0.005, 0.05] at a resolution of 0.005.

The total of 1960 images then comprised of 20 original, 320 rotated, 300 uniformly scaled, 600 non-uniformly scaled, 520 rotated and scaled images, and 200 Gaussian noised images.

4.3. Parameter settings for comparative corner detectors

All of the detectors were set with their default best parameter values. For a fair comparison, all boundary based detectors were implemented using the same edge extraction and contour tracking method. Selected parameters for the various detectors are summarized in Table 1.

In this section, an experiment is performed to determine appropriate parameter values for the proposed GCM detector. To extract good boundaries, we used the Matlab® implementation of Canny's edge detector, and the contour tracking method suggested by Mokhtarian [11], as the first two steps of the algorithm. The three parameters for Canny's edge detector were chosen as 1 (sigma), 0 (low threshold) and 0.35 (high threshold). The purpose of the third step of the proposed algorithm (Section 3.4) is to reduce the effect of noise, such as quantization and random noise. The sigma of the

![Figure 7](https://example.com/fig7.jpg)

Fig. 7. ACU and LE values under the six geometric transformations for the synthetic images: (a1)–(a6) ACU and (b1)–(b6) LE.
Gaussian function indicates the scale of the smoothed contours, and should be small enough both to retain real corners, and to leave their positions unchanged. In the experiments presented later, a sigma value between 2 and 3.5 was found to give good detection and localization performance, with 3 the most commonly used value.

Table 2
Mean metric values for the synthetic images under geometric degradations.

<table>
<thead>
<tr>
<th>Metric</th>
<th>GCM</th>
<th>CSS</th>
<th>CPDA</th>
<th>ATCSS</th>
<th>MSCP</th>
<th>Eigenvalue</th>
<th>Eigenvector</th>
<th>Wavelet</th>
<th>Harris</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACU</td>
<td>83.491</td>
<td>75.228</td>
<td>79.786</td>
<td>76.460</td>
<td>81.267</td>
<td>74.135</td>
<td>65.898</td>
<td>65.502</td>
<td>58.914</td>
</tr>
<tr>
<td>LE</td>
<td>1.406</td>
<td>1.410</td>
<td>1.441</td>
<td>1.376</td>
<td>1.385</td>
<td>1.472</td>
<td>1.752</td>
<td>1.732</td>
<td>1.741</td>
</tr>
</tbody>
</table>

Gaussian function indicates the scale of the smoothed contours, and should be small enough both to retain real corners, and to leave their positions unchanged. In the experiments presented later, a sigma value between 2 and 3.5 was found to give good detection and localization performance, with 3 the most commonly used value. We believe this is a relatively small sigma value, which proves...
effective due to the strong "cornerness" response function given in Eq. (6). In step four of the GCM algorithm, the ROS radius was set to 1 for calculating the cornerness response of curves as per the discussion in Section 3.3, and hence the ROS width was equal to 3. The gradient operator in Eq. (13) was implemented using central differencing, with the central difference for the sequence \( x_i \) given by \( \Delta x = 0.5(x_{i+1} - x_{i-1}) \).

The corner response threshold is the most important parameter for all detectors, because the ACU value is highly sensitive to the selected threshold. The smaller the threshold, the more corners detected. At low thresholds however, the detector may detect too many weak/noisy false corners, resulting in a low ACU and a high localization error (LE). At high thresholds, the detector may miss too many true corners, again resulting in a low ACU and a high localization error. Figs. 6(a, b) show the average effect of threshold changes on the GCM corner detector for the original synthetic and real images respectively. From Fig. 6(a), for the synthetic images the ACU value increases to a stable maximum in the interval \([0.002, 0.003]\), with a small LE value in this interval. From Fig. 6(b), for the real images the stable interval of the ACU value was \([0.005, 0.007]\), again with a small LE value in this interval. Therefore, we empirically choose 0.003 and 0.006 as the thresholds for the

Fig. 8. ACU and LE values under the six geometric transformations for the real images: (a1)-(a6) ACU and (b1)-(b6) LE.

\[
D_{xi} = \frac{0.5(x_{i+1} + x_{i-1})}{\sqrt{2}}.
\]
proposed GCM detector for the synthetic and the real images respectively. The mean of these two thresholds (0.0045) was used for the images disturbed by noise, which included the synthetic and the real images. These parameters are summarized in Table 1.

4.4. Experimental results

Figs. 7(a1–a6) show the ACU metric for each detector for the synthetic images under each geometric degradation, while

<table>
<thead>
<tr>
<th>Metric</th>
<th>GCM</th>
<th>CSS</th>
<th>CPDA</th>
<th>ATCSS</th>
<th>MSCP</th>
<th>Eigenvalue</th>
<th>Eigenvector</th>
<th>Wavelet</th>
<th>Harris</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACU</td>
<td>60.302</td>
<td>60.013</td>
<td>54.706</td>
<td>55.391</td>
<td>60.227</td>
<td>54.494</td>
<td>50.532</td>
<td>44.694</td>
<td></td>
</tr>
<tr>
<td>LE</td>
<td>1.629</td>
<td>1.625</td>
<td>1.725</td>
<td>1.639</td>
<td>1.622</td>
<td>1.765</td>
<td>1.892</td>
<td>1.894</td>
<td>1.855</td>
</tr>
</tbody>
</table>

Table 3
Mean metric values for the real images under geometric degradations.
Figs. 7(b1–b6) show the LE metric. The horizontal axes for these figures describe the various kinds of geometric transformations. Different line markers are used to show the results of each of the nine compared detectors. Each node in each line is the ACU or LE value for the transformation parameter. Numeric values in the figure legends denote the average of the ACU or LE, corresponding to the particular transformation.

