

# Existence Theorem for Vector Quasi-Variational-like Inequalities

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## Abstract

In this paper, using the maximal element theorem, we prove some existence theorem for vector quasi-variational-like inequalities without monotonicity and without compactness.

*Keywords* : Set-valued mapping; vector quasi-variational-like inequality;  $L$ - $\eta$ -condition, maximal element theorem; locally convex Hausdorff topological space; upper semi-continuous; open lower sections;  $0$ - $C(x)$ -diagonally convex.

## 1 Introduction

In 1980, Giannessi [8] extended the classical variational inequality for vector valued functions, called vector variational inequality, with further applications to alternative theorems. Since then, vector variational inequalities and their generalizations have been used as tools to solve vector optimization problems. For the details on vector variational inequalities and their generalizations, we refer to [2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 19, 20, 21].

Motivated and inspired by recent works [4, 5, 6, 7, 9, 13, 14, 16, 18, 19, 21] on vector quasi-variational inequalities and vector variational-like inequalities we establish some existence result of solutions for our problem without monotonicity and without compactness.

## 2 Preliminaries

Let  $Y$  be a Hausdorff topological vector space and  $X$  be a nonempty convex subset of a real locally convex Hausdorff topological vector space  $E$ . We denote  $L(E, Y)$  the space of

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all continuous linear mappings from  $E$  into  $Y$  and  $\langle u, y \rangle$  be the evaluation of  $u \in L(E, Y)$  at  $y \in E$ . Let  $\sigma$  be the family of all bounded subsets of  $X$  whose union is total in  $E$ , i.e., the linear hull of  $\cup\{S : S \in \sigma\}$  is dense in  $X$ . Let  $\beta$  be a neighbourhood base of  $0$  in  $Y$ . When  $S$  runs through  $\sigma$  and  $V$  through  $\beta$ , the family

$$M(S, V) = \{l \in L(E, Y) : \bigcup_{x \in S} \langle l, x \rangle \subset V\}$$

is a neighbourhood base of  $0$  in  $L(E, Y)$  at  $x \in E$  and  $L(E, Y)$  is the locally convex topological vector space under the  $\sigma$ -topology [8].

Let  $\text{int}A$  and  $\text{co}(A)$  denote the interior and convex hull of a set  $A$ , respectively. Let  $C : X \rightarrow 2^Y$  be a set-valued mapping such that  $C(x)$  is a closed pointed and convex cone with  $\text{int}C(x) \neq \emptyset$  for each  $x \in X$ . Let  $\eta : X \times X \rightarrow E$  and  $H : X \times X \rightarrow Y$  be vector valued mappings, and  $D : X \rightarrow 2^X$  and  $T : X \rightarrow 2^{L(E, Y)}$  be set-valued mappings. Now, we introduce a new model of vector quasi-variational-like inequality (VQVLI), which is to find  $x_0 \in X$  such that for all  $y \in D(x_0)$ , there exist  $v_0 \in T(x_0)$  with

$$\langle A(x_0, v_0), \eta(y, g(x_0)) \rangle + H(g(x_0), y) \notin -\text{int}C(x_0),$$

where  $A : X \times L(E, Y) \rightarrow L(E, Y)$  and  $g : X \rightarrow X$  are mappings. It is easy to see that  $x$  is a solution of (VQVLI) is equivalent to finding  $x \in X$  with

$$\langle A(x_0, T(x_0)), \eta(y, g(x_0)) \rangle + H(g(x_0), y) \not\subseteq -\text{int}C(x_0), \text{ for } y \in D(x_0),$$

where

$$\langle A(x_0, T(x_0)), \eta(y, g(x_0)) \rangle = \bigcup_{v_0 \in T(x_0)} \langle A(x_0, v_0), \eta(y, g(x_0)) \rangle - \text{int}C(x_0).$$

### Special Cases:

- (i) If  $A(x_0, v_0) = v_0$  and  $g$  are identity mappings, then (VQVLI) is equivalent to finding  $x_0 \in D$  such that  $x_0 \in D(x_0)$  and for all  $y \in D(x_0)$  there exists  $v_0 \in T(x_0)$

$$\langle v_0, \eta(y, x_0) \rangle + H(x_0, y) \notin -\text{int}C(x_0) \quad (2.1)$$

is considered by Peng and Yang [16].

- (ii) If  $H(x_0, y) \equiv 0$  for all  $x, y \in X$  then problem (2.1) reduces to finding  $x_0$  in  $X$  such that  $x_0 \in D(x_0)$  and for all  $y \in D(x_0)$  there exists  $v_0 \in T(x_0)$ :

$$\langle v_0, \eta(y, x_0) \rangle \notin -\text{int}C(x_0)$$

is considered and studied by Ding [4].

