Design of Adaptive Equalizer Based on Variable Step LMS Algorithm

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Abstract—Adaptive equalizer is important in transmission of wireless communication. The equalizer using least mean square (LMS) algorithm is adopted. Simulation results show that step size influences the algorithm convergence and stability, which will significantly affect the performance of adaptive equalizer. The requirement of step for convergence speed, time-varying tracking accuracy and convergence precision is contradictory. Therefore, a variable step LMS algorithm is presented in this paper. Simulation results show that the convergence speed and stability of the variable step algorithm are superior to ordinary LMS algorithm; moreover, the variable step algorithm is proper to be applied in channel equalization in low SNR.

Index Terms—Adaptive Equalizer, LMS, Variable step

I. INTRODUCTION

Multi-path effect exists in transmission of wireless data communication. The signal through different paths of different delay, is received in the same time, causing intersymbol interference. Moving communications carrier and surrounding objects (vehicles, etc.), result in the dissemination of environmental changes with time, that is, ISI caused by multi-path effects also changes with time. Adaptive equalization is a technology used to resolve inter-symbol interference.

Adaptive equalizer is used to make attents to time-varying unknown channel, so it needs a special algorithm to update the equalizer coefficients to track the channel changes. A detailed study of adaptive algorithm is a complex work.

As is described in [1]:

The disadvantage of zero forcing algorithms is that great noise gain may be appeared in the deep channel n frequency. As zero forcing equalizer completely ignores the effect of noise, it is not commonly used in wireless link.

The equalizer using least mean square (LMS) algorithm is more stable than the zero forcing equalizer. The criterion is that the mean square error (MSE) between the desired output value and the actual output value of the equalizer minimizes.

II. TRADITIONAL LMS ADAPTIVE EQUALIZATION ALGORITHM

The linear equalizer [2] is showed in Fig.1.

It can be seen, the signal x (n) after the AGWN channel is used as the input signal through the filter with different w (n), e(n) is the error between the output of the filter response y(n) and the expectation signal d(n) with delay of signal x(n). The filter adjust the values of w(n), according to feedback error e(n) and adaptive algorithm. With the sample x (n) of the continuously is updated, e(n) becomes smaller and smaller, and w (n) gradually is close to the nominal value, similar to the ideal channel characteristics. The filter plays the role of an inverse filter in the whole process actually.

The structure of the adaptive filter is showed in Fig.2.

The iterative formulas of steepest descent method based on least mean square algorithm (LMS algorithm) are defined as follows:

\[ y(n) = \mathbf{w}(n)^T x(n) = x^T(n)\mathbf{w}(n) \]

(1)

Where, x (n) is the filter input; y (n)is filter output and w (n) is filter weights

\[ e(n) = d(n) - y(n) \]

(2)
Where, \( d(n) \) is a reference signal and \( e(n) \) is the error between \( d(n) \) an \( y(n) \).

\[
w(n + 1) = w(n) + 2\mu e(n)x(n)
\]

(3)

Where, \( \mu \) is the step size.

Convergence condition of the LMS algorithm is limited to \( 0 < \mu < 1/\lambda_{\text{max}} \). \( \lambda_{\text{max}} \) is the largest eigenvalue of the autocorrelation matrix for input signal. The performance of the algorithm is influenced by the value of \( \mu \).

In LMS adaptive algorithm, the requirement of step factor for convergence rate, time-varying tracking accuracy and convergence precision is contradictory. In the range of convergence, convergence speed is faster with greater \( \mu \). But the \( \mu \) value is too large, oscillation will occur during the convergence; smaller \( \mu \) value can reduce steady-state the noise, improve the accuracy of convergence. However, the decrease of \( \mu \) value will reduce the convergence speed and tracking speed.

III. NEW VARIABLE STEP SIZE LMS ALGORITHM

1. Principle of variable step size LMS algorithm

The contradiction between the convergence speed and the convergence precision fixed step LMS algorithm can be solved in the variable step LMS algorithm. In the initial stages of adaptive and tracking phase, a larger step size is used in order to have fast convergence speed, when the algorithm is in the steady state, smaller step is used for a small steady-state error.

Some approximation is used as a measure to control step size in adaptive processes. Simple and effective method is to use the adaptive error signal in the process, trying to establish some kind of function between the step size and the error signal.