As shown in Fig. 7(a1), the ACU of the proposed GCM method is greater than each of the other detectors under each rotation angle, with the ACU mean of the GCM method exceeding 89%.

Figs. 7(a2), (a3) and (a4) show the ACU under the scale transformations in the x, y and x–y directions, respectively. The GCM method obtained good detection results over the interval [0.5, 1.5] in the x and y directions, while giving the best detection results in the smaller interval [0.5, 1.3]. Additionally, the ACU means of the GCM method are higher than any of the other detectors under these scale transformations. As shown in Figs. 7(a5, a6), the ACU mean for the GCM method is better than the other detectors under affine transformations, with the ACU means of the GCM exceeding 80%. The total average ACU and LE.
values of the nine detectors with respect to all geometric transformations are shown in Table 2, where the ACU of the GCM method outperformed each of the existing detectors. Table 2 also shows that the localization performances of the GCM, CSS, MSCP, ATCSS, Eigenvalue, and CAPD detectors are quite similar for the synthetic images.

Fig. 8 shows the ACU and LE for the real images for each detector for each geometric degradation. In Figs. 8(a1–a6), the ACU of the GCM, CSS, and MSCP detectors are quite similar, and the average ACU of these three detectors under each class of geometrical transformations are all around 60%, outperforming each of the other six detectors. Table 3 shows the average ACU of the nine detectors for the real images with respect to all geometric transformations, with the proposed GCM detector reporting the largest ACU mean. Figs. 8(b1–b6) show that the localization errors of the GCM, CSS, MSCP, and ATCSS detectors are again quite similar. The total average LE for the nine detectors is given in Table 3, which shows that the localization performances of the GCM, CSS, MSCP, and ATCSS detectors are quite similar for the real images.

Fig. 10. Detection results of the GCM detector for 20 original test images.
Figs. 9(a, b) show the results of ACU and LE for the synthetic and real images perturbed by Gaussian noise. The ACU of the proposed GCM detector outperformed the other eight detectors, with a localization error that was similar to the CSS, ATCS and MSCP detectors.

All experiments were performed on an Intel Pentium (R) 2.5 Ghz processor, Microsoft Windows XP operation system running Matlab. The average computational time in seconds for each detector under all geometric transformations, for both the
real and synthetic images, were: 1.0745 (GCM), 1.1436 (CSS), 1.2192 (CPDA), 1.0511 (ATCSS), 1.1384 (MSCP), 1.5022 (Eigenvector), 1.5697 (Eigenvector), 1.1388 (Wavelet), and 1.6777 (Harris).

The proposed GCM detector required less computational time than each of the other detectors with the exception of the ATCSS detector, while the Harris detector required the longest time due to the calculation of the eigenvalues of the structure tensor based on image intensities.

Fig. 10 shows the original images and the corresponding detection results for the GCM detector. The GCM detector used the parameters listed in Table 1 in all test images. In the thirteen synthetic images shown in Fig. 10, almost all true corners were detected with few erroneous corners (false positives), resulting in the highest ACU average of all of the detectors. Some true corners were missed in the seven real images shown in Fig. 10; however the ACU average of the GCM is the highest of all of the detectors. The GCM detector also reported the smallest LE average for the original real images, as can be seen from any of Figs. 8(b1–b6), with comparable results to the best of the other detectors across the entire range of degradations for both synthetic and real images. The GCM detector can then be said to have a high detection rate and good localization performance in the performed experiments.

4.5. Discussion

In this paper we have proposed a novel and robust GCM corner detector based on the structure tensor of planar curve gradients. It is vital that the structure tensor reflects the local structure
features of planar curves, which is ensured because of the GCM’s intuitive geometric characteristics discussed in Section 3. As a result, the proposed GCM detector obtained high detection rates while maintaining good localization performance.

In the performed experiments, over 80% of the GCM detector’s computational expense was spent performing edge detection, while calculating the cornerness measure was relatively efficient. Hence, the GCM approach is significantly faster than approaches based on image intensities, such as the Harris detector.

The planar curves were smoothed using a Gaussian function with a relatively small sigma before calculating the first order derivatives, which makes the GCM detector more robust to noise while maintaining good localization performance. The small ROS radius (one in the performed experiments) used by the proposed detector also ensures good localization performance, and avoids missing conjoint corners. Additionally, the GCM is formulated using least squares and Lagrange multipliers, and was shown to be robust to the various degradations performed.

5. Conclusion

In this paper we have presented a novel corner detection algorithm using gradient correlation matrices (GCMs) of planar curves, which was shown to outperform existing methods in terms of ACU during experiments on our image dataset. The GCM approach is based on an intuitive observation of the distribution of gradient vectors of planar curves. Gradient correlation matrices of planar curves were derived using least squares and Lagrange multipliers, and are defined as the standard structure tensor of planar curve gradients. It was shown that the determinant of the GCMs is an effective cornerness response function using the $I$ and END corner models. The determinant of the GCMs then forms the basis of our simple and efficient corner detection algorithm. The GCM approach has several advantages over more commonly used detectors. Both the Gaussian smoothing parameter and the ROS radius can be kept small, enhancing the detection and localization performance. Additionally, only the first order derivative of the planar curve is required, reducing the impact of noise. It has been demonstrated experimentally that the proposed detector has a high detection rate under different scalings, rotations, planar affine transformations, and noise disturbances, while maintaining good localization performance.

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References


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