- (iii) If  $T : K(\subset X) \rightarrow 2^{L(X, Y)}$  is a zero operator, then the problem (2.1) reduces to the quasi-equilibrium problem, which is to find  $x_0$  in  $X$  such that  $x_0 \in D(x_0)$  and

$$H(x_0, y) \notin -\text{int}C(x_0), \text{ for all } y \in D(x_0).$$

In order to prove the main result, we need the following definitions and lemmas.

**Definition 2.1.** [15], Let  $E$  and  $Y$  be real topological vector spaces,  $X$  be a nonempty convex subset of  $E$  and  $C : X \rightarrow 2^Y$  be a set-valued mapping such that  $C(x)$  is a closed pointed and convex cone with apex at 0 for each  $x \in X$ . Let  $\eta : X \times X \rightarrow E$  be a mapping,  $T : X \rightarrow 2^{L(E,Y)}$  is said to satisfy the generalized  $L$ - $\eta$ -condition if any finite set  $\{y_1, y_2, \dots, y_n\}$  in  $X$ ,  $x = \sum_{j=1}^n \alpha_j y_j$  with  $\alpha_j \geq 0$  and  $\sum_{j=1}^n \alpha_j = 1$ , there exists  $v \in T(x)$  such that

$$\langle v, \sum_{j=1}^n \alpha_j \eta(y_j, x) \rangle \notin -\text{int}C(x).$$

**Remark 2.1.** In considering our result, we need the following more modified  $L$ - $\eta$ -condition

$$\langle A(x, v), \sum_{j=1}^n \alpha_j \eta(y_j, g(x)) \rangle \notin -\text{int}C(x)$$

for  $T$  by adding two mappings  $g : X \rightarrow X$  and  $A : X \times L(E, Y) \rightarrow L(E, Y)$  in Definition 2.1.

**Remark 2.2.** If  $\eta(y, x)$  is affine in the first argument and for all  $x \in X$  there exists  $v \in T(x)$  such that

$$\langle v, \eta(x, x) \rangle \notin -\text{int}C(x),$$

then  $T$  satisfies the generalized  $L$ - $\eta$ -condition.

If  $\eta(y, x) = y - x$ , for all  $x, y \in X$ , then we have

$$\langle v, \sum_{j=1}^n \alpha_j (y_j - x) \rangle = \langle v, x - x \rangle = 0 \notin -\text{int}C(x), \text{ for all } v \in T(x),$$

hence  $T$  also satisfies the generalized  $L$ - $\eta$ -condition.

**Definition 2.2.** [18], Let  $X$  and  $Y$  be topological vector spaces and  $T : X \rightarrow 2^Y$  a set-valued mapping, then

(i)  $T$  is said to be upper semicontinuous, if for any  $x_0 \in X$  and for each open set  $U$  in  $Y$  containing  $T(x_0)$ , there is a neighbourhood  $V$  of  $x_0$  in  $X$  such that  $T(x) \subseteq U$  for all  $x \in V$ .

(ii)  $T$  is said to have open lower sections if the set  $T^{-1}(y) = \{x \in X : y \in T(x)\}$  is open in  $X$  for each  $y \in Y$ .

(iii)  $T$  is said to be closed, if the set  $\{(x, y) \in X \times Y : y \in T(x)\}$  is closed in  $X \times Y$ .

**Definition 2.3.** [21], Let  $C : X \rightarrow 2^Y$  be a mapping.  $H : X \times X \rightarrow Y$  is said to be 0- $C(x)$  diagonally convex with respect to the second argument, if for any finite subset  $\{y_1, y_2, \dots, y_n\}$  in  $X$  and any  $x \in X$  with  $x = \sum_{j=1}^n \alpha_j y_j$  ( $\alpha_j \geq 0$ ,  $\sum_{j=1}^n \alpha_j = 1$ ), we have

$$\sum_{j=1}^n \alpha_j H(x, y_j) \in C(x).$$

**Lemma 2.1.** [18], Let  $X$  and  $Y$  be topological spaces. Suppose  $T : X \rightarrow 2^Y$  and  $K : X \rightarrow 2^Y$  are mappings having open lower sections, then

- (i) the mapping  $F : X \rightarrow 2^Y$  defined by for each  $x \in X$ ,  $F(x) = \text{co}(T(x))$  has open lower sections;
- (ii) the mapping  $\alpha : X \rightarrow 2^Y$  defined by for each  $x \in X$ ,  $\alpha(x) = T(x) \cap K(x)$  has open lower sections.