Currently, the main variable step size algorithm is to establish the nonlinear relationship between the step size and the error signal.

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2. New variable step size LMS algorithm

In the process of convergence, \( e(n) \) decreases and approaches zero value gradually; \( \mu \) value changes similar to \( e(n) \); and when \( e(n) = 0, \mu = 0 \). Therefore, monotone and smooth curve of mathematical function between \( e(n) \) and \( \mu \) can be concluded. The curve is through origin with \( \mu \) changing by adjust \( e(n) \). It is studied that arc-Tangent curve is consistent with the variation of step factor. Therefore, variable step size LMS algorithm based on arc-tangent function is presented in this paper, called atan-LMS algorithm.

The relationship between \( \mu \) and \( e(n) \) is established as follows:

\[
\mu(n) = \text{atan} \left( e(n) \right)
\]

(4)

For better variation, factors of \( \alpha, \beta \) and \( \gamma \) are introduced:

\[
\mu(n) = \beta \tan \left( \alpha e(n)^\gamma \right)
\]

(5)

Where, the shape of the curve is controlled by \( \alpha \) which influences the increase speed of \( \mu \); the range of function is controlled by \( \beta \); the speed of decline curve is determined by \( \gamma \).

3. Parameters Analysis

(1) \( \alpha \) for different values, \( \beta = 0.024, \gamma = 2 \)

As can be seen from Fig.3 that the step increases with the increase of \( \alpha \) in the same error case, which can speed up the convergence speed of the adaptive algorithm. But, when the error is small, the step changes largely cause the poor stability of the algorithm. It can be seen from the figure, the step when \( \alpha = 4 \) is close to the corresponding step when \( \alpha = 8 \), but better stability is achieved. So \( \alpha = 4 \) is adopted.

(2) \( \beta \) for different values, \( \alpha = 4, \gamma = 2 \)

As can be seen from Fig.4 that the step increases with the increase of \( \beta \) in the same error, it can accelerate the convergence speed of the adaptive algorithm. The initial step size and range of the step is determined by \( \beta \).

(3) \( \gamma \) for different values, \( \alpha = 4, \beta = 0.024 \)

As can be seen from Figure 5, step changes with less difference in the initial stages, but when the error is small, the step changes gently with the increase of \( \gamma \). The larger is \( \gamma \) value, the stronger is the complexity of the algorithm. Usually \( \gamma = 1 \) or 2 is adopted.

4. Simulation

\( x[n] \) is a random bipolar sequence, which value is randomly +1 and -1. Random signal is transmitted by channel which property may be a FIR filter with three-
coefficient \([0.3, 0.9, 0.3]\). Channel output is added white Gaussian noise with the variance of 1. the response of the FIR adaptive equalizer with 11 order is \(x[n-7]\). An evaluation of the merits of adaptive equalization algorithms, mainly focused on the convergence speed and tracking capability for time-varying channel.

Experiments are performed by choosing the reasonable parameters. The \(\mu\) value of the LMS algorithm is 0.06. The parameters of the atan-LMS algorithm are defined as: \(\beta = 0.04, \alpha = 4, \gamma = 2\). The length of the training sequence is 1000. The curve value is confirmed by the mean value of the error in 20 independent experiments.

1. **Convergence speed**

Simulation results in Fig.6 prove both algorithms have the almost same convergence speed, but the atan-LMS algorithm is more stable than the LMS algorithm.

![Figure 6](image)

**Figure 6** Convergence performance of LMS and atan-LMS

2. **Tracking capability**

Channel parameters change from \([0.3, 0.9, 0.3]\) to \([0.8, 0.6, 0.4]\) in the first 300 iterations number. Tracking capability for time-varying channels is studied in different signal to noise ratio.

Fig.7 shows that when the SNR is low, the tracking capability of the atan-LMS algorithm is superior to the LMS algorithm.

![Figure 7](image)

**Figure 7** Tracking capability of atan-LMS with different SNR

IV. CONCLUSION

In this paper, the algorithm of the adaptive equalizer is studied. Through analyzing the principle of the LMS algorithm, a variable step algorithm is presented in which step factor is amended by arc-tangent function. Simulation results show that the variable step algorithm is superior to the ordinary LMS algorithm. The variable step algorithm is suit for channel equalization in mobile communication technology in low SNR.

V. REFERENCE