**Lemma 2.2.** [1], Let  $X$  and  $Y$  be topological spaces. If  $T : X \rightarrow 2^Y$  is a upper semicontinuous with closed values, then  $T$  is closed.

**Lemma 2.3.** [1], Let  $X$  and  $Y$  be topological spaces and  $T : X \rightarrow 2^Y$  be a upper semicontinuous mapping with compact values. Suppose  $\{x_\alpha\}$  is a net in  $X$  such that  $x_\alpha \rightarrow x_0$ . If  $y_\alpha \in T(x_\alpha)$  for each  $\alpha$ , then there exist a  $y_0 \in T(x_0)$  and a subnet  $\{y_\beta\}$  of  $\{y_\alpha\}$  such that  $y_\beta \rightarrow y_0$ .

**Lemma 2.4.** ([10], Maximal Element Theorem) Let  $X$  be a nonempty convex subset of a Hausdorff topological vector space  $E$  and  $S : X \rightarrow 2^X$  be a mapping such that for each  $x \in X$ ,  $x \notin \text{co}(S(x))$  and for each  $y \in X$ ,  $S^{-1}(y)$  is open in  $X$ . Further assume that there exist a nonempty compact subset  $N$  of  $X$  and a nonempty compact convex subset  $B$  of  $X$  such that  $\text{co}(S(x)) \cap B \neq \emptyset$  for all  $x \in X \setminus N$ . Then there exists a point  $x_0 \in X$  such that  $S(x_0) = \emptyset$ .

### 3 Main Result

**Theorem 3.1.** Let  $Y$  be a real Hausdorff topological vector space,  $X$  be a nonempty and convex set in a real locally convex Hausdorff topological vector space  $E$  and  $L(E, Y)$  be equipped with the  $\sigma$ -topology. Let  $D : X \rightarrow 2^X$  be a set-valued mapping such that for all  $x \in X$ ,  $D(x)$  is nonempty and convex,  $D^{-1}(y)$  is open in  $X$ , for all  $y \in X$  and the set  $W = \{x \in X : x \in D(x)\}$  is closed in  $X$ . Let  $C : X \rightarrow 2^Y$  be a set-valued mapping such that  $C(x)$  is a closed pointed and convex cone with  $\text{int} C(x) \neq \emptyset$  for each  $x \in X$ . Assume that:

- (i) the mapping  $M = Y \setminus (-\text{int}C) : X \rightarrow 2^Y$  is upper semicontinuous on  $X$ ;
- (ii) a mapping  $\eta : X \times X \rightarrow E$  is continuous with respect to the second argument;
- (iii)  $T : X \rightarrow 2^{L(E, Y)}$  is a upper semicontinuous mapping with compact values and it satisfies the modified  $L$ - $\eta$ -condition;
- (iv)  $A : X \times L(E, Y) \rightarrow L(E, Y)$  is upper semicontinuous;
- (v)  $g : X \rightarrow X$  is continuous;
- (vi)  $H : X \times X \rightarrow Y$  is continuous with respect to the first argument and  $0$ - $C(x)$  diagonally convex with respect to the second argument;
- (vii) there exist a nonempty compact subset  $N$  of  $X$  and a nonempty compact convex subset  $B$  of  $X$  such that for all  $x \in X \setminus N$ , there exists  $y \in B$  such that  $y \in D(x)$  and

$$\langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \in -\text{int}C(x), \quad \text{for all } v \in T(x).$$

Then, there exists a point  $x_0 \in X$  such that  $x_0 \in D(x_0)$  and for all  $y \in D(x_0)$ , there exists  $v_0 \in T(x_0)$ :

$$\langle A(x_0, v_0), \eta(y, g(x_0)) \rangle + H(g(x_0), y) \notin -intC(x_0).$$

*Proof.* Define a mapping  $P : X \rightarrow 2^X$  by, for  $x \in X$ ,

$$\begin{aligned} P(x) &= \{y \in X : \langle A(x, T(x)), \eta(y, g(x)) \rangle + H(g(x), y) \subseteq -intC(x)\} \\ &= \{y \in X : \langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \in -intC(x), \text{ for all } v \in T(x)\}. \end{aligned}$$

We first prove that  $x \notin co(P(x))$  for all  $x \in X$ . To see this, suppose way of contradiction, that there exists some point  $x \in X$  such that  $x \in co(P(x))$ . Then there exists finite points  $y_1, y_2, y_3, \dots, y_n \in X$  and with  $\sum_{j=1}^n \alpha_j = 1$  and  $\alpha_j \geq 0$  such that  $x = \sum_{j=1}^n \alpha_j y_j$  and  $y_j \in P(x)$  ( $j = 1, 2, \dots, n$ ). That is

$$\langle A(x, v), \eta(y_j, g(x)) \rangle + H(g(x), y_j) \in -intC(x)$$

for all  $v \in T(x)$  and  $j = 1, 2, \dots, n$ .

Since  $intC(x)$  is a convex set, we obtain

$$\langle A(x, v), \sum_{j=1}^n \alpha_j \eta(y_j, g(x)) \rangle + \sum_{j=1}^n \alpha_j H(g(x), y_j) \in -intC(x) \text{ for all } v \in T(x). \quad (3.2)$$

From the 0- $C(x)$  diagonal convexity with respect to the second argument of  $H$ , we have

$$\sum_{j=1}^n \alpha_j H(g(x), y_j) \in C(x). \quad (3.3)$$

By (3.2) and (3.3) we get, for all  $v \in T(x)$ ,

$$\begin{aligned} \langle A(x, v), \sum_{j=1}^n \alpha_j \eta(y_j, g(x)) \rangle &\in - \sum_{j=1}^n \alpha_j H(g(x), y_j) - intC(x) \\ &\subseteq -C(x) - intC(x) \subseteq -intC(x), \end{aligned}$$

which contradicts the fact that  $T$  satisfies the modified  $L$ - $\eta$ -condition. Therefore  $x \notin co(P(x))$  for all  $x \in X$ .

Now we prove that the set

$$\begin{aligned} P^{-1}(y) &= \{x \in X : \langle A(x, T(x)), \eta(y, g(x)) \rangle + H(g(x), y) \subseteq -intC(x)\} \\ &= \{x \in X : \langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \in -intC(x) \text{ for } v \in T(x)\} \end{aligned}$$

is open for each  $y \in X$ . That is,  $P$  has open lower sections in  $X$ . Consider the set-valued mapping  $Q : X \rightarrow 2^X$  defined by

$$\begin{aligned} Q(y) &= \{x \in X : \langle A(x, T(x)), \eta(y, g(x)) \rangle + H(g(x), y) \notin -intC(x)\} \\ &= \{x \in X : \exists v \in T(x); \langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \notin -intC(x)\}. \end{aligned}$$

We only need to prove that  $Q(y)$  is closed for all  $y \in X$ . In fact, consider a net  $\{x_t\} \in Q(y)$  such that  $x_t \rightarrow x$ . Then  $g(x_t) \rightarrow g(x) \in X$ . Since  $x_t \in Q(y)$ , there exists  $v_t \in T(x_t)$  such that

$$\langle A(x_t, v_t), \eta(y, g(x_t)) \rangle + H(g(x_t), y) \notin -intC(x_t).$$

By the upper semicontinuity and compact valuedness of  $T$ , from Lemma (2.3) we have a subnet  $\{v_{t_j}\}$  which converges to some  $v \in T(x)$ . By the continuity of  $\langle \cdot, \cdot \rangle$  we have

$$\langle A(x_{t_j}, v_{t_j}), \eta(y, g(x_{t_j})) \rangle + H(g(x_{t_j}), y) \rightarrow \langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y).$$

By Lemma (2.2) and the upper semicontinuity of  $M$ , we have

$$\langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \notin -\text{int}C(x).$$

Hence  $x \in Q(y)$ , which implies that  $Q(y)$  is closed. Therefore  $P$  has open lower sections of  $X$  and by Lemma (2.1), we know that  $\text{co}(P(x)) : X \rightarrow 2^X$  also has open lower sections. Also define another set-valued mapping  $S : X \rightarrow 2^X$  by

$$S(x) = \begin{cases} D(x) \cap \text{co}(P(x)) & \text{if } x \in W \\ D(x) & \text{if } x \notin W \end{cases}$$

Then, it is clear that for all  $x \in X$ ,  $S(x)$  is convex and  $x \notin S(x) = \text{co}(S(x))$ . Since for all  $y \in X$ ,

$$\begin{aligned} S^{-1}(y) &= \{x \in X : y \in S(x)\} \\ &= \{x \in W : y \in D(x) \cap \text{co}(P(x))\} \cup \{x \in X \setminus W : y \in D(x)\} \\ &= (W \cap D^{-1}(y) \cap \text{co}(P^{-1}(y))) \cup [(X \setminus W) \cap D^{-1}(y)] \\ &= [(W \cap D^{-1}(y) \cap \text{co}(P^{-1}(y))) \cup (X \setminus W)] \cap [(W \cap D^{-1}(y) \cap \text{co}(P^{-1}(y))) \cup D^{-1}(y)] \\ &= \{X \cap [(D^{-1}(y) \cap \text{co}(P^{-1}(y))) \cup (X \setminus W)]\} \cap [(W \cup D^{-1}(y)) \cap (D^{-1}(y))] \\ &= [(D^{-1}(y) \cap \text{co}(P^{-1}(y))) \cup (X \setminus W)] \cap D^{-1}(y) \\ &= (D^{-1}(y) \cap (\text{co}(P^{-1}(y))) \cup ((X \setminus W) \cap (D^{-1}(y))), \end{aligned}$$

and  $D^{-1}(y)$ ,  $\text{co}(P^{-1}(y))$  and  $X \setminus W$  are open in  $X$ ,  $S^{-1}(y)$  is open in  $X$ .

Condition (vii) implies that there exist a nonempty compact subset  $N$  of  $X$  and a nonempty compact convex subset  $B$  of  $X$  such that

$$\text{co}(S(x)) \cap B = S(x) \cap B \neq \emptyset \text{ for all } x \in X \setminus N.$$

Hence, by Maximal Element Theorem, there exists  $x_0 \in X$  such that

$$S(x_0) = \emptyset.$$

Then  $x_0 \in W$ . In fact, if  $x_0 \notin W$ , then  $D(x_0) = S(x_0) = \emptyset$ , which contradicts the definition of  $D$ . Hence

$$\emptyset = S(x_0) = D(x_0) \cap \text{co}(P(x_0)),$$

so  $D(x_0) \cap P(x_0) = \emptyset$ . It is obvious that  $x_0 \in D(x_0)$ . In fact, if  $x_0 \notin D(x_0)$ , then  $x_0 \notin W$ , which is a contradiction. Moreover, if  $y \in D(x_0)$ , then  $y \notin P(x_0)$ , so there exists  $v_0 \in T(x_0)$  satisfying

$$\langle A(x_0, v_0), \eta(y, g(x_0)) \rangle + H(g(x_0), y) \notin -\text{int}C(x_0).$$

□

By Theorem (3.1) and Remark (2.2), we have:

**Corollary 3.1.** *Let  $Y$  be a real Hausdorff topological vector space,  $X$  be a nonempty convex set in a real locally convex Hausdorff topological vector space  $E$  and  $L(E, Y)$  be equipped with the  $\sigma$ -topology. Let  $D : X \rightarrow 2^X$  be a mapping such that for all  $x \in X$ ,  $D(x)$  is nonempty and convex,  $D^{-1}(y)$  is open in  $X$ , for all  $y \in X$  and the set  $W = \{x \in X : x \in D(x)\}$  is closed in  $X$ . Let  $C : X \rightarrow 2^Y$  be a set-valued mapping such that  $C(x)$  is a closed pointed and convex cone with  $\text{int } C(x) \neq \emptyset$  for each  $x \in X$ . Assume that:*

- (i)  $A : X \times L(E, Y) \rightarrow L(E, Y)$  is upper semicontinuous;
- (ii)  $g : X \rightarrow X$  is continuous;
- (iii) the mapping  $M = Y \setminus (-\text{int}C) : X \rightarrow 2^Y$  is upper semicontinuous on  $X$ ;
- (iv) the set-valued mapping  $T : X \rightarrow 2^{L(E, Y)}$  is upper semicontinuous on  $X$  with compact values and  $\eta : X \times X \rightarrow E$  is continuous with respect to the second argument and affine with respect to the first argument such that for all  $x \in X$  there exists  $v \in T(x)$  satisfying
 
$$\langle A(x, v), \eta(x, g(x)) \rangle + H(g(x), x) \notin -\text{int}C(x);$$
- (v)  $H : X \times X \rightarrow Y$  is continuous with respect to the first argument and  $0$ - $C(x)$  diagonally convex with respect to the second argument;
- (vi) there exist a nonempty compact subset  $N$  of  $X$  and a nonempty compact convex subset  $B$  of  $X$  such that for all  $x \in X \setminus N$ , there exists  $y \in B$  such that  $y \in D(x)$  and

$$\langle A(x, v), \eta(y, g(x)) \rangle + H(g(x), y) \in -\text{int}C(x), \quad \text{for all } v \in T(x).$$

Then, there exists a point  $x_0 \in X$  such that  $x_0 \in D(x_0)$  and for all  $y \in D(x_0)$ , there exists  $v_0 \in T(x_0)$

$$\langle A(x_0, v_0), \eta(y, g(x_0)) \rangle + H(g(x_0), y) \notin -\text{int}C(x_0).$$

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